A CASE AGAINST THE FIRST QUANTIZATION

Nicolae Mazilu, PhD
Silver Lake, Ohio 44224, USA
NicolaeMazilu@sbcglobal.net

Abstract. The blackbody radiation is an open problem, in the sense that there is no classical counterpart to describe the spectrum in physical terms. The usual derivation of Planck implicitly assumes that the spectral density is *a mean* for a special kind of exponential distributions. There is however a case where the classical statistics makes sense for the spectrum, if we consider the spectral density *as a probability density* for the values of the frequency at a certain temperature. We show here this case and illustrate it on contemporary COBE–FIRAS data, pointing out to a special tensor nature of the cosmic background radiation. Some conclusions about the measurement process and about the progress in general are drawn.

INTRODUCTION: A FORGOTTEN CASE

One usually tries not to identify the spectral density of radiation with a probability density, because of insurmountable mathematical problems arising with such a consideration. In general the Fourier analysis, which was the first approach of the spectrum of "complete radiation" signal, has no closed form solution here (Houstoun, 1914a,b). In the cases where a function can be found in order to approximate the spectrum, usually that approximation is either obliterated by the impeccability of Planck's spectrum or leads to physically unsound results, or both. One such case is the work of G. W. Walker (Walker, 1914a,b). In keeping with the oscillator structure of the matter Walker found a very close approximation of the blackbody spectrum, having all of the required physical characterisics. However, Walker's function is way under Planck's when it comes to fitting quality of the data. Moreover, it suggests that the physical structure explaining the "complete" radiation from mechanical point of view is a critically damped harmonic oscillator – an aperiodic structure. This is not in accordance with the idea of quantization of the energy levels in an atom – the assumed physical structure generating the radiation – and with the the concept of frequency closely connected to radiation.

There were, of course, in history, many cases explaining one side or another of the problem, however not explicitly addressed to considering *the spectral density as a probability density for frequency in the spectrum* – the natural inspiration brought about by the classical Boltzmann factor. In cases where the necessity occurred to consider a probability in founding the blackbody spectrum, that probability was downright Gaussian. For example in the classical case of Einstein and Hopf calculations (Einstein, Hopf, 1910) the probability density of the spectral components in the complete signal representing radiation was assumed to be a Gaussian, and the conclusion of calculations was that *Planck's spectrum is physically unsustainable*. As known, this raised immediate criticism on the validity of such a statistical assumption (see Varró, 2006; Irons, 2000 for the some points of view upon the history of quantization). However, *in a certain way* it is indeed sound enough to asume normality, in a statistical sense, of the spectrum of complete radiation when compared to data, and we try now to explain this in detail.

It is quite a regular statistical practice (see Efron, 1982) to use transformations of the statistics involved in the analysis of the data, in order to bring their distribution to a Gaussian form, and thus unveil their clean statistical content. Quite obviously – at least from mathematical point of view – the blackbody radiation spectrum can be considered as a probability density: it is the probability of finding a certain frequency at a given temperature in the radiation spectrum. The shape of the curve representing this distribution even looks like a probability density curve. It should come then as no surprise that it can be reduced to a Gaussian form by the transformation

$$\frac{\nu}{T} \rightarrow \sqrt[3]{\frac{\nu}{T}}$$

This fact was discovered in 1919 by Irwin G. Priest, who showed that the spectral density of black-body radiation can be closely represented this way as a Gaussian (Priest, 1919a,b).

The present author is not aware of any other mention of this exquisite representation in the literature he surveyed. Probably, not having any physical interpretation, the representation remained at the level of a statistical trick. Thus, the first sound thing to do was to check the truth of Priest's claim using, of course, the old original data (Lummer, 1900). The result was, indeed, amazing: just as Priest claims, the Gaussian fit is visibly better than the Planck function when referred to those data. But, for the present purpose we illustrate the issue with fresher, readily available data, even though obtained and handled under the spell of Planck distribution.

Priest's basic equation is written in the old manner, using the wavelength instead of frequency:

$$E_\lambda = D_1 T^5 e^{-D_2\left[A^{-1/3} - (\lambda T)^{-1/3}\right]^2}$$

Dividing here by the maximum spectral density at a given temperature – which is more convenient for fitting purposes – we have

$$\frac{E_\lambda}{E_m} = e^{-D_2\left[A^{-1/3} - (\lambda T)^{-1/3}\right]^2}$$

The notations are as follows: E_λ is the spectral density of radiation energy al wavelength λ, which has a maximum E_m; further on, A and $D_{1,2}$ are constants depending on the absolute temperature T. To bring a present-day example of the quality of this function in fitting the data, we used here the equation (41) in order to fit the COBE–FIRAS official data with it (Fixsen et al., 1996). In Figure 1 (at the end of paper) it is shown what Priest means by the fact that data represent a Gaussian curve: simply plotting the data points versus cubic root of frequency gives a symmetric curve. The line is the Gaussian of best fit using EXCEL's SOLVER. Even the dipole data seem to respect this symmetry (Figure 2). In Figure 3, the result of curve fitting is given for the monopole data, in terms of frequency. The two curves – Planck's and Priest's are, at least visibly, very close to one another. There are no reasons to reject Priest's proposal, for instance reasons of the nature of those that led to the rejection of the Rayleigh-Jeans or Wien's formulas as representing the whole spectrum of radiation. The only reasons of rejection would remain those of physical interpretation of the formula, in which respect Planck's formula has the benefit of statistical mechanics interpretation (Boya et al., 2000).

