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The list of references ;" My Numbers, My Friends Popular Lectures on Number Theory" (By Paulo Ribenboim)
"Prime Numbers The Most Mysterious Figures in Math" (By David Wells)
& some on-line blogs on math.

On a New Method of Purely Generating Prime Numbers and the Application of It to the ' Goldbach's Conjecture'

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《Preface》

There seems to be no perfect method of the generation of primes by using the explicit functions which many researchers have tried to utilize but proved to be of no practical use.

But here, we will find the genuine mathematical expressions which can generate pure complete prime numbers without mixing non-prime numbers that can be factorized into primes(such numbers as 25=5×5 or 57=19×3).

《 How can it be possible to generate 'pure' primes? 》

To do so, we must introduce numbers called 'complementary numbers' to show above mentioned things.

What are 'complementary numbers' ? 'Complementary numbers' are the numbers that can not be expressed as values that can be calculated and expressed by the 'explicit functions', or also can be defined as the natural numbers which remain intact after we remove the values created by the 'explicit functions' from all of the natural numbers(1,2,3,4,5………to ∞).

Then, what sort of 'explicit functions' should we prepare ?

We call the explicit functions 'dual mathematical expressions' defined as below;

$$f(p,q) = 2pq + p + q \quad \cdots\cdots① $$
$$g(p,q) = 2pq + p + q + 1 \quad \cdots\cdots② $$

(p , q ; natural numbers)

Easily observed, there exist relations between f and g mentioned below.

$$2\times f+1=(2p+1)(2q+1) \quad \cdots\cdots③$$
$$2\times g-1=(2p+1)(2q+1) \quad \cdots\cdots④$$
$$f+g=(2p+1)(2q+1) \quad \cdots\cdots⑤$$
$$g-f=1 \quad \cdots\cdots⑥$$

What are meant by the above expressions? We answer in the following way.

For example, if we substitute $p=5$ and $q=7$ into the expression ①, we obtain $f=82$. Therefore from ③ we can get $2\times 82+1=165$, and when we substitute p=5 and q=7 into the right side of the expression ③, we can get $(2\times 5+1)\times(2\times 7+1)=165$. This is an evident identity.

But if we think about the number that can not be expressed by the right side of ①, for example, 20. We can have the expression $2\times 20+1=41$, which means that we obtained the value that cannot be factorized; i.e. we obtained the 'prime number'.

The fact above is quite a matter of course.

Originally 'f' is the function of p and q, therefore so long as 'f' is expressed with p and q, if we double the number and add 1, according to the expression ③, then factorization becomes possible.
But if we double and add 1 to the number that absolutely cannot be expressed by the expression ①, we can not have the number that can be factorized in the way of the expression ③.

Therefore we can have the number that can not be factorized into primes; i.e. hereby we declare "the prime number is purely generated!"

Consequently we call the number that cannot be expressed by the expression ① like above mentioned 20 'the complementary number'.
We describe the number or a set of these numbers as f^c.

Let us describe the set of 'complementary numbers' f^c in the ascending order. 《See Note 1.》

1,2,3,5,6,8,9,11,14,15,18,20,21,23,26,29,30,33,35,36,39,41,44,48………………

Concerning above numbers, if we double the numbers and add 1, then we can find the odd prime number series (from 3 to 97) is generated without mixing any numbers that can be factorized.

Generally,

$(2\times f^c+1)$ ……………………………………………⑦

will generate all the odd prime number series, and the nature of which is that if f^cs are even numbers, then they will generate type $(4k+1)$ prime number series, and if f^cs are odd numbers, then they will generate type $(4k-1)$ prime number series.

(The reason above is that if we substitute $(2k+1)$ into f^c, then we can see the expression ⑦ showing type $(4k+3)$ prime number series, which is just equal to the type $(4k-1)$ prime number series. 《See Note 2.》

Let us think about another 'complementary number' g^c regarding the function $g=g(p,q)$, according to the same procedure mentioned above.

