A new face of the Multiverse hypothesis: bosonic-phononic inflaton quantum universes

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Abstract

The boson-phonon duality due to inflaton energy is presented in the context of quantum universes discussed recently by the author. The duality leads to *bonons*, i.e. the bosonic-phononic quantum universes. This state of things manifestly corresponds to the Lewis–Kripke modal realism, and physical presence of Multiverse in Nature.

1 The Beckwith inflaton entropy

Recent deductions due to Dr. Andrew Beckwith [1] for inflaton potential

$$V_{inflaton}(\phi) \sim \phi^2, \tag{1}$$

led him to interestingly looking entropy formula

$$[\Delta S] = \frac{\hbar c}{T}\left\{2k^2 + 8\left(\sqrt{\frac{3}{8\pi}}\frac{1}{\ell_P}\frac{M_P c/\hbar}{\eta\phi}\right)^2\right\}^{\frac{1}{2}}, \tag{2}$$

which has arisen from the simple relation $\Delta S = E/T$, for which the energy E is

$$E_{inflaton} = \sqrt{2\hbar^2 k^2 c^2 + 2\left(2\sqrt{\frac{3}{8\pi}}\frac{1}{\ell_P}\frac{M_P}{\eta\phi}\right)^2 c^4}, \tag{3}$$

where $M_P = \sqrt{\frac{\hbar c}{G}}$ is the Planck mass, and $\ell_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length. Let us study this inflaton energy formula in some detail.

2 The quantum universes

Let us sketch the simple Multiverse model. The author [2] introduced the auxiliary field $\phi = \phi(\tau)$, which expressed in terms of the Planck length is

$$\phi(\tau) = \sqrt{\frac{3}{8\pi}}\frac{a(\tau)}{\ell_P}, \tag{4}$$

where $a(\tau)$ is the Friedmann conformal scale factor and τ is the conformal time describing the quantum universe.

With using of the Dirac lapse $N(t)$, the Einstein–Hilbert action of General Relativity computed for the conformally flat Friedmann–Robertson–Walker metric $g_{\mu\nu} = \mathrm{diag}[N^2(t), -a^2(t)\delta_{ij}]$, and integrated by parts, takes the form

$$S[\phi] = \int_{\tau_0}^{\tau} d\tau \left\{ -V\left(\frac{d\phi}{d\tau}\right)^2 - \frac{\phi^4}{\phi_0^4}E(\tau) \right\}, \tag{5}$$

where $\tau = \int_0^t N(t')/a(t')dt'$ is the conformal time, $E(\tau) = \int d^3x \mathscr{H}_{Matter}(x,\tau)$ is Matter fields energy, $V = \int d^3x < \infty$, and $\phi_0 = \phi(\tau_0) = \sqrt{\frac{3}{8\pi}}\frac{1}{\ell_P}$ (with $a(\tau_0) = 1$). Application of the conjugate momentum $P_\phi = \frac{\delta S[\phi]}{\delta(d\phi/d\tau)} = -2V\frac{d\phi}{d\tau}$ leads to the canonical form of the action

$$S[\phi] = -\int_{\tau_0}^{\tau} d\tau \left\{ P_\phi \frac{d\phi}{d\tau} - H(\tau) \right\}, \tag{6}$$

where $H(\tau) = \frac{1}{4V^2}P_\phi^2 - \frac{\phi^4}{\phi_0^4}E(\tau) \equiv 0$ is the Hamiltonian constraint of the cosmological model (vanishing is automatic due to Dirac's method). The solution of this constraint, i.e. $P_\phi = \pm 2V\frac{\phi^2}{\phi_0^2}\sqrt{E(\tau)}$, is the Hubble law

$$\frac{\phi(\tau)}{\phi_0} = \frac{1}{1+z(\tau_0,\tau)}, \tag{7}$$

where z is the redshift given by

$$z(\tau_0,\tau) = \pm\frac{1}{\phi_0}\int_{\tau_0}^{\tau}\sqrt{E(\tau')}d\tau'. \tag{8}$$