The things remained apparently with no consequence. As mentioned, we are not aware of the fact that Priest's observations have raised more than a momentary curiosity when they were communicated in 1919. It seems however that Priest himself thought otherwise. We quote:

> Equation (1) (*our equation (40) here, A/N*) has precisely the form of the well-known equation of the "probability curve" which suggests that the proposed equation *may have some significance, other than a mere empiric relation*. (Priest, 1919a; *our italics*)

and

> The author has no theoretical basis for proposing this equation; but recalling that the Planck equation was first an *empiric* relation to which theory was later *forced* to conform, he has thought that it might be worthy of notice and perhaps of some theoretical consideration by others. (Priest, 1919b; *italics in the original*)

Obviously, Priest attached a great physical significance to the fact that there is a transformation which reduces the blackbody radiation spectrum to a Gaussian. Perhaps he even tried to *force* some physical consideration based upon this, in the good old manner of Planck, but apparently with no success. "Others" did not seem to be interested either, so that we found the works of Priest quite accidentally. Priest's Gaussian gives us an experimentally legitimate "Boltzmann factor". In the case of classical Boltzmann factor there is the statistical idea of ensemble of a given energy at a certain temperature, which implicitly assumes that the temperature is related to a sufficient statistic. Here, however, the things get complicated: inasmuch as the "Priest factor" is revealed experimentally, one needs a physical explanation, because the statistical one is already at hand. Priest's Gaussian comes as a probability related to frequency, and reminds us that such a statistic is indeed necessary. In this direction the analysis reveals some suggestions to be added in the study of the blackbody radiation to the already existing ones.

THE MECHANICAL MODEL AND THE PHYSICS OF MEASUREMENTS

Let us first follow the logic along the idea that we need a statistic related to frequency, as we do in the case of temperature. This statistic can be justified and characterized as follows. Let's take the equation of a classical damped harmonic oscillator with constant coefficients, written in the form

$$M\ddot{q} + 2R\dot{q} + Kq = 0 \tag{1}$$

with obvious notations for the first and second time derivatives of the coordinate q. The solutions of this equation form a 2D manifold depending on three parameters. They can be written in the form

$$q(t) = e^{-\lambda t}(ze^{i(\omega t+\phi)} + z^*e^{-i(\omega t+\phi)}), \quad M^2\omega^2 \equiv MK - R^2, \quad M\lambda \equiv R \tag{2}$$

Therefore the solution of equation (1) represents actually *an ensemble of oscillators*, in which the element is identified by three parameters z, z* and $e^{i\phi}$, z* denoting the complex conjugate of z. One can say that the frequency – the mechanical frequency we know – might be somehow statistically related to this ensemble of oscillators, assuming of course that it is possible to find

such a statistics. The assertion can even be made more specific by showing that the frequency is invariant with respect to a certain group which "scans" the ensemble of oscillators.

Indeed, the ratio of any two linearly independent solutions of the differential equation (1), ξ say, is a solution of the following differential equation

$$\{\xi, t\} = 2\omega^2 \tag{3}$$

where the curly brackets denote the so-called Schwartzian derivative of ξ with respect to time, defined by

$$\{\xi, t\} \equiv \frac{d}{dt}\left(\frac{\ddot{\xi}}{\dot{\xi}}\right) - \frac{1}{2}\left(\frac{\ddot{\xi}}{\dot{\xi}}\right)^2 \tag{4}$$

The left hand side of equation (3) is invariant with respect to the linear fractional transformation (Needham, 2001)

$$\xi \leftrightarrow \xi' = \frac{A\xi + B}{C\xi + D} \tag{5}$$

with A, B, C, D four parameters, which we assume to be real. The set of all transformations (5) corresponding to all the possible values of these parameters is the group SL(2, R).

Thus the ensemble of all oscillators corresponding to the same frequency is in a one-to-one correspondence with the transformations of SL(2, R). This allows us to construct a "personal" parameter ξ for each oscillator of the ensemble, guided by the form of the general solution of equation (3). This solution can indeed be written as

$$\xi(\omega, t) = u + v \tan(\omega t + \phi) \tag{6}$$

where u, v and ϕ are constants, characterizing a given oscillator from the ensemble as before. Identifying the phase from (6) with that from (2), we can write that "personal" parameter of an oscillator in the form

$$\xi = \frac{z + z^* k \xi_t}{1 + k \xi_t}; \quad \xi_t \equiv e^{2i\omega t}; \quad \begin{matrix} z \equiv u + iv \\ z^* = u - iv \\ k \equiv e^{2i\phi} \end{matrix} \tag{7}$$

These equations reveal the fact that the parameter ξ is uniquely tied up with the specific oscilator from the ensemble, having the initial conditions represented by parameters u, v and ϕ. In other words, to a given set of three values (M, R, K) there is an associated ensemble of harmonic oscillators, scanned by a matrix with entries *depending exclusively on the initial conditions*. This matrix is

$$\begin{pmatrix} z^* k & z \\ k & 1 \end{pmatrix} \tag{8}$$

and can be considered as the representative of a damped harmonic oscillator from the ensemble corresponding to a given frequency. Our statistics should be conditioned by a certain measure defined on the set of such matrices. Let's build such a measure.