When we arrange the numbers in the ascending order, we can have the following number series of g^c;

1,2,3,4,6,7,9,10,12,15,16,19,21,22,24,27,30,31,34,36,37,40,42,45,49…………….

Removing the first number 1, we can generally obtain all the prime number series (except the number 2) by the following expression;

$(2\times g^c-1)$ ……………………………………………⑧

In this case, if 'complementary numbers' g^cs are even numbers, then we can have type $(4k-1)$ prime number series, and if g^cs are odd numbers, then we can have type $(4k+1)$ prime number series.

《 About the 'Goldbach's Conjecture' and the method of describing even numbers 》

First of all, we ascertain the 'Goldbach's Conjecture' before we give the demonstration of it.

The 'Goldbach's Conjecture' insist that all the even numbers not less than 6 can be divided into two odd prime numbers (not less than 3); for example $6=3+3$, $8=5+3$ $12=7+5$.

We call the above division 'Goldbach's decomposition' and in order to demonstrate it, we divide the even numbers into type(A) and type(B).

Type(A) is the twice even numbers, i.e. 4,8,12,16,20,24…….
Type(B) is the twice odd numbers, i.e. 6,10,14,18,22,26…….

Type(A) can be described as $4M$ mathematically.
Type(B) can be described as $4M+2$ mathematically.

(M; natural numbers)

Then concerning type(A) we can have the following expressions;

$$4M=(2M+2n+1)+(2M-2n-1) \quad \cdots\cdots ⑨$$

or

$$4M=(2M+2n-1)+(2M-2n+1) \quad \cdots\cdots ⑩$$

(n; non-negative integers)

and concerning type(B);

$$4M+2=(2M+2n+1)+(2M-2n+1) \quad \cdots\cdots ⑪.$$

We can describe all the even numbers by the expressions ⑨ through ⑪.

《 Concerning the 'decomposition into complementary numbers' 》

Here, we substitute the complementary number f^c into $(M+n)$ of $(2M+2n+1)$ and $(M-n)$ of $(2M-2n+1)$ in the expressions ⑨ through ⑪, while substituting g^c into $(M+n)$ of $(2M+2n-1)$ and $(M-n)$ of $(2M-2n-1)$ in the expressions ⑨ and ⑩. What happens if 2M can be divided into two 'complementary numbers' ? (This division of even numbers into two complementary numbers is what we call the 'decomposition into complementary numbers'.)

For example, if we choose $50(=2M)$, we can divide the number into $48(=M+n;\ f^c)$ and $2(=M-n;\ g^c)$. According to the expression ⑨, we can obtain $100(=4M)=(2\times48+1)+(2\times2-1)=97+3$.

If we choose 50 again, as $50=36\ (=M+n;\ g^c)+14\ (=M-n;\ f^c)$, we can get $100(=4M)=(2\times36-1)+(2\times14+1)=71+29$, according to the expression ⑩.

If we also choose $50=29(=M+n)+21(=M-n)$ (both f^c s), we can get $4M+2=102=(2\times29+1)+(2\times21+1)=59+43$, according to the expression ⑪.

We can find out the fact that 'decomposition into complementary numbers' will lead to the 'Goldbach's decomposition'.

We refer to the most credible following examples about the fact mentioned above.

When $4M=100$, there are 6 ways of 'Goldbach's decomposition'. In the case, complementary numbers f^c s and g^C s are following;

f^c	g^C	('Goldbach's decomposition')
26	24	(53 and 47)
44	6	(89 and 11)
20	30	(41 and 59)
14	36	(29 and 71)
8	42	(17 and 83)
48	2	(97 and 3)

Different complementary numbers (though 2M=50) are;

		with the same 'Goldbach's decomposition'
f^c 41	g^C 9	(83 and 17)
35	15	(71 and 29)
29	21	(59 and 41)
23	27	(47 and 53)
5	45	(11 and 89)
1	49	(3 and 97)

We can see the both ways of 'decomposition into complementary numbers' with the same values of 'Goldbach's decomposition'.

Confirming the above fact that 'Goldbach's decomposition' is possible when an arbitrary even number can be divided into two 'complementary numbers' , we see the demonstration of the possibility of 'the decomposition of arbitrary even numbers into complementary numbers' in the next chapter.

