# EVER-PRESENT LAMBDA AND THE QUANTUM POTENTIAL OF SPACETIME

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**Abstract**
An approach to dark energy is presented that combines ideas of causal set theory with a Machian perspective and a treatment of spacetime as a condensate, yielding a "quantum potential of spacetime" Q whose density $\rho_Q$ acts as an effective $\Lambda$ that satisfies the uncertainty relation $\Delta V \Delta\Lambda \sim 1$ of unimodular relativity. In contrast to the ever-present $\Lambda$ of causal set theory, $\rho_Q$'s value is always non-negative, and the nonlocality of Q ensures that $\rho_Q$ is spatially homogeneous, in accordance with observation.

## I. INTRODUCTION

In causal set theory and its application to cosmology [6, 28, 38, 57, 64], the four-volume $V$ of the universe corresponds to, or reflects, the number N of causal-set ("causet") elements, existing at the Planck scale, that constitute the spacetime of the universe. This correspondence, however, is not exact; i.e., for $V$ measured in Planck units, the equality $N=V$ holds only up to fluctuations. Now, N acts as a time-parameter in an expanding universe; and just as one does not sum over time in ordinary quantum mechanics, so likewise one expects N to be held fixed when performing a gravitational path integral, or sum over histories. Given the macroscopic correspondence between N and $V$, this amounts to holding $V$ fixed, so that it is natural to employ unimodular gravity here [6]. (Unimodular relativity has also been motivated by adopting a thermodynamic approach to spacetime, an approach which, like causal set theory, takes spacetime to be fundamentally discrete [65].) Even with N held fixed, however, $V$ is subject to fluctuations that reflect the non-exactness of the correspondence between N and $V$; these fluctuations are of Poisson type, so that $\Delta V \sim \pm\sqrt{V}$. These $V$-fluctuations give rise to fluctuations of the vacuum energy density $\Lambda$ in accordance with the uncertainty relation $\Delta V \Delta\Lambda \sim \hbar$, a relation that follows from unimodular relativity and is similar to time-energy uncertainty [6]. Since the $V$-fluctuations are stochastic (being of Poisson type), the $\Lambda$-fluctuations are stochastic as well. This stochasticity, however, does not preclude the possibility that the value of $\Delta\Lambda$ is determined by the value of $\Delta V$. Such a determination of $\Delta\Lambda$ would resemble the situation associated with the "exact uncertainty principle" of [27], in which random momentum-fluctuations of a quantum ensemble are determined by the uncertainty of the conjugate position-variable. We propose here that $\Delta\Lambda$'s value is in fact determined; and we explore this proposal in the context of causal set theory, in particular the cosmological model of Ahmed, Dodelson, Greene and Sorkin [6]. Because $\Delta\Lambda$'s value is determined, however, this value does *not* exhibit random-walk behavior, in contrast to the model of [6]; and this fact, together with the strong nonlocality that characterizes the determination of $\Delta\Lambda$, ensures a spatially uniform distribution of $\Lambda$ at any given time t. As a result, the present account of $\Lambda$ avoids a problem noted by both Barrow and Zuntz [9, 68] with respect to [6]'s model: namely, this model as it stands, despite being strongly nonlocal itself, entails spatial inhomogeneities of $\Lambda$ that are not in agreement with observation. In addition, our account of $\Delta\Lambda$ alleviates some of the "tension" surrounding the choice of a suitable value for the important fluctuation-parameter $\alpha$ of [6].

As will be seen, we deviate somewhat from the idea of exact uncertainty by allowing the possibility that $\Delta V$ is not always the only factor determining $\Delta\Lambda$. As a result, the uncertainty relation $\Delta V\Delta\Lambda \sim \hbar$ may fail to hold under certain conditions. This fact does not pose any problems, however, as explained in part IV, below.

In the next section, i.e. part II, we discuss the $\Delta V\Delta\Lambda$ uncertainty relation, particularly as it relates to volume-fluctuations, as well as arguing for the existence of a cosmological preferred frame. In part III, we focus on the determination of $\Delta\Lambda$ by $\Delta V$, and we argue that dark energy is in fact a "quantum potential of spacetime" induced by the fluctuations $\Delta V$. In part IV, we address some other topics suggested by our main argument.

## II. VOLUME-FLUCTUATIONS AND THEIR MUTUAL CANCELLATION

Following [6], we use units with $c=\hbar=8\pi G=1$; hence, for $m_p$ the reduced Planck mass and $E_p$ the reduced Planck energy, we have $m_p=E_p=1$. In addition, and again following [6], we adopt the standpoint of causal set theory, according to which a given spacetime volume v is viewed as having n causet elements sprinkled into it at unit density, with the "sprinkling" here being a Poisson process. "Unit density" means that there is, on average, one causet element per volume $V_p$, where $V_p=l^4$ and $l$ is of Planck order [63, 28]. When n is held fixed (see Part I, above, for the case n=N), deviations from unit density are due solely to (Poisson) volume-fluctuations $\Delta$v. Hence, and writing "$\rho$" for the number of causet elements in a given Planck-size volume $V_p$, $\rho$'s average value $\langle\rho\rangle$ is equal to 1, *up to volume-fluctuations*.

Consider a volume v consisting of n causet elements, with n=v up to volume-fluctuations (which implies that v here is measured in Planck units). If n is held fixed, a given fluctuation $\Delta v \sim \sqrt{v}$ can be described by saying that the volume-fluctuations (or, alternatively, the density fluctuations, or $\rho$-fluctuations) of all but $\sim \sqrt{n}$ elements cancel each other out in v, and the uncancelled remnant consisting of $\sqrt{n}$ fluctuating elements is such that $\sqrt{n} \sim \sqrt{v} \sim \Delta v$. Now, for N the total number of causet elements in the universe at a given time t, and for $V$ the total volume of the universe at t (where $V$ is measured by the past light-cone of a suitable observer), the uncertainty relation $\Delta V\Delta\Lambda \sim 1$ implies that the value of $\Delta\Lambda$ is inversely related to the total magnitude $\Delta V$ of the volume-fluctuations at t that are uncancelled in $V$, with this magnitude being $\sim \sqrt{N} \sim \sqrt{V}$. In other words, these uncancelled fluctuations collectively constitute, at t, a volume-fluctuation $\Delta V \sim \sqrt{V}$ of the universe, with $\Delta V\Delta\Lambda \sim 1$; and this yields a suitable value of $\Lambda$, i.e. a value that matches the observed dark energy. These considerations naturally suggest the following question, which Y. Kuznetsov asked in [41]: what ensures that the volume v with respect to which $\Lambda$ fluctuates is in fact the volume $V$ of the entire universe, rather than some smaller volume? Or, in more general terms, what ensures that the value of "v" in "$\Delta v\Delta\Lambda \sim 1$" is such as to yield a suitable value of $\Lambda$?