Start from the fact that ξ_t is unique for it, in exact the same way in which the thermodynamic equilibrium is uniquely defined by a certain energy density. If we agree that this can be expressed by the equation $d\xi_t = 0$, then we can write here the following Riccati equation for ξ:

$$d\xi = \Omega_1\xi^2 + \Omega_2\xi + \Omega_3 \tag{9}$$

giving this parameter as a function of initial conditions. The differential forms $\Omega_{1,2,3}$ can be now written as

$$\Omega_1 = \frac{kdz}{z - z^*}; \quad \Omega_2 = \frac{dk}{k} + \frac{dz + dz^*}{z - z^*}; \quad \Omega_3 = \frac{dz^*}{k(z - z^*)} \tag{10}$$

In real variables u, v, ϕ, these reveal a Bäcklund transformation of the hyperbolic plane (Rogers, Schief, 2002; Reyes, 2003):

$$\omega_2 = d\phi + \frac{du}{v},$$

$$\omega_1 = \cos\phi\frac{du}{v} + \sin\phi\frac{dv}{v}, \qquad \omega_3 = -\sin\phi\frac{du}{v} + \cos\phi\frac{dv}{v} \tag{11}$$

The discriminant of the quadratic polynomial from the right hand side of the equation (9) gives the following Lorentz metric

$$\omega_1^2 - \omega_2^2 + \omega_3^2 = -(d\phi + \frac{du}{v})^2 + \frac{(du)^2 + (dv)^2}{v^2} \tag{12}$$

which characterizes our ensemble of oscillators of the same frequency. The volume element of this metric is given by

$$\Omega_1 \wedge \Omega_2 \wedge \Omega_3 = d\phi \wedge \frac{du}{v} \wedge \frac{dv}{v} \tag{13}$$

and constitutes an a priori measure for any probability distribution. Of course, the space is not compact, so whatever statistic we may find has to be on finite intervals of the parameters.

The parameter ξ, which is a frequency, has indeed the statistical interpretation of the mean of an ensemble with quadratic variance distribution function (Morris, 1982). The Riccati equation (9) has the meaning of the equation of variance as a function of the mean, characteristic for a family of such distributions, whose parameter is the affine parameter of the geodesics of metric (12). The differential forms from equation (11) then represent some conservation laws in a geometrical sense.

However, in order to better grasp the object of such a statistic, we will quit the geometrical abstract path, and use here a kind of WKB method, inspired from that used in quantum mechanics. One of the practical definitions of the frequency in mechanically unassisted interpretation of measurements of signals is that of *instantaneous frequency*, i.e. the time derivative of the phase of signal (Mandel, 1974). If we define the recorded signal in the form

$$q(t) = A(t)e^{i\psi(t)} \tag{14}$$

where A is the instantaneous amplitude and ψ is the instantaneous phase, then the instantaneous frequency will be given by the first derivative of ψ. Now, we want to see to what extent this definition corresponds to our notion of physical frequency, as derived from the equation of the

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classical harmonic oscillator. Thus, if the signal is to represent a harmonic oscillator like the one represented by equation (1), then its amplitude and phase ought to satisfy the joint differential equations:

$$\frac{\ddot{A}}{A} + 2\lambda\frac{\dot{A}}{A} + \omega_0^2 = \dot{\psi}^2; \quad \frac{\dot{A}}{A} + \frac{\dot{\psi}}{2\psi} + \lambda = 0 \tag{15}$$

The first one of these equations is a definition of instantaneous frequency for our complex signal. Working the algebra related to the second equation (15), we get from the first equation the result:

$$\dot{\psi}^2 = \omega_0^2 - \lambda^2 - \frac{1}{2}\{\psi, t\} \tag{16}$$

This shows that there is a part of the frequency given by its mean, which is determined by mechanical properties of the oscillator (and therefore should contain information on those mechanical properties), and a part that may contain interacting properties – and thus may include ensemble characteristics as presented before – given by the Schwartzian derivative of the phase. That part can be associated with the bandwidth of the signal (see (Cohen, 1995) for details on the time-frequency analysis). Therefore the frequency *a priori defined* as instantaneous frequency, i.e. without knowing the supporting mechanical structure, contains indeed a part depending on the ensemble. One can read on equation (16) the fact that the instantaneous frequency is sharply defined by some material parameters only in cases where the phase depends on time through a linear fractional function:

$$\phi = \frac{\alpha t + \beta}{\gamma t + \delta} \tag{17}$$

where α, β, γ, δ are four constants. This is the classical case where the frequency is defined as the inverse of a period of time. In that case the amplitude equation in (15) becomes

$$\ddot{A} + 2\lambda\dot{A} + \lambda^2 A = 0 \tag{18}$$

giving the general solution

$$A(t) = (at + b)e^{-\lambda t} \tag{19}$$

This equation reveals a critically damped oscillator, like in Walker's work (Walker, 1914b) but only for the amplitude of our signal. The whole signal has its frequency modulated in time. Indeed, if we collect the results above in order to express the relevant mechanical coordinate as written in the formula (14) we have:

$$q(t) = (at + b)e^{-\lambda t}\exp\left\{i\frac{\alpha t + \beta}{\gamma t + \delta}\right\} \tag{20}$$

Suppose now that we have such a signal, and want to discover its physical structure, like in the old procedure of analyzing the complete radiation. The regular modern procedure to discover a material structure underneath a signal is to find its Laplace transform: this gives us the idea of that physical structure in terms of some mechanical parameters. The Laplace transform of this signal can be evaluated based on the example 3.324.1 from the Tables of Gradshteyn and Ryzhik (Gradshteyn, Ryzhik 1994). Taking, for convenience, a = b = 1, we get the following expression for the Laplace transform of the signal (20), *involving the square root of the frequency*:

$$\frac{\Delta}{\lambda + \omega} K_0\left((1+i)\frac{\sqrt{\Delta}}{\gamma}\sqrt{\lambda + \omega}\right) + \frac{\gamma^{3/2}\sqrt{\Delta}}{(\lambda + \omega)^{3/2}} K_1\left((1+i)\frac{\sqrt{\Delta}}{\gamma}\sqrt{\lambda + \omega}\right) \tag{21}$$

where $\Delta \equiv \alpha\delta - \beta\gamma$ is assumed positive, γ and λ are supposed to have the same sign and $K_{0,1}$ are the modified Bessel functions of order 0 and 1 respectively. This function is, of course, a far cry from the Priest's Gaussian, but still, it may reflect some other phenomena. For instance, it can give a distribution of the kind describing the electromagnetic perturbations from close disordered sources, like the microwave oceanic background. It is known indeed, that this kind of distribution is a limiting case of the Planck's negative binomial distribution (Jakeman, 1980, 1999).

AN ALTERNATIVE EXPLANATION

Coming back to our main subject – the "Priest factor" – the presence of the cubic root of frequency in Priest's Gaussian seems to point out in an entirely another direction even from the realm of the K-distributed noise. This direction is suggested by the definition of the frequency as a solution of the Schwartzian derivative in equation (6): it should be the root of an algebraic equation of degree three. This would mean that the frequency is the eigenvalue of a 3×3 matrix, rather than a scalar. In fact, when the Planck's formula was theoretically established, the frequency was considered as the magnitude of a Cartesian vector. Let's get into a few memories.

There doesn't seem to be here a better reference than the works of Timothy Boyer (Boyer, 1969) on the blackbody radiation. Boyer makes it quite obvious that, in deducing the Planck spectrum there are two distinctive theoretical moments. The most important of these, which caught almost exclusively the attention of the physicists, is that of description of an oscillator. When rounding up this description in modern terms, Planck's contribution can be characterized as follows. Let's denote by u the spectral density of energy of radiation, as measured in a certain way. This density fluctuates around a certain value u_0, which is the value of density at thermal equilibrium. Let's characterize the fluctuations of energy density by the equation

$$u_0 = \frac{Au + B}{Cu + D} \tag{22}$$

where the parameters A, B, C, D depend of the apparatus of measurement. The measurement process can thus be physically characterized as a kind of "evolution" of the measurement apparatus in order to accommodate the spectrum energy fluctuations. Specifically the three independent ratios A:B:C:D, characterizing the physical structure of the apparatus, vary in such a way that the thermal equilibrium value of the spectral density of energy is maintained constant. Then equation (22) can be put in a Riccati differential form

$$du = \Omega_1 u^2 + \Omega_2 u + \Omega_3 \tag{23}$$

where $\Omega_{1,2,3}$ are three differential 1-forms in the parameters A, B, C, D. The equation (23) represents, in cases where the forms $\Omega_{1,2,3}$ are exact differentials in one parameter, the defining condition of a family of distributions with quadratic variance function. Planck's arguments is based on a particular case of the Riccati equation (23), where the parameters A, B, C, D depend

continuously *on the inverse of the equilibrium temperature*. Mention should be made that the algebra characterizing the three differential 1-forms from equation (23) is that of the group SL(2, R). Therefore Planck's arguments are related to the evolution along the geodesics of the hyperbolic plane, whose "time parameter" is the inverse temperature.

The second theoretical moment in deducing the blackbody radiation spectrum, amounts to "counting the oscillators" having the frequency located in a certain frequency interval. Here, everybody, on the classical as well as on the quantal side, agrees that *the frequency has to be a vector*, just like the wavelength. This is how the Rayleigh-Jeans law of radiation came up, as an expression of the classical equipartition of the energy. As well-known, this radiation law couldn't possibly represent the whole spectrum of the blackbody radiation, because it is in contradiction with the theoretically well-established Stefan-Boltzmann law of the whole radiation. And so the quantum hypothesis made its way into physics.

