Symmetrically reproducing quark and lepton mass, charge, and generation

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It is shown that a particle set possessing electric charges, masses, and weak couplings that coincide with those of the quarks and leptons can be produced with the aid of the symmetry of the cuboctahedron. Specifically, it is shown that small powers of 4.1, in combination with the constants 0.1 and 3, are useful in economically reproducing the quark and lepton masses, and that these small powers—and thereby the masses they represent—can be joined automatically with their correct values for charge and generation with the aid of cuboctahedral symmetry.

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I. THE QUARK AND LEPTON MASS RATIOS

It is possible with the aid of the symmetry of the cuboctahedron to generate a particle set possessing charges, masses, and generations that coincide with those of the quarks and leptons [1]. Note that the twelve vertices of a cuboctahedron form a surface comprised of four hexagonal rings, six squares, and eight triangles. In Fig. 1 these vertices are labeled so as to assign values for G, Q, and R to each quark and lepton, where:

- $G = \{\ -1,\ 0,\ +1\ \}$, values that determine particle generation and thereby weak coupling.
- $Q = \{\ 0,\ -1/3,\ +2/3,\ -1\ \}$, the values for electric charge.
- $R = \{\ 0,\ 1,\ 1,\ 2,\ 3,\ 5\ \}$, values that will serve as parameters to a mass formula used to generate eight key quark and lepton mass ratios.

Notice that the cuboctahedron of Fig. 1 exhibits the following symmetry:

- Each of its four hexagonal rings contains values for R that equal $\{\ 0,\ 1,\ 1,\ 2,\ 3,\ 5\ \}$, values for G that equal $\{\ -1,\ -1,\ 0,\ 0,\ +1,\ +1\ \}$, and three quarks and three leptons.
- Each generation G contains the charges $\{\ 0, -1/3, +2/3, -1\ \}$.
- Each of its six squares has absolute values for Q that sum to 2.
- Each of its eight triangles has values of R that sum to 6.

The above observations demonstrate that the assignments of Fig. 1 are significantly constrained by symmetry. These assignments will now be used to generate a set of twelve particles whose masses closely approximate those of the quarks and leptons, where it is the symmetrical interplay of the values for generation G, charge Q, and the mass formula parameter R that will assure that all charges, masses, and generations are correctly joined.

The rule for weak coupling via generation G will be that particles can couple only if their values for G sum to 0. So Fig. 1 yields weak couplings of $u-d$, $b-t$, $s-c$, $e-\nu_1$, $\mu-\nu_2$, and $\tau-\nu_3$. But note that only μ and τ, or ν_2 and ν_3, can exchange *differing* values for G without affecting the various forms of symmetry. Either swap of values for G alters leptonic coupling to $\mu-\nu_3$ and $\tau-\nu_2$, flexibility that can be seen as reflecting maximal mixing between the $+1$ and -1 leptonic generations. In this way the experimental weak couplings are approximated by G.

Before generating the quark and lepton masses (or, more precisely, eight mass ratios), it is necessary to assign values to an additional parameter T. Let $T = 1$ for the u-, d-, c-, and b-quarks (which in the cuboctahedron of Fig. 1 reside in a single plane), while letting $T = 0$ for all other particles. Also let $m = 1$ for all heavy particles, and $m = 2$ for all light particles. These assignments, along with values for R taken from Fig. 1, allow the empirical mass formula

$$M\left(\Delta R, \Delta T, m\right) = 4.1^{\Delta R/m} \times 0.1^{\Delta T} \times 3^{1/m} \qquad (1)$$

to produce these mass ratios for heavy quarks and leptons

$$\begin{aligned} m_\tau/m_e &= M\left(R_\tau - R_e, T_\tau - T_e, 1\right) \\ &= 4.1^5 \times 3\ , \end{aligned}$$

$$\begin{aligned} m_\mu/m_e &= M\left(R_\mu - R_e, T_\mu - T_e, 1\right) \\ &= 4.1^3 \times 3\ , \end{aligned}$$

$$\begin{aligned} m_t/m_c &= M\left(R_t - R_c, T_t - T_c, 1\right) \\ &= 4.1^1 \times 3 \times 10\ , \end{aligned}$$

$$\begin{aligned} m_b/m_c &= M\left(R_b - R_c, T_b - T_c, 1\right) \\ &= 4.1^0 \times 3\ , \end{aligned}$$