Although the relation $\Delta V\Delta\Lambda \sim 1$ comes from unimodular relativity, it is important to realize that the underlying issue or problem here is independent of relativity and depends only on the assumption that spacetime fluctuations are endemic to the Planck scale and have an energy density $\Lambda$ that may gravitate. This assumption itself makes it necessary to ask about the cancellation of these fluctuations on larger scales, and about the precise extent of such cancellation; for in the absence of any cancellation, one might expect the energy (density) of a given Planck-size fluctuation to be Planckian as well, in clear conflict with observation. If these fluctuations do not cancel at all, then the volume v with respect to which $\Lambda$ fluctuates, i.e. the volume that is conjugate to $\Lambda$, is very small, making $\Lambda$ very large; and so, in asking about the presence and extent of such cancellation, one is in effect asking about the size of this v, and hence about the value of $\Lambda$ itself.

We seek to take the above questions seriously here; and though we will propose ideas that are somewhat speculative, even controversial, these proposals are motivated by the fact that they address these questions and help provide answers to them. Actually, we wish to broaden the question posed in [41] to include the following question: why not take the uncertainty relation $\Delta V\Delta\Lambda \sim 1$ to be satisfied by $\Delta V \sim V$, so that $\Delta V$ represents the entire growth of the universe from an initial time prior to inflation, a time at which $V$ was negligible? Such an understanding of $\Delta V$, given the relation $\Delta V\Delta\Lambda \sim 1$, entails a negligible post-inflation value of $\Lambda$ [10], since $V$ ($\sim \Delta V$) is enormous. Of course, the fact that this interpretation of $\Delta V$ gives the "wrong" value of $\Lambda$ would seem to disqualify it as a viable possibility. It may be, however, that the observed value of $\Lambda$ results from factors other than the $\Delta V\Delta\Lambda \sim 1$ relation itself (though this relation, with the "correct" value of $\Lambda$, could still hold as an *effective* relation). This is the possibility explored in the remainder of this paper, with the effective, observed $\Lambda$ given by the density $\rho_Q$ of the quantum potential Q of spacetime, as explained in part III. The idea that $\Lambda$'s observed value is not a direct result of the $\Delta V\Delta\Lambda \sim 1$ relation is, in any case, suggested by the arguments of [9, 68], which indicate that obtaining this value from the relation $\Delta V\Delta\Lambda \sim 1$, so that $\Delta V \sim \sqrt{V}$, entails a spatially inhomogeneous distribution of $\Lambda$ that is not in accordance with observation. Hence, the idea that the $\Delta V\Delta\Lambda \sim 1$ relation is satisfied simply by $\Delta V \sim V$, and not by $\Delta V \sim \sqrt{V}$, should not be dismissed out of hand, but should be taken seriously as a legitimate possibility.

### II.1 Darabi's Machian proposal

Accordingly, we suggest that the mass/energy obtained from the $\Delta V\Delta\Lambda \sim 1$ relation, for $\Delta V \sim V$, is indeed a genuine physical quantity. For the purposes of the present work, at least, it is convenient to treat this quantity as the mass m of spacetime itself; the significance of this treatment will become clear from the discussion in part III of the quantum potential of spacetime. In addition, we adopt a Machian, "holistic" view of the spacetime universe (a view that fits in well with a holographic perspective [39]); in particular, we make use of some suggestive Machian ideas proposed by F. Darabi [17] in connection with Planck-size vacuum fluctuations arising from energy-time uncertainty. Darabi's account appeals to Mach's principle – specifically, a mutual correlation and interaction between Planck-size regions – to explain why these virtual fluctuations, or new "baby universes," are not annihilated, but instead are integrated or "socialized" into the larger "mother" universe of spacetime consisting of "older" baby universes (or causet elements, to adopt the language and framework of causal set theory) that have already been socialized. The "socialization" here means that only the total "mother" universe satisfies energy-time uncertainty, rather than this uncertainty relation being satisfied by each baby universe individually. As a result, the new baby universes do not have their own (Planckian) energy scale; the only operative energy scale here is that of the (mother) universe *as a whole*, with this energy scale being determined in Machian fashion and corresponding (according to [17], at least) to the observed dark energy.

Attempting to exploit these ideas of Darabi's for our purposes, however – in particular, by replacing time-energy uncertainty with the related $\Delta V \Delta \Lambda$ uncertainty – creates certain problems. For one thing, the claim that the operative energy scale is that of the universe as a whole seems to imply that the volume with respect to which $\Lambda$ fluctuates is the volume $V$ of the entire universe. As noted, however, the resulting value of $\Lambda$ is far too small to serve as dark energy. Now, one way around this problem is to suppose that the socialization of baby universes consists of the nonlocal, mutual *cancellation* of the (Planck-scale) volume-fluctuations involving the N causet elements that constitute the spacetime of the universe. (Using the idea of a quantum-gravitational path integral, or sum over histories, we can speak of these fluctuations as being "canceled in the propagator" of the causal set that constitutes the spacetime volume $V$ of the universe.) Taking these fluctuations to be random in character, the fluctuation-cancellation occurring at a given time will leave an uncancelled remnant of roughly $\sqrt{N}$ ($\sim \sqrt{V}$) elements. As a result, we have a net volume-fluctuation $\Delta V$ of the universe, such that $\Delta V \sim \sqrt{V}$; and if we take *this* $\Delta V$ to satisfy the $\Delta V \Delta \Lambda \sim 1$ relation and hence generate stochastically a corresponding $\Delta\Lambda$, we get a value of $\Lambda$ suitable for serving as dark energy. In that case, however, the above-mentioned argument of [9, 68] applies – namely, the argument that, due to the fact that $\Lambda$ is generated by a *stochastic* process, spatial inhomogeneities of $\Lambda$ arise that conflict with observations of the CMB. There is also an additional problem: if we suppose that, at a given time t, the overwhelming majority of volume-fluctuations mutually cancel each other, then we are led to the conclusion that the density $\rho$ of causet elements, at t, is such that $\rho = 1$ *almost everywhere*. Such a near-uniform, lattice-like distribution of causet elements, however, violates a fundamental tenet of causal set theory; for, it entails the existence of a preferred frame and a consequent violation of Local Lorentz Invariance (LLI), in sharp contrast to causal set theory's LLI-satisfying Poisson distribution of causet elements [55].