《Regarding the demonstration of the possibility of the decomposition of even numbers into complementary numbers》

【Preliminary consideration】

Now we classify the cases where f and f^c are included and 2M is divided into (M+n) and (M−n).

case 1 (M+n) ∈ f and (M−n) ∈ f
case 2 (M+n) ∈ f^c and (M−n) ∈ f
case 3 (M+n) ∈ f and (M−n) ∈ f^c
case 4 (M+n) ∈ f^c and (M−n) ∈ f^c

We give concrete examples of the decomposition of the even number 2M=100 into complementary numbers as follows.

In the case 1, we have
(M+n) =60、(M−n) =40 (n=10),
in the same case (M+n) =72、(M−n) =28 (n=22),
in the case 2 (M+n) =54、(M−n) =46 (n= 4),
in the same case (M+n) =63、(M−n) =37 (n=13),
in the case 3 (M+n) =52、(M−n) =48 (n= 2)
in the same case (M+n) =59、(M−n) =41 (n= 9),
in the case 4 (M+n) =56、(M−n) =44 (n= 6),
in the same case (M+n) =86、(M−n) =14 (n=36).

Concerning the values of fs and f^cs, we can see their distribution as follows; the distribution of fs becomes dense as the value of M becomes large, but the distribution of f^cs becomes sparse.

Therefore, we can see the case 1 through 3 easily hold good, but the case 4 does not easily hold good.

Nevertheless, we can see the case 4 hold good by choosing the value of n in the following way.

【The demonstration concerning f s and complementary numbers f^cs】

We assume the value of M to be arbitrary, and exclude the case 1 ,where there are no values of complementary numbers f^cs.

In the cases 2 through 4, we assume that case 4 does not hold true and there are only cases of 2 and 3 if the value of M is large enough. 《See Note 3》

We demonstrate that the case 4 is true by employing the 'indirect method of the demonstration' or the proof by contradiction (absurdity) even if the value of M becomes so large.

Now if we assume $(M+n) \in f^c$ in the case 2 and define $Q=(M+n)$, then the value of $(M-n)$ becomes $(Q-2n)$.
Or when we assume $(M-n) \in f^c$ in the case 3, and if we define $P=(M-n)$, then the value of $(M+n)$ becomes $(P+2n)$. Therefore, the values of $(P+2n)$ and $(Q-2n)$ are to be given by $f=f(p,q)=2pq+p+q$.

But the values of $(Q-2n)$ and $(P+2n)$ can not always be given as values determined by $f=f(p,q)=2pq+p+q$.
Therefore, through the change of the value of n, $(Q-2n) \in f^c$ or $(p+2n) \in f^c$ becomes possible. 《See Note 4》

Consequently, the assumption that there only exist the case 2 and case 3 where either $(M+n)$ or $(M-n)$ always takes the value of f is wrong, and the case 4 where $(M+n)$ and $(M-n)$ can take both values of f^cs exists by choosing the value of n.
As the result of the above, $(M+n) \in f^c$ and $(M-n) \in f^c$ can firmly hold true, which means that the 'decomposition into complementary numbers' is possible concerning $2M=(M+n)+(M-n)$.

Q.E.D.

【The demonstration of the possibility of the 'decomposition of 2M into complementary numbers' concerning f^c, g and g^c.】

We can also give the demonstration that 2M can be described as the sum total of the values of f^c and g^c in the same way as mentioned above when the value of M is arbitrary.

We treat the cases where f^c, g and g^c are included in the expressions, but the cases where f^c is included only in either $(M+n)$ or $(M-n)$ are described in the following way.

case 5 $(M+n) \in f^c$ and $(M-n) \in g$
case 6 $(M+n) \in g$ and $(M-n) \in f^c$
case 7 $(M+n) \in g^c$ and $(M-n) \in f^c$
case 8 $(M+n) \in f^c$ and $(M-n) \in g^c$

We can easily see the cases of 5 and 6 hold good as the value of M becomes large (more than some hundred) by referring to Note 3. Now we assume that only the cases 5 and 6 hold good as the value of M becomes large, but if we employ the 'indirect demonstration' mentioned before, $(M-n)=Q-2n$ in the case 5 and $(M+n)=P+2n$ in the case 6 can not always take the value of $g=g(p,q)=2pq+p+q+1$ with the arbitrary value of n.