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as well as their light particle counterparts

$$m_{\nu_3}/m_{\nu_1} = M\left(R_{\nu_3} - R_{\nu_1}, R_{\nu_3} - R_{\nu_1}, 2\right) = 4.1^{5/2} \times 3^{1/2} ,$$

$$m_{\nu_2}/m_{\nu_1} = M\left(R_{\nu_2} - R_{\nu_1}, R_{\nu_2} - R_{\nu_1}, 2\right) = 4.1^{3/2} \times 3^{1/2} ,$$

$$m_s/m_u = M\left(R_s - R_u, T_s - T_u, 2\right) = 4.1^{1/2} \times 3^{1/2} \times 10 ,$$

$$m_d/m_u = M\left(R_d - R_u, T_d - T_u, 2\right) = 4.1^{0} \times 3^{1/2} ,$$

which are readily summarized as follows

$$\left(\frac{m(\nu_3)}{m(\nu_1)}\right)^2 = \frac{m(\tau)}{m(e)} = 4.1^5 \times 3 , \quad (2)$$

$$\left(\frac{m(\nu_2)}{m(\nu_1)}\right)^2 = \frac{m(\mu)}{m(e)} = 4.1^3 \times 3 , \quad (3)$$

$$\left(\frac{m(s)}{m(u)} \times 0.1\right)^2 = \frac{m(t)}{m(c)} \times 0.1 = 4.1^1 \times 3 , \quad (4)$$

$$\left(\frac{m(d)}{m(u)}\right)^2 = \frac{m(b)}{m(c)} = 4.1^0 \times 3 . \quad (5)$$

II. COMPARISON AGAINST EXPERIMENT

Tables I–III reveal that these equations closely reproduce their corresponding experimental values, where for the tau- and muon-electron mass ratios the fit is especially precise: to roughly 1 part in 2,000 and 1 part in 40,000, respectively. However, for the neutrinos the following ratio between squared-mass splittings should also hold

$$\frac{m(\nu_3)^2 - m(\nu_1)^2}{m(\nu_2)^2 - m(\nu_1)^2} = \frac{4.1^5 \times 3 - 1}{4.1^3 \times 3 - 1} \approx 16.8 \quad . \quad (6)$$

Data exist for both of these splittings, namely [2]

$$|\Delta m^2_{31}| = 2.40 \times 10^{-3}\ \Delta \mathrm{eV}^2 \quad (7)$$

and [2]

$$|\Delta m^2_{21}| = 7.65 \times 10^{-5}\ \Delta \mathrm{eV}^2 , \quad (8)$$

which combine to yield

$$\frac{|\Delta m^2_{31}|}{|\Delta m^2_{21}|} = \frac{2.40 \times 10^{-3}\ \Delta \mathrm{eV}^2}{7.65 \times 10^{-5}\ \Delta \mathrm{eV}^2} \approx 31.4 \quad . \quad (9)$$

This ratio is about twice its calculated value and constitutes the sole major discrepancy.

TABLE I: The heavy lepton mass ratios of Eqs. (2) and (3) compared against experiment.

Ratio	Calculated	Experimental
m_τ/m_e	$4.1^5 \times 3 = 3475.686\ldots$	3477.15 ± 0.31 [a]
m_μ/m_e	$4.1^3 \times 3 = 206.763$	206.768 282 3 [b]

[a] Derived from the tau and electron masses taken from Ref. [3]. Fit to roughly 1 part in 2000.
[b] Ref. [4]. Fit to roughly 1 part in 40 000.

TABLE II: Equations (4) and (5) allow the b- and c-quark masses to be calculated from the top quark's experimental mass of 173 300 ± 1100 MeV [5]; below these calculated masses are compared against experiment.