Before dealing with these problems and questions, however, there is another issue that needs to be addressed. One might object that the sort of Machian cancellation of Planck-scale volume-fluctuations described above is physically counterintuitive, since it seems to require an exquisite degree of precision that is implausible from a physical standpoint. In response to this objection, we invoke the idea – which has its origins in non-equilibrium physics, and which has been employed specifically in cosmological contexts by the authors of [35, 36, 15] – that in highly fluctuating, disordered environments, a phase transition may occur that can be characterized as "freezing by heating," or "jamming," and that is marked by the emergence of a crystalline state in which fluctuations are largely absent. Appealing to this idea, we propose here that nonlocal interactions of volume-fluctuating causet elements produce a (temporary) jamming or freezing of these elements' fluctuations, of the sort described in the above references. Since the Planck-scale volume-fluctuations at a given time t are random in character, there is not a perfectly equal division between volume-expanding fluctuations and volume-contracting fluctuations at t; specifically, for N the total number of causet elements, there are roughly $\sqrt{N}$ more fluctuations of one kind than of the other (and we use the letter "k" to refer to whichever kind of fluctuation is more prevalent at a given t). This fact, together with the general phenomenon of fluctuation-induced screening (for an example of which see [47]), motivates a further hypothesis: at a given t, there are $\sim \sqrt{N}$ fluctuations of kind k that are not jammed, due to the fact that they are screened by the other fluctuations of kind k that *are* jammed (for large N, of course, these latter fluctuations are much more numerous than the unjammed ones). These unjammed fluctuations constitute the above-noted net fluctuation $\Delta V$ of the universe's volume $V$, with $\Delta V \sim \sqrt{V} \sim \sqrt{N}$. As we will see in part III, the combination of these (relatively scarce) unjammed fluctuations with a predominantly crystalline state, a state resulting from the jamming or freezing of fluctuations, gives rise to a quantum potential Q of spacetime. This quantity of energy Q is a source of new causet elements; and the N-fluctuations that reflect the creation of these new elements cause the crystalline state to break up, with nonlocal interactions between these fluctuations leading once again to a widespread jamming or cancellation of fluctuations and the consequent re-emergence of a predominantly crystalline state, and so on in a continually repeated cycle. (We leave until section IV a consideration of the possible origin of the *nonlocality* required for a large-scale cancellation of volume-fluctuations.)

Given, then, the existence of a nonlocal cancellation of spacetime volume-fluctuations on a cosmological scale, we argue in what follows that the idea of such a cancellation can lead to a viable account of dark energy, despite the problems described above. On this account, the uncertainty relation $\Delta V \Delta\Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$, is *not* taken as a fundamental principle from which dark energy in the form of $\Delta\Lambda$ is obtained. Rather, we start from the idea mentioned earlier that a cosmically pervasive and nonlocal cancellation of volume-fluctuations may result in causet elements being distributed in such a way that $\rho = 1$ almost everywhere; and we claim that such a distribution itself gives rise to energy in the form of the above-mentioned quantum potential Q of spacetime, with the density $\rho_Q$ of this energy acting as an effective $\Lambda$ by virtue of satisfying (in general) the relation $\Delta V \Delta\Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$. (We say "in general" because, as explained later, some deviations from $\Delta V \Delta\Lambda \sim 1$ may occur at certain times, specifically in early cosmological epochs.) The emergence of Q thus allows the relation $\Delta V \Delta\Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$, to be recovered (in general) as an *effective* principle. At the same time, as indicated earlier, we do take the relation $\Delta V \Delta\Lambda \sim 1$, *for* $\Delta V \sim V$, as basic; and we associate the resulting energy scale with the mass m of spacetime, obtaining the value $m \sim 1 = m_p$, since the relevant energy E is just $E = V\Lambda = \Delta V \Delta\Lambda \sim 1 = E_p = m_p$. This value of m implies – given the general expression for the quantum potential, which contains a mass term in the denominator – that the mass of spacetime has no effect on the value of the spacetime quantum potential Q. (Taking spacetime's mass to be simply its inertia, i.e. its resistance to expansion or contraction, this lack of effect of m on Q indicates that this inertia is negligible and hence may be disregarded.)

As explained later, the nonlocal nature of Q entails a spatially homogeneous distribution of dark energy, thereby circumventing the arguments of [9, 68]. Furthermore, as just indicated (and as shown in part III, below), Q has the right magnitude to serve as dark energy. Hence, we see how two of the problems mentioned above in connection with our use of Darabi's ideas can be resolved. We propose to address the remaining problem, namely the violation of LLI, by arguing that the existence of a preferred frame, and the consequent violation of LLI, should be accepted after all.

### II.2 Smeared time and a preferred frame

Consider first causal set theory's idea of a quantum-gravitational sum over histories. As noted, in taking such a sum over histories, the time-parameter N is held fixed, which however does not eliminate all volume-fluctuations, since N=$V$ only up to fluctuations [6]. A crucial point here is that $V$ itself can serve as a time-parameter; there is nothing special about N that makes it uniquely suited to parameterize time. This suggests that, for any relevant sum over histories, time cannot be held *completely* fixed, due to the existence of volume-fluctuations $\Delta V$; in other words, time is necessarily subject to a certain amount of "smearing." To put it another way, the parameterization of time by N – or more specifically, by a series of "timesteps," each of which is associated with a particular value of N – is relatively coarse-grained, since it ignores the smearing of N (or of time) by volume-fluctuations $\Delta V$. The resulting "smeared-out" time interval is associated not only with the uncancelled volume-fluctuations $\Delta V \sim \sqrt{V}$; it is marked as well by the nonlocal cancellation, mentioned above, of the vast majority of volume-fluctuations.

We turn now to an argument for a cosmological preferred frame that has been given by Arlen Anderson [7]. Anderson notes the conjugacy, in unimodular gravity, between cosmological time and $\Lambda$; and he goes on to observe that, in order to accommodate the many-fingered time formalism of quantum field theory and the extra degrees of freedom that it entails, it is desirable to replace the scalar $\Lambda$ of standard general relativity with a cosmological stress tensor $\Lambda_{\mu\nu}$. This $\Lambda_{\mu\nu}$ represents a preferred frame that may be equated, for practical purposes, with the rest frame of the CMB. (On the general rationale for equating a cosmological preferred frame with the CMB rest frame, see [37].) As [7] makes clear, this preferred frame does not lead to a loss of general covariance.

The relevance of all this to the present discussion is that the same sort of motivation for $\Lambda_{\mu\nu}$ that is found in [7] is available even without appeal to the many-fingered time formalism. For, the volume-fluctuations that "smear" (N-parameterized) time, as described above, occur on various small (i.e., sub-cosmological) scales and are not everywhere identical, which indicates that the amount of smearing can vary from region to region, all the way down to regions near the Planck scale in size; and this in turn suggests that there are multiple "local times" that vary from region to region, with each such time parameterized by the magnitude of the volume-fluctuations occurring in the relevant region. And the resulting impossibility of completely eliminating the existence of different times in different regions provides the same sort of motivation for representing $\Lambda$ by a cosmological stress tensor $\Lambda_{\mu\nu}$ as the many-fingered time formalism does. This $\Lambda$, however, being conjugate to cosmological time [7], is just the $\Lambda$ such that $\Delta V \Delta\Lambda \sim 1$, *for* $\Delta V \sim V$, since the volume $V$ of an expanding universe is a measure of cosmological time; and as noted, this $\Lambda$ has far too small a value to serve as dark energy. The model of [7], therefore, cannot be viewed as a complete model of gravity, since as it stands it fails to account for dark energy. This does not mean, however, that [7]'s $\Lambda_{\mu\nu}$ should simply be discarded. For, the energy scale such that $\Delta V \Delta\Lambda \sim 1$, with $\Delta V \sim V$, does exist (its existence is noted in [51], e.g., though it is not described there in terms of $\Delta V$); and, as indicated earlier, we propose that it represents the energy scale of the mass m of spacetime. Hence, we conclude that there is genuine motivation for accepting the existence of a cosmological preferred frame, or rest frame of the universe, even from the standpoint of causal set theory and its idea of a quantum-gravitational sum over histories.