Applying the Dirac quantization $P_\phi = -i\frac{d}{d\phi}$ of the Hamiltonian constraint, one obtains the Wheeler–DeWitt equation governing the quantum universes

$$\left(\frac{d^2}{d\phi^2} + E_\phi^2\right)\Psi(\phi) = 0, \tag{9}$$

where E_ϕ is the quantity

$$E_\phi = 2V\frac{\phi^2}{\phi_0^2}\sqrt{E(\tau)}, \tag{10}$$

and one sees that when Matter fields energy is constant then

$$E_\phi \sim V_{inflaton}(\phi), \tag{11}$$

i.e. one can treat the quantum universes as the result of inflation. Let us call such quantum universes by the special name - inflaton quantum universes.

3 The matter-sound duality

Let us see what happens in inflaton quantum universes. Employing the auxiliary field (4) to the inflaton energy (3) one obtains

$$E_{inflaton} = \sqrt{2\hbar^2k^2c^2 + 2\left(\frac{2M_P}{\eta a}\right)^2 c^4}. \tag{12}$$

However, by using of the Einstein relation

$$E = \sqrt{p^2c^2 + m^2c^4}, \tag{13}$$

and Planck–Einstein wave-particle duality $E = \hbar\omega$, $p = \hbar k$ one sees that Eq. (12) describes the particle-universe possessing momentum and mass

$$p = \sqrt{2}\hbar k, \tag{14}$$

$$m = \sqrt{2}\frac{2}{\eta a}M_P, \tag{15}$$

or equivalently that the Friedmann scale factor can be established by mass of a particle as $a = \dfrac{\sqrt{8}}{\eta}\dfrac{M_P}{m}$. One can derive easy the group velocity $v_g = \dfrac{d\omega}{dk}$ of the particle-universe

$$v_g = \frac{\sqrt{2}c}{\sqrt{1+\left(\dfrac{mc}{p}\right)^2}} = \frac{\sqrt{2}c}{\sqrt{1+\left(\dfrac{2/\eta}{k\ell_P a}\right)^2}}, \tag{16}$$

and similarly also the phase velocity $v_{ph} = \dfrac{\omega}{k}$ can be obtained

$$v_{ph} = \sqrt{2}c\sqrt{1+\left(\frac{mc}{p}\right)^2} = \sqrt{2}c\sqrt{1+\left(\frac{2/\eta}{k\ell_P a}\right)^2}. \tag{17}$$

On the other hand one can use also the dispersion relation for sound quanta in solids, i.e. phonons

$$\omega_k = \sqrt{2\omega^2(1-\cos(k\ell_P a))}, \tag{18}$$

with $\omega = kc$, where now a is the lattice spacing. Then quantum universe is a phonon, and the equation holds

$$(k\ell_P a)^2\cos(k\ell_P a) = -\left(\frac{2}{\eta}\right)^2. \tag{19}$$

Solutions of Eq. (19) must be found the only numerically. According to the quantization rule for phonons, the wave vector and the lattice spacing are related by the following relation

$$k\ell_P a = \frac{n}{N}\pi, \tag{20}$$

where N is a number of identical atoms, $n = 0, \pm 1, \ldots, \pm N$, i.e. ka takes integer values in $\left[-\frac{\pi}{N}, \frac{\pi}{N}\right]$. However, for the quantum universe, ka has integer values which are solutions of (19). One can derive straightforwardly the group $v_g = \frac{d\omega_k}{dk}$ and the phase $v_{ph} = \frac{\omega_k}{k}$ velocities of the phonon-universe

$$\begin{aligned} v_g &\equiv \frac{c}{\sqrt{2}}k\ell_P a\sqrt{1+\cos(k\ell_P a)} = \frac{c}{\sqrt{2}}\sqrt{(k\ell_P a)^2 - \left(\frac{2}{\eta}\right)^2} && (21)\\ v_{ph} &\equiv \sqrt{2}c\sqrt{1-\cos(k\ell_P a)} = \sqrt{2}c\sqrt{1+\left(\frac{2/\eta}{k\ell_P a}\right)^2}, && (22)\end{aligned}$$

where we have applied the constraint (19), which after using the phonon quantization (20) result in the quantization of the velocities

$$\begin{aligned} v_g &= \frac{c}{\sqrt{2}}\sqrt{\left(\frac{\pi}{N}\right)^2 n^2 - \left(\frac{2}{\eta}\right)^2}, && (23)\\ v_{ph} &= \sqrt{2}c\sqrt{1+\left(\frac{2}{\eta}\frac{N}{\pi}\right)^2\frac{1}{n^2}}, && (24)\end{aligned}$$