We would like to say that Priest's proposal is germane to this second moment in the theoretical definition of the blackbody radiation spectrum, whereby the distribution characterizing the radiation is a Gaussian. This time, however, the proposal cannot be rejected, like the Rayleigh-Jeans one, or any other proposal for that matter, inasmuch as the representation of data is exquisite, *over the whole frequency range*. All it can be objected is the fact that we don't have a physical interpretation for the cubic root of the frequency and that is, indeed, a challenge.

However, we have a hint, in the fact that equation (6) has the form of the root of a third degree polynomial with real roots. This would suggest that the *frequency is not a vector*, like in the classical case, *but a tensor*, whose eigenvalue is (6) and two other like values, differing by a phase from each other. They still can have a physical interpretation in terms of the harmonic oscillator, as follows. If one considers the equation (1) as merely a mechanical model of a physical structure which represents what is experimentally recorded, then that equation should be taken a little bit differently. Specifically, whatever we take into consideration as frequency is given, in the good classical manner, by the two roots of the characteristic equation of (1). These are

$$\nu_+ \equiv -\lambda + i\sqrt{\lambda^2 + \omega_0^2}; \quad \nu_- \equiv -\lambda - i\sqrt{\lambda^2 + \omega_0^2} \tag{24}$$

We don't know how to define the theoretical frequency otherwise, but by the means of the harmonic oscillator. However, what we are actually looking for *is the energy of an electromagnetic field*, i.e. a quantity related to the eigenvalues of the corresponding Maxwell tensor. Usually, the component fields are assumed to be directly measured (Wilhelm, 1985, 1993) in such instances. But, at least for the case of thermal radiation, Priest's result suggests that the complex frequencies from equation (24) serve in construction of something else which is measured, in some connection with those real eigenvalues. This "something" can still be theoretically described by a tensor, whose eigenvalues are given by the following algebraic recipe:

$$\sqrt[3]{\nu_+} + \sqrt[3]{\nu_-}; \quad \varepsilon\sqrt[3]{\nu_+} + \varepsilon^2\sqrt[3]{\nu_-}; \quad \varepsilon^2\sqrt[3]{\nu_+} + \varepsilon\sqrt[3]{\nu_-} \tag{25}$$

The data then seem to show simply that a *certain algebraic combination of these eigenvalues is normally distributed*, which is quite a regular condition in statistical physics. Specifically which combination we have in mind, can actually be shown very simply by the following procedure of V. V. Novozhilov, who was referring originally to the stress tensor of continuum mechanics (Novozhilov, 1952).

When we measure a tensor, we actually measure its components on a plane in space, and normal to that plane. If the process represented by that tensor is spatially isotropic, as the cosmic background radiation appears to be, then we can safely assume that the measurement *in a point in space* yields actually two averages: one of the normal components of the tensor on all planes through that point, the other of the corresponding plane components of that tensor. If the eigenvalues of our tensor are denoted by $r_{1,2,3}$ then the two averages are

$$\frac{r_1 + r_2 + r_3}{3}; \quad \sqrt{\frac{(r_2 - r_3)^2 + (r_3 - r_1)^2 + (r_1 - r_2)^2}{15}} \tag{26}$$

Applying this recipe to our eigenvalues from equation (25), the average of normal components is zero, while the average in-plane components is

$$\sqrt{\frac{2\sqrt[3]{2\lambda^2 + \omega_0^2}}{5}} \quad \xrightarrow{\lambda \to 0} \quad \sqrt{\frac{2}{5}}\sqrt[3]{\omega_0} \tag{27}$$

According to this scenario the data for cosmic background radiation – and perhaps for the thermal radiation in general – tell us that this radiation is an electromagnetic field mechanically represented by a slightly damped harmonic oscillator, whose equilibrium thermodynamics is dictated by a "superstatistic" which is Gaussian in the isotropic average (27). The mean and variance of this distribution are determined by the equilibrium temperature, according to Wien's displacement law, as revealed by Irwin G. Priest. Mention should be made that such a result can very well be specific to the way the measurements of the radiation are usually made, i.e. by a bolometer.