For further elaboration of the nature of this preferred frame, consider the idea, found in [5], that the rest frame of the universe is defined by hypersurfaces of constant mean extrinsic curvature (CMC). This leads to the view that the preferred frame consists of CMC hypersurfaces, a view that matches Afshordi's account of a preferred frame or "gravitational aether" $T'_{\mu\nu}$ [1, 3, 4]. (Such a preferred frame is also a feature of Horava-Lifshitz gravity in the low energy limit [2]; and see [40] on how ascribing viscosity to the aether can yield observationally acceptable values of G.) This aether, on Afshordi's account, is such that it is always coupled to some other field; in the present context, the aether may be coupled to the mass m of spacetime, which can be treated as a 3-tensor $m_{ij}$. (See [8] for such a treatment of mass in connection with Bose-Einstein condensates; this treatment is in accordance with the present approach to spacetime as a condensate.) In describing the *dark energy* associated with this aether, however, we depart from Afshordi's framework in order to develop an account that recovers the $\Delta V \Delta \Lambda$ uncertainty relation of unimodular GR (albeit as an *effective* relation). Nonetheless, it is possible that aether pressure may make some sort of non-negligible contribution to the current cosmic acceleration.

Regarding the issue of LLI, we take the position that, as long as time is parameterized by N alone – as in causal set theory – so that the "smearing" of time is disregarded or abstracted from, then causet elements exhibit a Poisson distribution, no preferred frame is evident, and LLI is upheld. It is only when we regard volume-fluctuations as an actual smearing of time itself – in contrast to causal set theory, which views volume-fluctuations as essentially instantaneous, i.e. as occurring at a particular "unsmeared" moment at which N is held fixed – that we see evidence of a preferred frame, with a concomitant deviation from both LLI and a Poisson distribution of causet elements, and are in a position to describe or take account of the effects of this deviation. Thus, we view LLI as an approximation that holds only when time is parameterized by N alone and is therefore treated as "unsmeared;" such a view of LLI conforms with [56], where it is argued that LLI is not a strict consequence of general relativity and is valid only as an approximation.

The considerations mentioned above regarding preferred frames motivate us to explore the consequences of adjoining the idea of a preferred frame to that of causal sets. One consequence is that a distribution of causet elements such that $\rho = 1$ almost everywhere – a distribution that represents a particular configuration of the scalar field $\psi$ described above, a configuration in which the vast majority of causet elements correspond to Planck-size "cells" of spacetime volume distributed in a uniform or homogeneous manner – must be accepted as physically allowable, despite the fact that it indicates the existence of a preferred frame. This consequence is, in fact, crucial to obtaining a satisfactory account of ever-present $\Lambda$, or dark energy, since, as indicated earlier, the very existence of a distribution of causet elements such that $\rho = 1$ almost everywhere entails the existence of energy, in the form of a quantum potential Q of spacetime, that can serve as an effective $\Lambda$, due to the fact that the density $\rho_Q$ of this energy satisfies (in general) the relation $\Delta V \Delta \Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$.

Both (i) the Machian idea that volume-fluctuations are subject to a nonlocal cancellation process involving nearly all causet elements in the universe, and (ii) the idea of $\rho_Q$ as an *effective* $\Lambda$, have a certain affinity with Gogberashvili's Machian model of gravity with a fundamental preferred frame [21], a model in which the key principles of general relativity (GR) – including LLI – are recovered in the IR as effective principles. At the same time, as is clear from [18], GR itself – in a "normalized" form – permits a nonlocal/Machian account of $\Lambda$. The value of this $\Lambda$, in fact, turns out to be zero, so that dark-energy effects need to be described in terms of something else, such as $\rho_Q$. Also worth noting is [18]'s conclusion that both the Machian $\Lambda$ of normalized GR and any effective $\Lambda$ are *non-negative*; this conclusion is compatible with the account of dark energy given here, but not with [6]. Thus, both [21]'s Machian model and the normalized GR of [18] can accommodate the present account of $\rho_Q$ as an effective $\Lambda$. We also note the existence of a quantum version of Mach's principle [19]; and analogously to this extension of Mach's principle to the quantum realm, the present account of $\Lambda$ in terms of Q may be viewed as (part of) a quantum/Planckian extension of [21]'s Machian model of gravity. In addition, both [21]'s model and the present account of Q and $\Lambda$ recall, in some respects, the cosmological model of [20], in which the universe is treated as a single "particle" formed by dynamical symmetry breaking of a (Weyl) vacuum. Particularly relevant is the notion that the "internal space" of such particles is not subject to any locality requirement [67], so that nonlocal relations may play an important role within the universe.

## III. SPACETIME AS A CONDENSATE, AND ITS QUANTUM POTENTIAL

To proceed, then, we adopt [6]'s view that $\Lambda$ reflects the action (per causet element) that is due to the bare existence of spacetime itself, irrespective of excitations such as matter and gravitational waves. We also accept [6]'s assumption that $\Lambda$'s "target value" – the value that $\Lambda$ tends to take, up to fluctuations – is zero; as noted, this assumption is supported by the normalized GR of [18]. Motivated by this conception of $\Lambda$, we ask whether there is a theoretical framework suitable for representing or expressing the idea of such a $\Lambda$. One attractive possibility for such a framework – suggested in part by its usefulness in connection with analogue gravity – is the theory of Bose-Einstein condensates [8]. It is natural to write the condensate wave function $\psi$ using the Madelung representation, which gives us $\psi = \sqrt{\rho}(\exp[iS/\hbar])$. Here $\rho = n/v$ is the particle density of the condensate, for a given volume v; in addition, $|\psi|^2 = \rho$, and S has the dimensions of an action [8]. As is well known, substituting the Madelung representation of $\psi$ into the Schrodinger equation yields the following two equations [8, 12, 13], in which we write "$\Delta$" for the Laplacian operator and "R" for $\sqrt{\rho}$:

$$\partial S/\partial t + (\nabla S)^2/2m + V - [(\hbar^2/2m)(\Delta R/R)] = 0$$

$$\partial \rho/\partial t + \nabla \cdot (\rho\, \nabla S/m) = 0.$$

The first of these is the so-called quantum Hamilton-Jacobi equation (QHJE), and the second is a continuity equation. The term "$[(-\hbar^2/2m)(\Delta R/R)]$" in the QHJE is the quantum potential Q, with V being a classical potential. (In the case of Bose-Einstein condensates, there are typically two kinds of potential V: an interaction potential between particles and an external confining potential.) Applying this condensate treatment to the spacetime of causal set theory, we identify "$\rho$" in the above equations with the causet-element density $\rho$ described earlier; and we express the idea that $\Lambda$ – i.e., the action due to the bare existence of spacetime - has zero as a target value by saying that (i) the value of the action S is essentially zero, or negligible, and (ii) each term in both the QHJE and the continuity equation has a value which, *up to fluctuations*, is negligible and hence may be treated as zero (of course, the terms containing "S" are zero already because of (i)). It follows from (i) that $\psi$ is a scalar field, with $\psi = \sqrt{\rho} = R$. Furthermore, the only significant fluctuations we assume here are $\rho$-fluctuations, the existence of which follows from the volume-fluctuations of causal set theory. (In particular, we do not suppose that stochastic spacetime fluctuations generate S-fluctuations of sufficient magnitude to constitute dark energy; for, we regard the idea that dark energy is generated by such stochastic fluctuations as ruled out observationally by the arguments of [9, 68] mentioned earlier.) These fluctuations give rise to a fluctuating quantum potential Q as explained below; and as a result of these $\rho$- and Q-fluctuations, we get deviations from the two equations above, deviations that can be viewed as quantum-gravitational corrections to a pure quantum spacetime.

(Regarding the potential V, we assume a negligible interaction potential between causet elements, an assumption that seems reasonable *prima facie*. Furthermore, for 4D spacetime, the lack of "confinement" indicated by the universe's (accelerated) expansion suggests that the confining potential is negligible as well.)

It is important to note that an individual $\rho$-fluctuation does not, by itself, give rise to a nonzero Q. The quantum potential Q of spacetime is a collective product of the entire pattern of $\rho$-fluctuations occurring at a given time t, and it takes a single value at t; hence, Q is delocalized, or nonlocal, in accordance with [12, 13], and consequently its density $\rho_Q = Q/V$ is spatially uniform or homogeneous. This $\rho_Q$ recalls the nonlocal $\Lambda$ of [18], except that in [18] $\Lambda$'s value is *temporally* as well as spatially homogeneous.

It is also important to realize that even though this $\rho_Q$ can be *represented*, using the field equations of unimodular GR, as a quantity $\Lambda$ such that $\Delta V \Delta \Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$, our account of this $\Lambda$ is not obtained from GR itself, unimodular or otherwise. Rather, we explain this $\Lambda$ or dark energy as a *quantum*-gravity effect; specifically, we explain it in terms of the fluctuating spacetime quantum potential Q mentioned above. By taking account of this Q, however, we can, as noted, effectively recover the $\Delta V \Delta \Lambda$ uncertainty relation of unimodular GR, with $\Delta V \sim \sqrt{V}$. (As indicated in part IV, a coupling of dark matter to spacetime may alter m's effective value, especially in early cosmological eras, leading to some deviation from the uncertainty relation $\Delta V \Delta \Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$. Since this relation is merely effective, however, this deviation raises no theoretical issues or problems.)

Returning now to a consideration of those $\psi$-configurations with the property that $\rho = 1$ almost everywhere (at time t), we seek to show that the (uncancelled) volume-fluctuations $\Delta V$ that form part of a given such $\psi$-configuration *determine* a definite value for $\Lambda$ at t. The fact that $\rho = 1$ almost everywhere implies that (only) approximately $\sqrt{N}$ causet elements are actually fluctuating at t – with these fluctuations collectively constituting a volume-fluctuation $\Delta V$ of the universe – and these fluctuating elements are taken to have a Poisson distribution, in accordance with [6]. Given this distribution and the largeness of N in relation to $\sqrt{N}$, and taking the roughly $\sqrt{N}$ causet elements that fluctuate at t to constitute a subset s of the set of all causet elements in the universe, we conclude that the members of s are overwhelmingly likely to be *sparsely distributed* throughout the universe. In other words, any two members of s are almost certain to be separated by a supra-Planckian spatial distance. Since each member of s is associated with a volume-fluctuation and hence with a fluctuation of $\rho$, it is obvious that these $\rho$-fluctuations are likewise sparsely distributed; i.e., they are each surrounded by a relatively large region where $\rho = 1$ uniformly. It is also evident that the $\rho$-fluctuations of the members of s produce nonzero values or fluctuations of $\partial\rho/\partial x^n$ ("n" here ranges over the three spatial dimensions) that are sparsely distributed as well. (We treat "$\partial$" here as a finite difference operator, with Planck-length finite differences.) Finally, note that the members of s, at a given time, *collectively* produce either a contraction or an expansion (as the case may be) of the four-volume *V* of the universe.

Our main argument, in what follows, is that the fluctuations of $\rho$ and $\partial\rho/\partial x^n$ associated with the members of s give rise to a (fluctuating) *quantum potential Q (or $\Delta Q$) of spacetime which plays the role of dark energy*, with Q's density $\rho_Q$ having the same magnitude as the $\Delta\Lambda$ of [6]. In other words, this $\Delta\Lambda$ manifests as $\rho_Q$, so that the uncertainty relation $\Delta V \Delta\Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$, is effectively satisfied. Before demonstrating by explicit calculation that $\rho_Q$ and [6]'s $\Delta\Lambda$ are in fact comparable, however, there are several other points to make.

### III.1 Some remarks on the quantum potential of spacetime

In the first place, since $\rho_Q$, as we shall see, always has the same sign regardless of whether the volume-fluctuation that determines it is a contraction or an expansion, we take $\rho_Q$'s value to be *positive*, in accordance with cosmological observations. (Since the usual sign convention is to give the quantum potential a negative value in order to indicate that it is a repulsive force, we should, strictly speaking, write the dark energy density as "$-\rho_Q$" to indicate that this density is positive. We will not always adhere to this convention, however, preferring to simply write "$\rho_Q$".) This contrasts with [6], where $\Lambda$ may fluctuate either positively or negatively. Treating $\rho_Q$, as exclusively positive does raises some issues concerning its value at early times, however, which will be addressed in part IV. In any case, as noted earlier, [18] contains an independent argument for the non-negativity of both the "true" cosmological constant $\Lambda$ and a time-varying dark energy, based on a "normalized" GR with field equations that resemble those of unimodular GR.

Something also needs to be said concerning our claim that the Q-fluctuations arising from volume-fluctuations entail deviations from the QHJE; the basic idea behind this claim is that the perturbations of spacetime represented by volume-fluctuations induce temporary violations of the QHJE, violations that take the form of fluctuations of the spacetime quantum potential Q. This claim itself seems reasonable enough, since (a) the QHJE follows from the Schrodinger equation, and (b) volume-fluctuations clearly are not part of, nor are they a consequence of, the process of Schrodinger evolution. Instead, they represent an external force capable of disturbing this process; in that respect, they are like the factors responsible for wave-function collapse. Hence, we should not expect the QHJE to describe their occurrence and effects, just as we do not expect the Schrodinger equation to describe the collapse of the wave function.