The group velocity, however, is real and nonzero if and only if

$$k\ell_P a > \frac{2}{\eta} \qquad or \qquad k\ell_P a < -\frac{2}{\eta}. \tag{25}$$

In this way the quantization is not arbitrary, but limited by the inequalities

$$n \geq \frac{N}{\pi}\frac{2}{\eta} \qquad or \qquad n \leq -\frac{N}{\pi}\frac{2}{\eta}. \tag{26}$$

It is easy to see that also the phase velocity is limited to $v_{ph} \in [-2\sqrt{2}c, 2\sqrt{2}c]$. In the limiting cases $k\ell_P a = \pm\frac{2}{\eta}$ the group velocity vanishes identically, and by (19) such a situation corresponds to $\cos\frac{2}{\eta} = -1$, i.e. $\frac{2}{\eta} = (r \pm 1/2)\pi$, where $r \in \mathbf{Z}/\{0\}$. Then the phase velocity has determined value $v_{ph} = 2c$. For the limit $k\ell_P a \gg \frac{2}{\eta}$ the velocities are $v_g = \frac{c}{\sqrt{2}}k\ell_P a = \frac{c}{\sqrt{2}}\frac{n}{N}\pi$, and $v_{ph} = \sqrt{2}c$.

One sees, however, that the phase velocities of the particle and the phonon are identical, while the group velocities are blatantly different. For full particle-phonon duality of the inflaton quantum universe, the most natural way is to put *ad hoc* the equality between the group velocities. It is easy to see that the duality condition can be presented as the two-quadratic equation

$$(k\ell_P a)^4 - 4(k\ell_P a)^2 - \frac{16}{\eta^4} = 0, \tag{27}$$

which is not difficult to solve

$$k\ell_P a = \sqrt{2 + 2\sqrt{1 + \frac{4}{\eta^4}}}. \tag{28}$$

Using, however, once again the phonon quantization (20) allows to establish the quantization of the parameter η

$$\frac{2}{\eta} = \left[\left(\frac{\pi}{2N}\right)^4 n^4 - 4\left(\frac{\pi}{2N}\right)^2 n^2\right]^{\frac{1}{4}}. \tag{29}$$

It is easy too see that for $n \gg 2N/\pi \approx 0.64N$ the quantization is linear

$$\frac{2}{\eta} = \frac{\pi}{2N}n. \tag{30}$$

Interestingly, in general the inflaton energy (34) after application of the phonon quantization can be presented in the following form:

$$E_{inflaton} = \sqrt{2\left(\frac{\pi}{N}\right)^2 n^2 + \frac{4}{\eta^2}}\frac{E_P}{a}, \tag{31}$$

i.e. in the most general case the inflaton energy is quantized and has behavior $1/a$. When one takes the limit $\eta \to \infty$ in Eq. (28) one can deduce

$$k\ell_P a = 2, \tag{32}$$

or after phonon quantization

$$\frac{n}{N} = \frac{2}{\pi} \approx 0.64 < 1, \tag{33}$$

i.e. such a limit is fully appropriate in the light of the phonon quantization. In the light of Eq. (3) it means that

$$\lim_{\eta \to \infty} E_{inflaton} = \frac{2\sqrt{2}E_P}{a}, \tag{34}$$

where $E_P = \sqrt{\frac{\hbar c^5}{G}}$ is the Planck energy. Moreover, if one uses the phonon quantization to Eq. (34) one obtains

$$\lim_{\eta \to \infty} E_{inflaton} = \frac{\sqrt{2}\pi n}{N}\frac{E_P}{a}. \tag{35}$$