Mass	Calculated	Experimental [a]
m_b	4227 ± 27 MeV [b]	4130 to 4370 MeV
m_c	1409 ± 9 MeV [c]	1180 to 1340 MeV

[a] Ref. [3]. It is important to recognize, however, that the experimental values for m_b and m_c are the "running" masses in the $\overline{\mathrm{MS}}$ scheme at $\mu = m_b$ and $\mu = m_c$, respectively.
[b] $173\,300 \pm 1100\ \mathrm{MeV}/(4.1^1 \times 3^0 \times 10) = 4227 \pm 27$ MeV.
[c] $173\,300 \pm 1100\ \mathrm{MeV}/(4.1^1 \times 3^1 \times 10) = 1409 \pm 9$ MeV.

TABLE III: The light quark mass ratios of Eqs. (4) and (5) compared against experiment.

Ratio	Calculated	Experimental [a]
m_s/m_d	$4.1^{1/2} \times 10 = 20.2\ldots$	17 to 22
m_u/m_d	$1/3^{1/2} = 0.577\ldots$	0.35 to 0.6
$\frac{m_s}{(m_d + m_u)/2}$	$25.6\ldots$	22 to 30

[a] Ref. [3].

III. SUMMARY AND CONCLUSION

It has been shown that the quark and lepton masses can be produced economically with the aid of an empirical mass formula that relies on small powers of 4.1, used in combination with the constants 0.1 and 3. It has also been shown that these small powers—and thereby the masses they represent—are readily joined with their correct charges and generations with the aid of the symmetry of the cuboctahedron, which also plays an important role in determining the values of these powers. All this supports the general conclusion that the mass formula's success derives from hidden symmetry among the quark and lepton charges, masses, and generatons. It is logical to ask what physics might underlie this mass formula and the symmetry it exploits.

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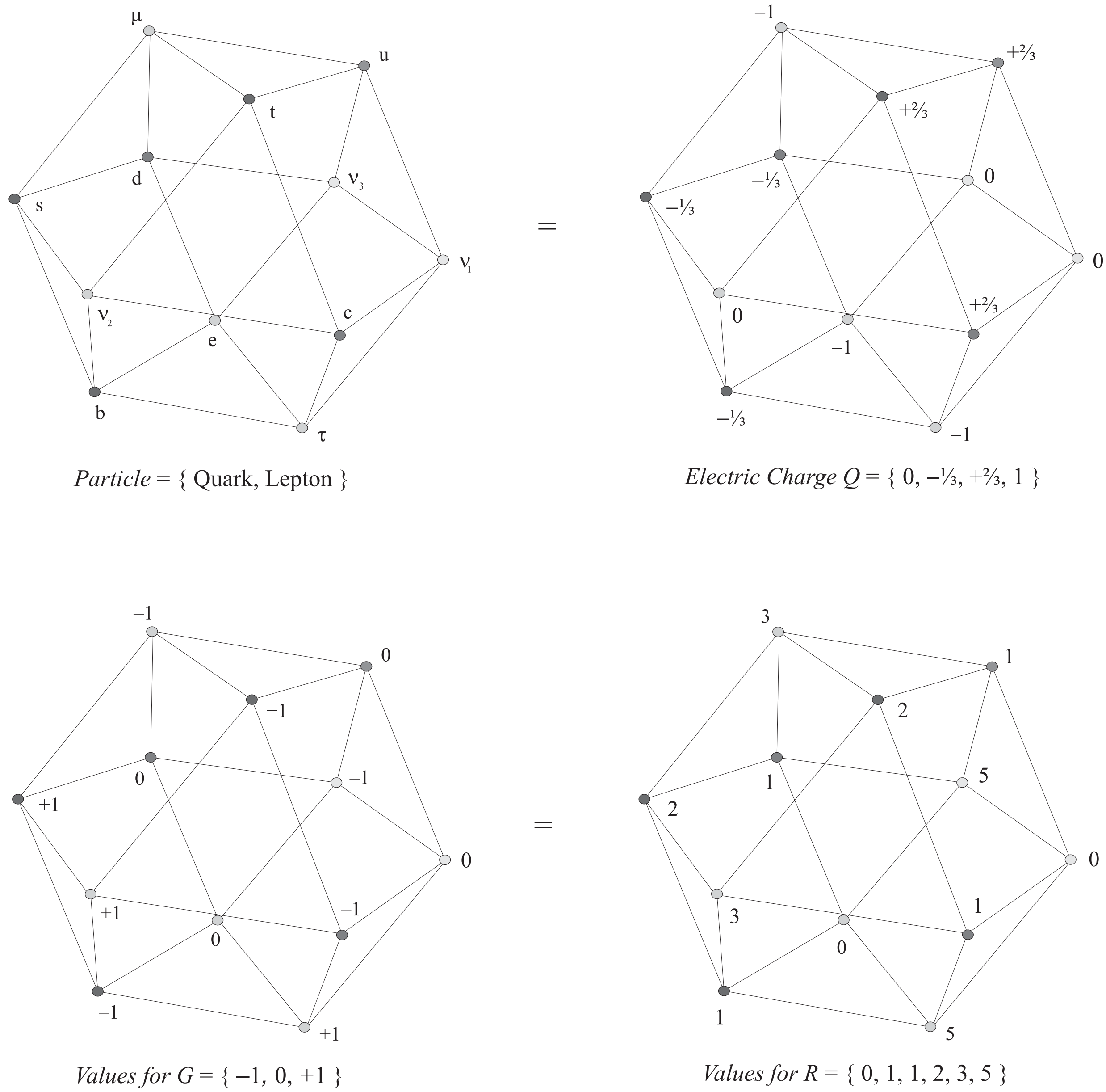