The resulting deviation from the QHJE is an externally-induced effect – “external,” that is, from the standpoint of quantum theory *per se*. Specifically it is an effect of the volume-fluctuations entailed by causal set theory (together with the $\Delta V \Delta \Lambda$ uncertainty relation of unimodular gravity). There are two main points to make concerning this. First, the existence of such a non-quantum effect does not mean that the QHJE is completely irrelevant to the Planck-scale spacetime of causal set theory. The QHJE is still relevant because volume-fluctuations are viewed or understood here as sources of deviation *from the QHJE*. Second, it seems strange that a *quantum*-theoretic concept like the quantum potential should lead to a deviation from a key equation of quantum mechanics, namely the QHJE. At the Planck scale, however, where quantum and gravitational factors interact, we cannot expect the concepts of either quantum theory or gravitational theory to be completely unaffected by this interaction; in particular, the emergence of “hybrid” concepts that owe something to *both* theories should be taken seriously as a distinct possibility. And the spacetime quantum potential proposed here is just such a hybrid concept: it has the familiar mathematical form of the quantum-theoretic quantum potential, but it owes its existence to fluctuations of *geometry* (specifically, volume-fluctuations), and hence it has a gravitational aspect as well.

Note that the idea of spacetime as a condensate – and the related view of the universe as a “macroscopic quantum phenomenon,” so that principles such as the QHJE are relevant to it – has been proposed by Hu [30, 31]. (A different, though not incompatible, motivation for regarding the spacetime universe as quantum is given in [23].) We can add here that such a view of spacetime finds some motivation in causal set theory itself; for, this theory employs the number density $\rho = n/v$ of causet elements, which (as noted) is the same kind of density used in connection with Bose-Einstein condensates. Thus, a condensate treatment of causal sets has a certain naturalness to it.

Finally, the idea of the quantum potential density $\rho_Q$ as an effective $\Lambda$ receives support from Groessing’s account of the quantum potential as a form of *vacuum energy*, specifically thermal vacuum energy [26, 14]; and given the idea of Q as *thermal*, Tiwari’s view of dark energy as thermodynamic in character [65] also provides some support for treating Q as dark energy. (In addition, the use of unimodular relativity in [65], which was noted earlier, is congruent with the present approach.)

### III.2 Determining the quantum potential of spacetime

We now turn to the task of calculating the value of Q produced by a given volume-fluctuation $\Delta V$; the calculation here is actually quite simple and straightforward. Since Q here is non-relativistic, we consider only the three spatial dimensions. It is worth noting, first, that the phenomenon of dimensional reduction at the Planck scale (on which see [47], e.g.), in which some dimensions are screened due to spacetime blurriness or fuzziness produced by geometric fluctuations, does not occur here, since we are considering the effects of a general *cancellation* of such fluctuations, a cancellation associated with the presence of a fixed number N of causet elements. When N is allowed to fluctuate, dimensional reduction may well occur; but in the case considered here, spacetime is relatively definite and "well-behaved," and so all of its dimensions may be assumed to be fully manifest.

Consider, then, the sparsely distributed, Planck-scale fluctuations of $\rho$ and $\partial\rho/\partial x^n$, occurring at a given time t, that collectively constitute (as described above) a particular volume-fluctuation $\Delta V$ of the four-volume $V$ of the universe. As noted, $\Delta V$ here may represent either an expansion or a contraction of $V$. For the case of a volume-*expansion*, the fact that $\Delta V \sim \sqrt{V}$, together with the fact that there are $\sim \sqrt{V}$-many fluctuating causet elements (i.e., causet elements whose fluctuations are not canceled), means that, on average, each of these fluctuating elements experiences an approximate *doubling* of its associated volume, and hence a *halving* of its density $\rho$. (For simplicity, we assume that *each* fluctuating element exhibits "average behavior," so that a single description can be applied to all of them; this assumption does not affect any of our substantive conclusions.) The case of a volume-*contraction* is not quite as straightforward, however. For, we cannot allow the volume associated with the fluctuating elements to shrink all the way to zero, even though this would satisfy the relation $\Delta V \sim \sqrt{V}$; the reason is that this would amount to the *disappearance* of these elements, thereby contradicting the assumption that the number of elements is held fixed. We therefore make the stipulation, which seems reasonable enough in any case, that the average density-fluctuation $\partial\rho/\partial x^n$ of each element undergoing a volume-contraction is just the *inverse* of the average value of $\partial\rho/\partial x^n$ for a volume-expansion. In other words, on average, each fluctuating causet element experiences an approximate *halving* of its associated volume, and hence a *doubling* of its density $\rho$. (Here too, we make the simplifying assumption that each fluctuating element is "average".)

Combining the points just made with [6]'s account of volume-fluctuations, we now proceed to determine the value of the quantum potential Q produced by a given volume-fluctuation; the hope, of course, is that Q's density $\rho_Q$ is comparable to the ever-present $\Lambda$ of [6]. First, we regard the universe's volume $V$ as partitioned into Planck-sized cells, such that each cell is the volume associated with a particular causet element, and no two cells are associated with the same element. Each of these cells, therefore, determines the density $\rho$ of its associated causet element. The cells of interest here, of course, are the *fluctuating* cells that collectively constitute, at a given time t, a volume-fluctuation $\Delta V$ of the universe. For a given t, there are roughly $\sqrt{V}$-many such fluctuating cells C; and using the simplifying assumptions of the preceding paragraph, we can say that *either* (a) each of these cells C is such that $\rho = 2.0$ at t, *or* (b) each of these cells C is such that $\rho = 0.5$ at t (depending on whether $\Delta V$ is a contraction or an expansion, respectively). Furthermore, for almost every one of these cells C, all of the cells in the vicinity of C satisfy $\rho = 1$, where the "vicinity" of C is a region approximately one order of magnitude above the Planck scale. (To say that the above condition holds for "almost" every such C is to say that the number n of cells C for which this condition holds is such that $n \sim \sqrt{V}$.) As a result of this uniform density of the cells surrounding a typical C, we have that $\nabla\rho = 0$ (i.e., $\nabla\rho$ is a null vector) in the vicinity of C, *except* in the case of C itself and those cells immediately surrounding (or adjacent to) C. A little reflection makes clear that, for almost every one of these $\sqrt{V}$-many cells C, if we take the value of the ratio $\Delta R/R$ associated with C itself and add to it the value of $\Delta R/R$ that is associated with each of the cells immediately surrounding C, then the sum is of order 1. For the entire universe, therefore, we have $(\Delta R/R) \sim \sqrt{V}$. To obtain a value for Q, however, we need to multiply $(\Delta R/R)$ by $-\hbar^2/2m$. Since $\hbar=1$ and $m\sim1$, as indicated earlier, we only need to divide by -2, which does not affect the order of magnitude. Hence, we have $-Q \sim (\Delta R/R)/2 \sim \sqrt{V}$, which is just what we need (see below).