However, in general also the value of the parameter η is not arbitrary. The phonon quantization applied to (28) leads to

$$\sqrt{2 + 2\sqrt{1 + \frac{4}{\eta^4}}} = \frac{n}{N}\pi \leq \pi, \tag{36}$$

or after elementary algebraic manipulations

$$\eta \geq \frac{2/\sqrt{\pi}}{\sqrt[4]{\pi^2 - 4}} \approx 0.725. \tag{37}$$

In general, however, one obtains quantization of the parameter

$$\eta_n = \frac{2}{\sqrt{\frac{n}{N}\pi \sqrt[4]{\left(\frac{n}{N}\pi\right)^2 - 4}}}, \tag{38}$$

i.e. the inflaton energy is quantized according to the rule

$$E^{(n)}_{inflaton} = \sqrt{2\left(\frac{n}{N}\pi\right)^2 + \frac{4}{\eta_n^2}}\frac{E_P}{a} = \frac{n}{N}\pi\sqrt{2 + \sqrt{1 - \left(\frac{2}{\pi}\frac{N}{n}\right)^2}}\frac{E_P}{a}. \tag{39}$$

The minimal value $\eta_{min} \approx 0.725$ is obtained for $n = N$, i.e. independently on the number N of identical atoms, which factually is arbitrary and unknown. In this manner one can take $n = N$. In this case $\eta = \frac{2/\sqrt{\pi}}{\sqrt[4]{\pi^2 - 4}}$, and the inflaton energy takes the maximal value independently on N

$$E_{inflaton}^{max} = \sqrt{2\pi^2 + \frac{4}{\eta_{min}^2}}\frac{E_P}{a} = \sqrt{2\pi^2 + \pi\sqrt{\pi^2 - 4}}\frac{E_P}{a} \approx 5.230\frac{E_P}{a}. \tag{40}$$

When one treats N as the number of the bonons in the Multiverse, then the interesting quantity is the mean inflaton energy. The general formula is easy to deduce

$$\left\langle E_{inflaton}\right\rangle_N = \frac{1}{N - n_{min}} \sum_{n=n_{min}}^{N} E_{inflaton}^{(n)}. \tag{41}$$

From the formula (39), however, it is clear that n_{min} is dependent on N via the positivity of the expression under the internal square root. It gives

$$n_{min} = \left\lceil \frac{2}{\pi} N \right\rceil, \tag{42}$$

and effectively one can obtain

$$\left\langle E_{inflaton}\right\rangle_N = \frac{Q_N}{a}, \tag{43}$$

where $Q_N = \Lambda_N E_P$ with Λ_N given by the formula

$$\Lambda_N = \frac{1}{N - \left\lceil \frac{2}{\pi} N \right\rceil} \sum_{n=\left\lceil \frac{2}{\pi} N \right\rceil}^{N} \sqrt{2\left(\frac{n}{N}\pi\right)^2 + \sqrt{\left(\frac{n}{N}\pi\right)^4 - 4\left(\frac{n}{N}\pi\right)^2}}. \tag{44}$$

The question is, however, the convergence of quantity Λ_N for the huge N limit, Λ_∞, i.e. when the Multiverse is full of bonons, for which $\left\langle E_{inflaton}\right\rangle_\infty = \Lambda_\infty \frac{E_P}{a}$. In the other words, the problem is whether Λ_∞ is an universal constant, and its value is also the computational problem. In general it is difficult to establish Λ_∞, but one can compute few values of Λ_N for large values of N. The relation (42) also predicts the minimal value $N = 3$. One can enlighten the situation by some examples

$$\begin{aligned}
\Lambda_3 &\approx 8.403883069,\\
\Lambda_{10^1} &\approx 5.785515161,\\
\Lambda_{10^2} &\approx 4.267547858,\\
\Lambda_{10^3} &\approx 4.155288355,\\
\Lambda_{10^4} &\approx 4.144199901,\\
\Lambda_{10^5} &\approx 4.142912567,\\
\Lambda_{10^6} &\approx 4.142812785,
\end{aligned}$$