FIG. 1: A cuboctahedron is used to assign *en masse* values for generation G, electric charge Q, and mass formula parameter R to all quarks and leptons, where the particles on the upper left each get their values for G, Q, and R from their corresponding positions on the right and below. Note that the cuboctahedron's vertices define four hexagonal rings, each containing (a) three quarks and three leptons, (b) values for G that equal $\{-1, -1, 0, 0, +1, +1\}$, and (c) values for R that equal $\{0, 1, 1, 2, 3, 5\}$. The cuboctahedron's surface is comprised of six squares, each containing absolute values of Q that sum to 2 (at the upper right), and eight triangles, each containing values of R that sum to 6 (at the lower right). Moreover, each generation G contains charges of $\{0, -1/3, +2/3, -1\}$. The rule for how generation G governs weak coupling *to a first approximation* is that particles can weakly couple only if their values for G sum to 0. But note that, uniquely, only μ and τ, or ν_2 and ν_3, can swap *differing* values for G while (a) maintaining charges of $\{0, -1/3, +2/3, -1\}$ in each generation, and (b) maintaining values for G of $\{-1, -1, 0, 0, +1, +1\}$ in each hexagonal ring. Either swap alters leptonic coupling from $\mu - \nu_2$ and $\tau - \nu_3$ to $\mu - \nu_3$ and $\tau - \nu_2$, where this distinctive flexibility can be regarded as describing maximal mixing between the +1 and −1 leptonic generations. It is interesting to note that the quark and lepton distinction is 2-valued, G is 3-valued, Q is 4-valued, and R is 6-valued, where G, Q, and R *symmetrically co-reside* on the same cuboctahedron to reproduce, fairly accurately, the experimental couplings, charges, and masses of the quarks and leptons. It is, of course, a direct consequence of this symmetry that the correct charges and generations are joined with the correct masses. But symmetry does not merely see to it that the values for G, Q, and R correctly align: it also plays a major role in determining the values themselves. In particular, there are few interesting "6-valued labelings" of the cuboctahedron to choose from, with {0, 1, 1, 2, 3, and 5}—the first six Fibonacci numbers—appearing to be the simplest. Yet these are precisely the numbers required by Eq. (1), the mass formula, to produce the eight mass ratios of Eqs. (2)–(5). (But note that the first six Fibonacci numbers in reverse of their normal direction {−3, 2, −1, 1, 0, and 1} can be used to generate identical mass ratios if the sign of ΔR in Eq. (1) is toggled.)