Thus, each of the above-described fluctuating cells C gives rise to a ratio $\Delta R/R$ such that $(\Delta R/R)/2 \sim 1$. Let us now determine this ratio more precisely for a typical C. We begin by considering with a volume-*expansion*, which as noted has the effect of making $\rho = 0.5$, and hence $\sqrt{\rho} = R = 0.7$ (approximately), for each C. Consider, for a particular C, a one-dimensional vector-component v that passes through C along a given coordinate axis x. Immediately adjacent to C along the x-axis, there are two Planck-sized cells $C_a$ and $C_b$,one on each side of C, such that $\rho = 1 = \sqrt{\rho}$ for each of these cells, and with v pointing from $C_a$ to $C_b$. We now evaluate $\partial R/\partial x$ and $\partial^2 R/\partial x^2$ by moving across C in the direction of v, starting at $C_a$. Using the above assumptions concerning C and the cells in its vicinity, we have for $C_a$ the following values: $\partial R/\partial x = -0.3 = \partial^2 R/\partial x^2$. For C itself, we have $\partial R/\partial x = 0.3$, and $\partial^2 R/\partial x^2 = 0.6$. And for $C_b$, we have $\partial R/\partial x = 0$, and $\partial^2 R/\partial x^2 = -0.3$. (For all other cells in the vicinity of C, $\partial^2 R/\partial x^2 = 0$, and we therefore ignore them.) Consider now the ratio $(\partial^2 R/\partial x^2)/R$. For $C_a$, the value of this ratio is -0.3, since $R = 1$; and the same is true for $C_b$, so that the total value for these two cells is -0.6. For C itself, we have $(\partial^2 R/\partial x^2)/R = 0.6/0.7 = 0.86$ (approximately). Hence, the overall value of $(\partial^2 R/\partial x^2)/R$ here is $0.86 - 0.6 = 0.26$. (Note that for all other cells in C's vicinity, $(\partial^2 R/\partial x^2) = 0 = (\partial R/\partial x)$, since by assumption $\rho = 1$ for each of these cells.) Repeating this procedure for the other two spatial dimensions and summing the results, we obtain $\Delta R/R = 3 \cdot 0.26 = 0.78$; and dividing by -2 gives us -0.39. Thus, each of the $\sqrt{V}$-many cells C contributes to Q's value an amount equal to -0.39, and hence the absolute value of each cell's contribution is of order 1. We therefore have $-Q \sim \sqrt{V}$.

For the case of a volume-*contraction*, we proceed in the same manner, except that now each C is such that $\rho = 2.0$, and hence $\sqrt{\rho} = R = 1.41$ (approximately). As a result, we have, for $C_a$, $\partial R/\partial x = 0.4 = \partial^2 R/\partial x^2$; and for C itself, we have $\partial R/\partial x = -0.4$, and $\partial^2 R/\partial x^2 = -0.8$. For $C_b$, we have $\partial R/\partial x = 0$, and $\partial^2 R/\partial x^2 = 0.4$. As for the ratio $(\partial^2 R/\partial x^2)/R$, its value for $C_a$ and $C_b$ combined is $0.4 + 0.4 = 0.8$. For C itself, we have $(\partial^2 R/\partial x^2)/R = -0.8/1.41 \approx -0.57$. Hence, the total value of $(\partial^2 R/\partial x^2)/R$ here is approximately $0.8 - 0.57 = 0.23$. Repeating this for the other two spatial dimensions and taking the sum, we obtain $(\Delta R/R) = 0.69$. Dividing this result by -2, we see that each of the $\sqrt{V}$-many cells C contributes to Q's value an amount $\approx -0.35$; here again, then, the absolute value of each cell's contribution is of order 1, and hence $-Q \sim \sqrt{V}$.

Thus, for a given volume-fluctuation $\Delta V$ occurring at time t, each cell C for which $\rho \neq 1$ contributes to Q an amount having a value of order 1, in Planck units. Hence, for the volume $V$ of the universe, where $V$ contains $V^{1/2}$-many such cells C at t, the quantum potential Q due to $\Delta V$ is such that $-Q \sim V^{1/2}$, with density $-\rho_Q \sim V^{-1/2}$. The magnitude of $\rho_Q$ is thus comparable to that of the (ever-present) $\Lambda$ of [6], which is just what is needed in order to justify treating $\rho_Q$ (or more precisely, $-\rho_Q$) as an effective $\Lambda$.

Given what was said earlier about the nonlocal nature and distribution of Q and $\rho_Q$, it is clear that, in contrast to the $\Lambda$ of [6], $\rho_Q$'s value at a given time t is *not* "selected" by a stochastic (random-walk) process; as a result, the spatial inhomogeneity of [6]'s $\Lambda$, which leads to effects that conflict with observations of the CMB [9, 68], is avoided. One could object that, even though $\rho_Q$'s value is not directly "chosen" stochastically, $\rho_Q$ does depend on a stochastic process, namely the process of volume-fluctuation.

Nonetheless, in view of the huge number of Planck-scale volume-fluctuations occurring in the universe at a given time t, it is reasonable to suppose, by virtue of the Law of Large Numbers, that these volume-fluctuations are distributed homogeneously, even between causally separated regions of spacetime. Hence, the element of stochasticity here should not lead to observationally problematic inhomogeneities of $\rho_Q$. (Note also that the early-time suppression of $\rho_Q$ described in part IV, below, provides an additional avenue for avoiding conflict with CMB observations.)

The general idea of a quantum potential that is nonlocal on cosmological scales – and hence has a "Machian" character – is not original with the present account, but has been advanced before by others [59, 60]; the same can be said of the idea of treating $\Lambda$ in Machian/nonlocal terms [18], as well as the idea that spacetime itself exhibits strong nonlocal connections of a Machian character [43-45, 53, 54]. The above account of Q is, therefore, not a completely novel one, though it does have some new features which, we feel, make it worthy of attention. This account of Q also has an affinity with: (i) the idea of spacetime as a condensate, according to which the universe as a whole is a "macroscopic quantum phenomenon" [31]; (ii) the conception of the universe itself as subject to quantum entanglement [22, 23, 29, 46]; and (iii) the view that the universe's fundamental degrees of freedom obey infinite statistics, and hence are strongly correlated and nonlocal. (On some of the motivations for using infinite statistics in cosmological and quantum-gravitational contexts, see [16, 32-34, 46, 48-50, 58].) Together these various theories and proposals, though none of them is free from controversy, suggest that the idea of Q as nonlocal on a cosmological scale should be taken seriously, especially in view of the fact that the model of [6] itself, as we have seen, requires a strong nonlocality.