i.e. at the first glance it is visible that the number $4.14\ldots$ is probably the good approximation of the exact limit Λ_∞. Because of the minimal number of identical atoms is $N = 3$, one can identify the atoms with spatial dimensions $N \equiv D$, and treat the Multiverse model presented above as the multidimensional Universe. The other interpretation is that N is the number of «atoms of space», and then the limit $N \to \infty$ is the classical space limit, i.e. the space which is solid-medium of the atoms - bonons. In the other words, also, the most stable mean inflaton energy is obtained for the Multiverse with $N = \infty$ identical atoms, i.e. the solid-medium which can be considered as the **Æther** model. The convergence of Λ_N in dependence on N is graphically showed on the Fig. (1). It is rather certain for the Figure that the huge N limit exists, while both the convergence procedure and the exact value of Λ_∞ are not clear. However, for precision, let us call Λ_N *the inflaton N-atomic constant*, and Λ_∞ *the inflaton constant*.

Interestingly, one can deduce the inflaton frequency constant ω_I as

$$\hbar\omega_I = \Lambda_\infty E_P, \tag{45}$$

which in good approximation has the value

$$\omega_I = \Lambda_\infty \frac{E_P}{\hbar} = \Lambda_\infty \sqrt{\frac{c^5}{\hbar G}} \approx 7.67 \cdot 10^{43} s^{-1}, \tag{46}$$

and can be treated as the constant frequency field, and identified as the Zero-Point Frequency. The Zero-Point Frequency (46) allows to establish the time

$$t_I = \frac{2\pi}{\omega_I} \approx 8.17 \cdot 10^{-44} s, \tag{47}$$

which can be treated as the characteristic time of the inflation.

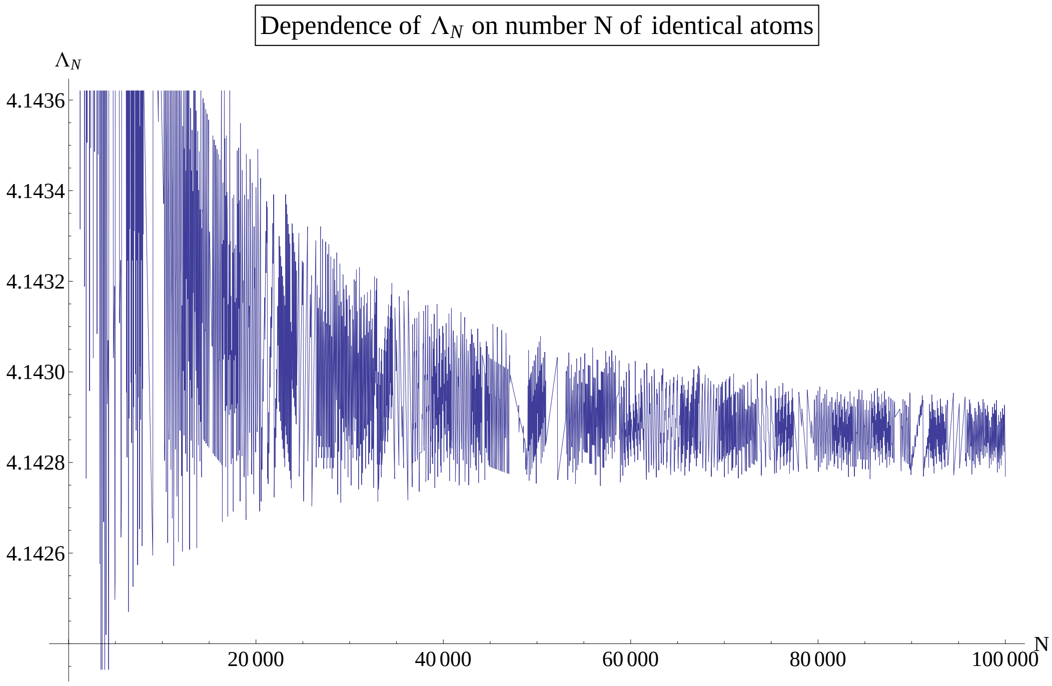

Figure 1: Dependence of Λ_N on the number N of identical atoms in phonon quantization, given by the formula (44), in the range $N \in [3, 10^5]$. The graphic shows manifestly that there is the finite value for $N \to \infty$ limit, $\Lambda_\infty = 4.142\ldots$ called the inflaton constant in this text, which has a nature of an universal constant. However, the exact value of the constant is both the computational and theoretical problem which should be solved.