CONCLUSIONS

Considering Priest proposal, the classical conclusion of Einstein and Hopf would have clearly changed, for the premise would be different. Specifically, knowing the spectral density as a function of frequency in the complete signal, one can only ask what is the physical structure sustaining the thermodynamic equilibrium. There would be no doubt on the law of radiation. If we assume the classical relation between a signal and its Fourier transform, then the Priest's function is actually the square of the Fourier transform of the signal. This means that the complex function representing the signal had to be estimated by an integral of the form

$$A \int_{-\infty}^{+\infty} e^{-a(\tau - b)^2 + i\tau^3 t} d\tau; \quad \tau \equiv \sqrt[3]{\nu} \tag{28}$$

where t is the time, ν is the frequency and A, a, b are three constants depending on the equilibrium temperature. This integral has obviously a close connection with an Airy function, whose representation is

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$$\text{Ai}(z) = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} e^{i(z\tau + \frac{\tau^3}{3})} d\tau \tag{29}$$

Indeed, the exponent in (28) can be put in the form

$$-a(\tau - b)^2 + i\tau^3 t \equiv i\left(\frac{\xi^3}{3} + z\xi + B\right) \tag{30}$$

with the following identifications

$$\xi = (3t)^{1/3}\left(\tau + i\frac{a}{3t}\right); z = \frac{a}{(3t)^{1/3}}\left(\frac{a}{3t} - 2ib\right); B = -\frac{2a^2 b}{3t} + i\left(b^2 - \frac{2a^3 b}{3(3t)^2}\right) \tag{31}$$

With these, the equation (29) becomes

$$A\int\limits_{-\infty}^{+\infty} e^{-a(\tau - b)^2 + i\tau^3 t} d\tau = \frac{2\pi A}{(3t)^{1/3}} \cdot \exp\left(-b^2 + \frac{2a^3 b}{3(3t)^2} - i\frac{2a^2 b}{3t}\right) \cdot \text{Ai}\left(\frac{a}{(3t)^{1/3}}\left(\frac{a}{3t} - 2ib\right)\right) \tag{32}$$

These are quite rough results, but they show that, certainly, the blackbody radiation is in the natural classical order of things, at least from a measurement point of view. Indeed, the Airy function has everything to do with the light intensity measurements in the neighborhood of the caustic (Airy, 1838, 1848). Therefore we would have some contemporary theoretical things ahead of time. The Airy packets, for instance, would have been introduced perhaps way earlier than it was done (Berry, Balasz 1979) and probably the recent discoveries of the Airy beams would be communicated way earlier. By the same token we would have discussed today in some other terms about lasers, the coherent states and the correlation between the Schrödinger equation and the heat equation (Albeverio, Høegh-Krohn, 1974), to mention only those subjects having an explicit connection with the Airy function. The heat equation itself had perhaps to be of third order in space variables (Widder, 1979), and probably the Schrödinger equation would too. Anyway, the issue would have certainly very much impacted the study of stochastic Airy processes (for a recent review of such processes see (Majumdar, Comtet, 2004)).

Obviously, Priest attached a great physical significance to the fact that there is a transformation which reduces the blackbody radiation spectrum to a Gaussian. Perhaps he even tried to *force* some physical consideration based upon this, in the good old manner of Planck, but apparently with no success. The sketchy calculation above, and the awareness of the importance of Airy function in optics, quantum mechanics, stochastic processes etc., forced us to pay a closer attention to the moments related to the development of the theory of blackbody radiation. If Priest's Gaussian gives a "Boltzmann factor" allowing us to repeat the previous streak of reasoning, a problem here still remains: because the spectrum is given by measurements, one needs to describe the physical structure *based on it*. In the case of classical Boltzmann factor there is statistical idea of ensemble of given energy at a certain temperature. Here, however, the things get complicated: because the "Priest factor" is revealed experimentally, one needs a physical explanation, because the statistical one is already at hand. Priest's Gaussian comes as a probability directly related to the mechanical frequency, which is itself a sufficient statistic just like the temperature, and reminds us that such a statistic is indeed necessary. In this direction the

analysis above reveals some suggestions to be added to the already existing ones, but raises an important question.

The results listed above in connection to the subject "Airy" are of quite recent date, and have come to light, in a way or another, as a result of quantization. The suggestion springing from our analysis is that the quantization could very well be bypassed, and we still would have those results. The question is: was quantization objectively necessary as today's textbooks show?