Therefore the assumption is denied. As the result of it, the cases 7 and 8 must hold good when the appropriate value of n is chosen.

Consequently we can see $(M+n) \in f^c$ or g^c and $(M-n) \in f^c$ or g^c, which means that the decomposition of arbitrary 2M into complementary numbers is possible if we choose the appropriate value of n.

Hence we can give the demonstration to the 'Goldbach's conjecture' that even numbers not less than 6 can be always divided into two prime numbers.

Q.E.D.

【On the stricter demonstration employing the "Ｅｒｄöｓ' theorem"】

The demonstration above , however, does not use the 'existence theorem' of prime numbers, does not consider the 'range' of them and does not investigate the guarantee of the existence of complementary numbers especially in the case where $M\to\infty$, lacking in the strictness.

Here, we will give the stricter demonstration, with mathematically logical consistency with the 'existence theorem' of prime numbers.
We give the final mathematical demonstration, employing the existence theorem i.e. the "Ｅｒｄöｓ' theorem".

The "Ｅｒｄöｓ' theorem" is that there exist at least one $(4k+1)$ prime number and one $(4k-1)$ prime number between the range $m(m>6)$ and $2m$, and the larger the value of m becomes, the more prime numbers exist in the same range.

The greatest problem of the last chapter is that there is no consideration of the range of the existence of prime numbers, the arguments are whether they can be effective only when the value of M is small enough, and not effective in the case where $M\to\infty$.
We must answer the above questions.
(These arguments have been discussed from the old days, and there is no solution by the computers' analysis, some mathematicians still doubt the Goldbach's conjecture.)

Now we define M as large enough and almost $M\to\infty$, we investigate $(M+n)=P+2n\in f^c$ or g^C is guaranteed if we take $P=(M-n)\in f^c$ or g^C .

According to the "Ｅｒｄöｓ' theorem", from the range M to 2M, from $M/2$ to M, from $M/4$ to $M/2$, from $M/8$ to $M/4$, from $M/16$ to $M/8$……
(in short from zero to 2M) , there is the guarantee of the existence of type $(4k+1)$ primes and type $(4k-1)$ primes.

While M is the large number, there is the guarantee of the existence of f^c or g^C between the range from $M/2$ to M, from $M/4$ to $M/2$, from $M/8$ to $M/4$, from $M/16$ to $M/8$, from $M/32$ to $M/16$...... (in short from zero to M) ,according to the application of the "Erdös' theorem", we take one of the numbers as $P=(M-n) \in f^c$ or g^C . (We diminished the range by half.)

On the other hand, $(M+n)=P+2n$ is not less than M and smaller than $2M$. (This is because $(M+n)=2M-P$, $P=(M-n)>0$ and $n \geqq 0$.)

So long as there exists the guarantee of the plural existence of type $(4k+1)$ and $(4k-1)$ prime numbers in the range from $2M$ to $4M$, by the same reason mentioned before, there exist f^cs or g^C s which satisfy $M \leqq f^c$ or $g^C < 2M$ that is the same range mentioned above. (We diminished the range by half.)

As we refer to Note 5, concerning the values of $(M+n)$ and f^c or g^C, if there is the guarantee of the existence of f^c or g^C which satisfies $(M-n) = P \in f^c$ or g^C in the range $0 < f^c$ or $g^C \leqq M$, there is the guarantee of satisfying $(M+n)=Q=P+2n \in f^c$ or g^C in the same range from M to $2M$ mentioned above, through the choice of the value of n. 《 See Note 5》

Therefore the doubt if there are no guarantees of the possibility of the existence of the number $(M+n) \in f^c$ or g^C in the range from M to $2M$ where $M \to \infty$ or if $M \to \infty$ then the existence itself becomes nothing and the 'Goldbach's conjecture loses ground will be wiped out.