4 Modal realism in physics and cosmology

The presented idea corresponds to the logics propagated by modal realism studied in recent years by D. Lewis [3] and S. Kripke [4]. Moreover, modal realism is the straightforward philosophical consequence of the Multiverse hypothesis introduced to philosophy by Epicurus [5], and adopted to psychology by W. James [6]. The syndrome of plurality of worlds is Multiverse cosmology, propagated recently by M. Tegmark [7], is straightforwardly related to Eastern cosmogonies, especially to Islamic views [8]. The first such a cosmology was proposed by A. Linde [9] in the context of chaotic, or eternal, inflation. Recently, however, the Multiverse hypothesis is propagated by string theorist M. Kaku [10] as parallel universes. Also W. Weinberg's [11] beliefs are related to Multiverse. In quantum physics the related subject is called Many-Worlds Interpretation (MWI) [12, 13], and results from the Everett's relative state interpretation of quantum mechanics. J.D. Barrow and F.J. Tipler [14], and Tipler [15] independently have studied MWI in the context of Cosmology, particularly the cosmological anthropic principle was considered by them. Interesting book edited by B. Carr [16] presenting opinions of numerous scholars and authors about the Multiverse hypothesis, was recently published. In the context of Multiverse cosmology also the articles presented recently by A. Jenkins and G. Perez [17], and J. Feng and M. Trodden [18] in *Scientific American* look like topically. Also some purely literate works, especially book of C. Bruce [19] discussing few issues due to the Schrödinger cat problem, were recently published.

5 Summary

Both the proposed duality and the conclusions presented above are non-trivial. Factually, from the one side we have identified the inflaton quantum universes with Einstein's relativistic particles, i.e. bosons, but on the other hand with elementary excitations in a solid, i.e. phonons. Thus we propose to call such very special quantum universes *bonons*, i.e. quantum objects following from the boson-phonon duality.

In this paper we have introduced the inflaton N-atomic constant, and the inflaton constant which has probably the nature of an universal constant. The problem is to study convergence of inflaton N-atomic constant in the limit of huge number of identical atoms, i.e. bonons. Anyway, measurement of the inflaton N-atomic constant could be excellent establishment of the number of the atoms. The problem for future studies is to establish the numerical value of the inflaton costant.

The presented duality between a particle and a sound has deep philosophical content. Namely, it manifestly expresses the relation between quantum matter (bonons) and quantum sound (phonons) in the context of inflaton quantum universe. This, however, can be interpreted as the syndrome of material creation of inflaton quantum universe due to sound quanta. Possible existence of the inflaton constant will be the proof of the Æther existence, where the Æther is understood as the solid-medium of bonons, which are the «atoms of space». This idea is somewhat similar to XIXth centaury Æther model proposed by Lord Kelvin, but Kelvin considered the mechanical model of classical solid, while in we propose to study the quantum solid and bosons taken together.

Recently, the author [20] had unpleasant necessity to announce within News of the GRG people associated by hyperspace@aei that the word "bononic" is the word "bosonic" mispronounced by incorrect publication. Factually, it was the mistake of the publisher, who also published the author's paper with no his written approval. The publisher did not inform the author about publishing of the paper, and the author did not send the paper for publication in the journal. This is the only coincidence. *Bonon* is a logical name in this situation. However, now both the noun *bonon* and the adjective *bononic* are fully justified and scientifically correct in the sense of the inflaton quantum universes, and the matter-sound duality that rules the Multiverse.

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