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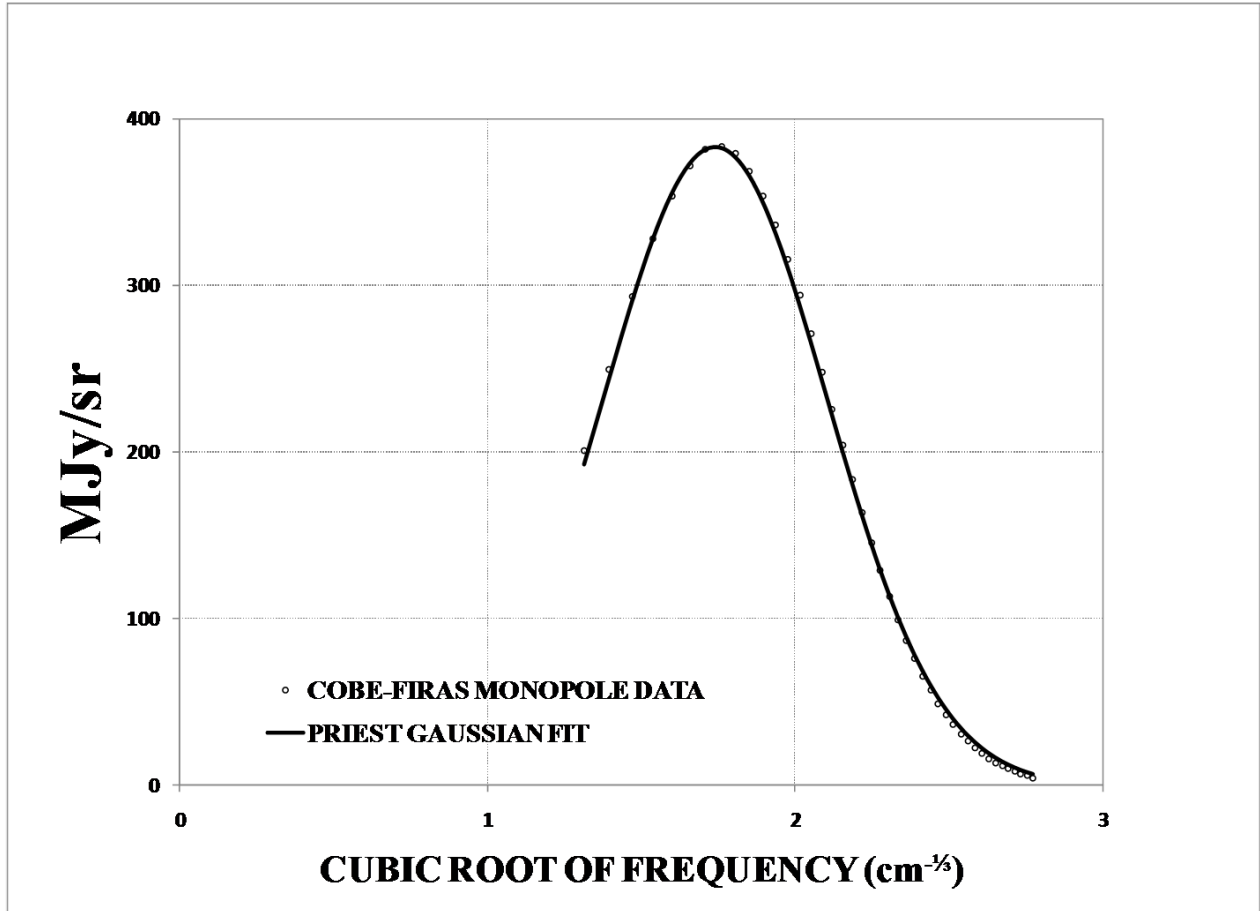
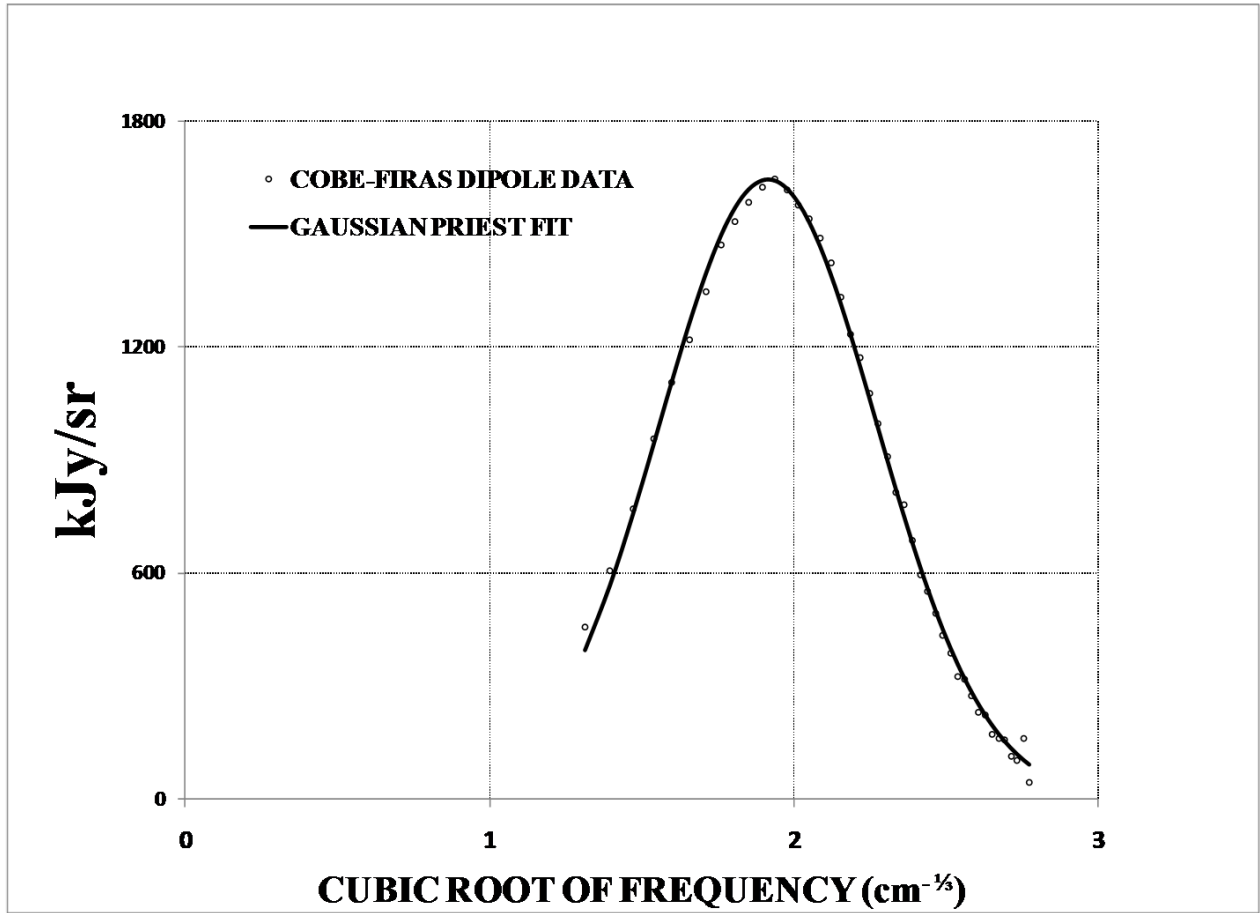**Figure 1. Priest's Gaussian representation of CMBR monopole data**