Hence, even in the case where $M \to \infty$, according to the "Erdös theorem", concerning every arbitrary value of $2M$, $(M+n) \in f^c$ or g^C and $(M-n) \in f^c$ or g^C i.e. the 'decomposition into complementary numbers' is perfectly proved to be possible. 《See Note 6》

Here we come to the conclusion that the 'Goldbach's conjecture' can be said to be totally demonstrated.

Q. E. D.

Note 1

Numbers that can be calculated by f=f(p,q)=2pq+p+q (less than 50)

p=1,q=1…….4
=2 7
=3 10
=4 13
=5 16
=6 19
=7 22
=8 25
=9 28
=10 31
=11 34
=12 37
=13 40
=14 43
=15 46
=16 49
=17 52
…………………………
p=2,q=2…….12
=3 17
=4 22
=5 27
=6 32
=7 37
=8 42
=9 47
=10 52
…………………………
p=3,q=3…….24
=4 31
=5 38
p=3,q=6 45
=7 52
…………………………
p=4,q=4…….40
=5 49
=6 58
…………………………
p=5,q=5…….60
……
……
……

Complementary numbers that can not be calculated by f=f(p,q)=2pq+p+q (less than 50; we describe the numbers as f^c s.) are as follows;

1,2,3,5,6,8,9,11,14,15,18,20,21,23,26,29,30,33,35,36,39,41,44,48··················.

If we double the numbers of f^c s and add 1 to them, then we can have the prime number series (except 2 which is the only one even prime.) from 3 to 97, which can be applied to all the f^c s from 1 to ∞ .

Note that original explicit function $f=f(p,q)=2pq+p+q$ consists of very simple number series; i.e. arithmetic progressions whose common differences are 3, 5, 7, 9, 11, 13, 15,…………………

Above facts will contribute to analyze the system of prime numbers, in the nature of which lies 'the regularity in the randomness of prime numbers', may be useful for the solution of such problems as 'The Riemann Hypothesis'.

Concerning the practical use, the distinguishment between the value of f and f^c on the computers' algorithm and technological innovation for the rapid determination of the huge value X(= an odd number) to be whether a prime or a non-prime number is very important .

(If $(X-1)/2$ belongs to f^c , then the number X is a prime.)

The quick and 100% accurate checking and judgment of primes of huge values will be very useful in every scientific field if such algorithm becomes possible and successful.

Furthermore, we will have the quite new method of systematically generating huge prime numbers on the computers by using f^c if the above- mentioned great technological innovation is realized after all.

Note 2

When we refer to the textbooks or on-line blogs, we often find that so many articles regarding prime numbers say that primes can be described as type $(6n+1)$ and type $(6n—1)$.

We described them as type $(4k+1)$ and type$(4k—1)$.
All the natural numbers can be described as $4k$, $4k+1$, $4k+2$, $4k+3$ (Here, k; integers not less than zero). All the primes (except 2) belong to either $(4k+1)$ or $(4k+3)$; the former consists of 5,13,17,29…….;the latter consists of 3,7,11,19,23,31…………which is just equal to type $(4k—1)$.

There are few articles which describe the relationship between type $(6n+1)$ & type $(6n—1)$ and type $(4k+1)$ & type$(4k—1)$.

Here, we research the relationship between above two ways of descriptions of prime numbers.

$(6n+1)$ can be transformed as $(6n+1)=4n+2\times(n+1)—1$, so if $(n+1)$ is an even number, above number is type $(4k-1)$,and if $(n+1)$ is an odd number$(=2m+1)$, then $4n+2\times(2m+1)—1$ becomes type$(4k+1)$.
On the other hand, $(6n—1)=4n+2\times(n—1)+1$, so if $(n—1)$ is an even number, it becomes the type $(4k+1)$ prime, and if $(n—1)$ is an odd number, then $4n+2\times(2m+1)+1=(4k+3)$,which is just equal to the type $(4k-1)$ prime number.