## IV. SOME ADDITIONAL ISSUES AND QUESTIONS

The fact that $\rho_Q$ is always positive eliminates the danger of negative $\Lambda$-fluctuations producing a negative total energy density $\rho_{tot}$ (on this danger, see [6]); at the same time, however, it raises another problem, namely, ensuring that $\rho_Q$'s early value is low enough to both allow structure formation to occur and satisfy the constraints associated with BBN and observed abundances. This problem is addressed in [6] by allowing negative fluctuations of $\Lambda$, but that route is obviously not available to us here. Instead, we propose that dark matter couples to the Q-fluctuations of the spacetime condensate – a rather natural idea in view of the fact that dark matter itself may well take the form of a Bose-Einstein condensate ([11, 42, 61, 62]; and see [66] on the coupling of BEC's to spacetime *fluctuations*). As a result of this coupling, dark matter may contribute positively to the value of the mass term "m" in the expression for Q, thereby leading to a possibly significant increase in m's value and, consequently, to a possibly significant decrease in Q. For a given time t, the extent to which m's value is affected at t by the coupling depends both on the strength of the coupling, which we assume to be constant over time, and on the dark matter density $\rho_m$. At early times, the relatively large value of $\rho_m$ lowers (or effectively suppresses) the values of Q and $\rho_Q$ significantly; hence, assuming that the coupling originates prior to Big Bang Nucleosynthesis, it is possible for the constraints of both structure formation and BBN to be satisfied. At late times, however, when dark matter is highly diluted, the effect of $\rho_m$ on Q and $\rho_Q$ becomes negligible, and the value of the "m" term in the expression for Q is ~ 1, as it is in the absence of any coupling with dark matter. This allows $\rho_Q$ to dominate $\rho_{tot}$, thereby making possible the universe's accelerated expansion.

Turning now to the uncertainty relation $\Delta V \Delta \Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$, it is apparent that the coupling described above can cause this relation (including the value of $\Delta V$ itself) to be modified at early times. Such a modification is not problematic from the present standpoint, however, since the above uncertainty relation is treated here as merely an effective principle. This is not to say that the relation $\Delta V \Delta \Lambda \sim 1$ should not be expected to hold in *some* form. The point is simply that this relation can always be satisfied by taking $\Delta V \sim V$, as noted earlier.

The changing effects of $\rho_m$ on $\rho_Q$ can be incorporated into the model of [6] *via* the fluctuation-parameter $\alpha$ of this model, which now gets reinterpreted as a time-varying parameter that reflects (in part) the extent to which $\rho_m$ effectively suppresses Q and $\rho_Q$. Such a reinterpretation of $\alpha$ is possible because the magnitude of dark energy fluctuations is no longer the product of a random walk, but is instead determined and constrained by the magnitude of both $\Delta V$ and $\rho_m$. (Note, by the way, that the random variable $\xi$ of [6] gets reinterpreted as well, becoming in effect a constant such that $\xi \approx 1$, which allows us to disregard it.) The upshot is that $\alpha$ can take a relatively low value at early times and a relatively high value at late times, thereby eliminating the "tension" regarding $\alpha$'s value – a tension described in [6] – which is due to the need to select a single value of $\alpha$ that holds at all times.

As $\rho_m$ decreases, the suppression of $\rho_Q$ by dark matter eventually becomes negligible, so that $\rho_Q$ approaches its "unsuppressed" value of ~ $H^2$, which together with the decrease of $\rho_m$ leads to accelerated cosmic expansion. This raises the question of cosmic coincidence: why has $\rho_Q$ become unsuppressed just now, i.e. in the present epoch? To answer this question, at least in qualitative terms, we note first that the length of time it takes for $\rho_Q$ to become unsuppressed depends on the amount by which $\rho_Q$ (and Q) is suppressed in the first place. The latter amount, in turn, depends on the strength of the coupling between dark matter and Q-fluctuations: the stronger the coupling, the greater the suppression of Q and $\rho_Q$, for a given value of $\rho_m$. The coupling, however, cannot be so strong as to suppress Q altogether, since in that case there would be nothing for dark matter to couple to and hence no coupling at all, contrary to hypothesis. As a result, there are significant limits on the strength of the coupling, and hence on the suppression of $\rho_Q$, even in the epoch of matter domination. It is not unreasonable, therefore, to expect that a substantial decrease in $\rho_m$, such as occurs between the matter-dominated era and the present, will have the effect of eliminating much if not all of this *limited* suppression of $\rho_Q$; and so we have an answer, albeit a rather rough and qualitative one, to the question raised above.

To conclude, let us return to the idea that Planck-scale volume-fluctuations may undergo mutual cancellation on a cosmological scale, and consider whether any physical motivation can be given for the strong nonlocality that this idea entails. One source of such motivation can be found by adopting the idea, advanced by a number of authors [33, 34, 46, 48-50, 58], that the fundamental particles or quanta constituting the vacuum obey infinite statistics. (The general idea of particles obeying infinite statistics comes from the quon model of Greenberg [24, 25] (cited in [32]).) As argued by Jackson and Hogan [32], the strong nonlocal correlations between particles obeying infinite statistics produce a cancellation of vacuum fluctuations and a consequent disappearance of vacuum energy. Furthermore, as argued by both Ng and Medved [48-50, 46], the fundamental particles here must have extremely long wavelengths – of the order of the Hubble radius – making the scale of cancellation truly cosmological. Now, if we take the further step of associating vacuum energy with spacetime foam or fluctuations of geometry (including volume-fluctuations), as in [48-50], then the cancellation here includes a cancellation of volume-fluctuations, up to an uncancelled remnant $\Delta V \sim \sqrt{V}$ whose presence reflects the random character of these fluctuations. And as we have shown here, the spatial distribution of these remnant fluctuations is such that they collectively give rise to an effective $\Lambda$ in the form of a quantum potential of spacetime, with this $\Lambda$ satisfying the uncertainty relation $\Delta V \Delta \Lambda \sim 1$, for $\Delta V \sim \sqrt{V}$.

From a Machian standpoint, the use of infinite statistics is actually quite natural. For, in the first place, infinite statistics and holography are closely related [16]; in particular, the nonlocality associated with infinite statistics appears related to the nonlocality that is a feature of theories in which holographic relationships play a significant role [49]. Second, Mach's principle itself, with its attendant nonlocality, is expressible in holographic form [39]; and taken together, these considerations suggest that Machian nonlocality is realized, at least in part, through infinite statistics.

Finally, we note an additional possible source of Machian nonlocality. Consider the idea that Planck-scale spacetime, or spacetime "foam," is a gas or lattice of Planckian wormholes, with the constituent elements of spacetime structure (i.e. the causet elements, to adopt the standpoint of causal set theory) existing at the interstices of this lattice [52]. The presence of a lattice structure here is in conformity with the ideas presented in Part II, above; and the wormholes themselves can be viewed as establishing nonlocal relations or connections between widely separated causet elements. This kind of nonlocality has been described in detail by Requardt [53, 54], and in the present context it can be taken to produce a general cancellation of cause elements' volume-fluctuations. The scalar field $\psi = \sqrt{\rho} = R$ here is defined only at the lattice interstices or "points;" and the nonlocal fluctuation-cancellation that is due to the Planckian wormholes – and possibly also to the existence of quanta that obey infinite statistics – gives rise to the quantum potential Q of spacetime, as we have seen.

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