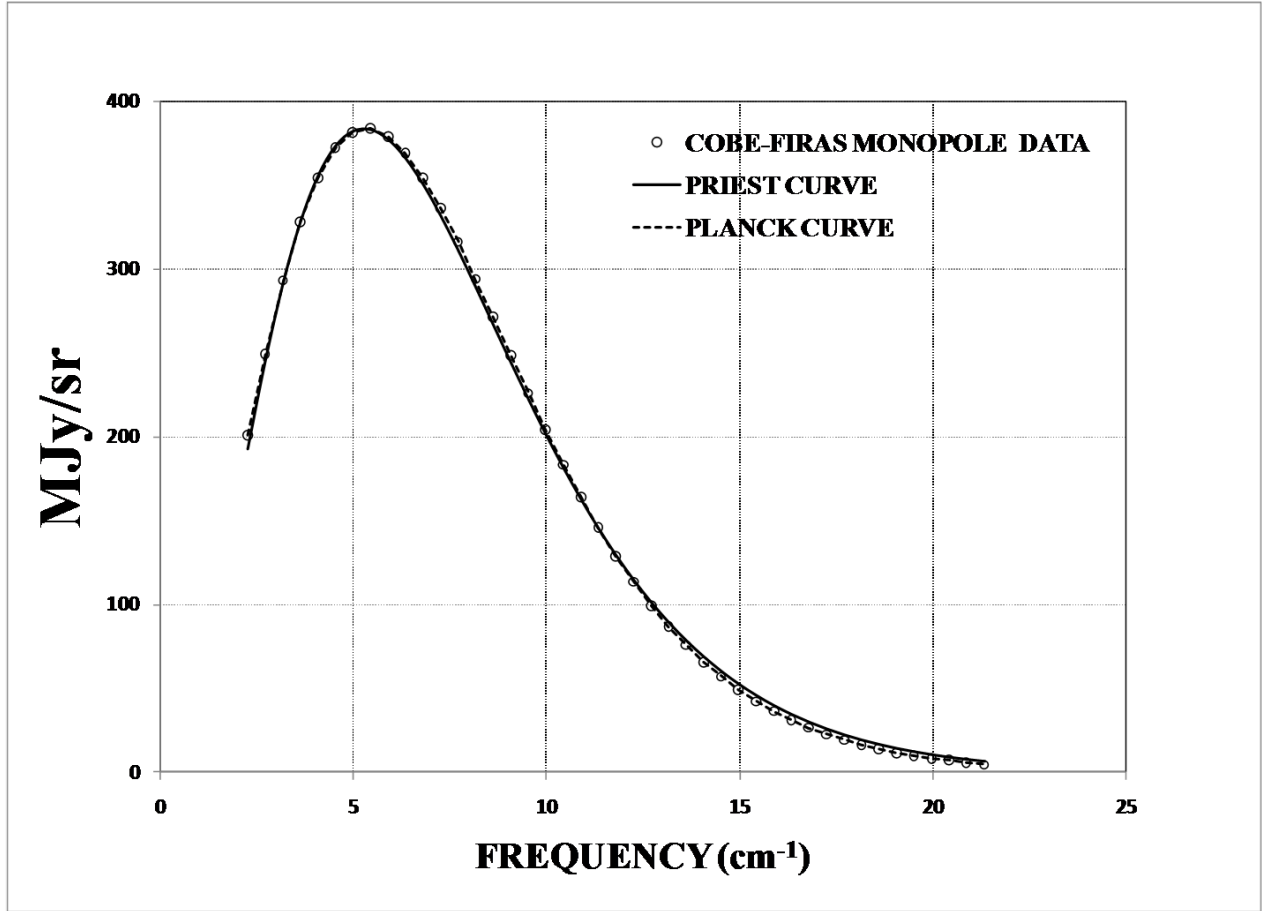**Figure 2. Priest's Gaussian representation of CMBR dipole data**

Figure 3. Priest's and Planck's representations of CMBR monopole data