The reason why the author adheres to type $(4k+1)$ and type$(4k—1)$ is that we have a purpose of considering the 'Goldbach's Conjecture' , using the "Erdös' theorem", in the case when $2M\rightarrow\infty$.

Note 3

Case 1 necessarily holds good when 2M is larger than a certain number. As M becomes larger more than some hundred, the distribution of the values of f s becomes dense, while that of the values of f^c s becomes sparse. And in that situation, Case 2 and Case 3 become self-evident.

But on the contrary ,Case 4 is not self-evident.

As M becomes larger, g^c also becomes hard to exist.
Since $(2\times f^c+1)$ or $(2\times g^c-1)$ can produce the prime number series, above facts can explain the situation where the larger the value of M is, the sparser the distribution of prime numbers become.

Concerning above facts, if we can create the mathematical expressions that give X which is the number of the values generated by the function $f=f(p,q)=2pq+p+q$ (duplicated values excluded) in the range between zero and M, then $M-X$ will give us the number of f^c s, which is just equal to the number of prime numbers in the range between zero and 2M. Therefore we may have the more accurate expression than PNT(=the prime number theory) if successfully.

Note 4

We should think about the 'proposition A' which insists that $(M+n) = P+2n$ $(P \in f^c)$ does not take the value of f^c and always takes the value of f.

What we should do is that we deny the above 'proposition A' by the existence of the 'counterexample' or more generally, we employ the method of the 'proof by contradiction' to deny it, inducing absurdity.

If we adopt the following examples;

adopting the cases of $M=12$, $P+2n=18(f^c)$, $P=6(f^c)$ and

$M=12$, $P+2n=16(f)$, $P=8(f^c)$, then

we can get 37 and 13 in the former case, 33 and 17 in the latter case. Therefore, we can deny the 'proposition A' by the existence of the counterexample.

More generally than above, we can induce absurdity as follows.

$P+2n = f = f(p,q) = 2pq+p+q$, where P takes the value of f^c.
Then, double the both sides of the above expression and add 1, we can have the 'absurdity' as follows;

$$4n+(2\times f^c+1) = 2\times f+1 = (2p+1)(2q+1),$$

which means that the prime number $(2\times f^c+1)$ is always equal to the number that can be factorized as $(2p+1)(2q+1) - 4n$, which is just the opposite case where the 'decomposition into complementary numbers' is possible (i.e. the prime = the prime — 4n).

Hence we have the 'absurdity', and the 'proposition A' is denied.

Therefore, we reach the conclusion that we can get $(M+n) = (P+2n) \in f^c$ $(P \in f^c)$, if we choose the appropriate value of n.

Above method is also applied to $(Q-2n)$ and g^c in the next chapter.

Note 5

Last but not least is the explanation about the demonstration of the 'Goldbach's Conjecture' relating to the value of n.

As we used the phrase 'choosing the value of n' as a matter of course, but without considering the existence of the value of n and the guarantee of it, the demonstration can not become perfect. And such a way is not a perfect explanation of 'the decomposition into complementary numbers'. So, let us explain the above as follows.

Now we think only about the complementary numbers of Q= f^c(larger) $=(M+n)$ and P= f^c(smaller)$=(M-n)$, and the primes are thought to exist as type $(4k+1)$ or type $(4k-1)$ only.
(If $n=0$, then $Q=P=M$)

Here we see what 'the decomposition into complementary numbers' means ; if we choose the appropriate value of n, then we can obtain the expression the prime($2\times f^c$(larger)+1)=the prime($2\times f^c$(smaller)+1)$+4n$ just as the opposite case mentioned in Note 4. (Because if Q=P+2n and Q=M+n, then $2M=2Q-2n=Q+Q-2n=Q+P$.)
In this case, we get $k-k'=n$ as $(4k\pm1)-(4k'\pm1)=4n$. ($(4k\pm1)$ and $(4k'\pm1)$ represent the primes.)

We consider the general cases where not only the values of f^c s but also g^c s are used as complementary numbers.
Regarding the combinations of f^c and g^c,we give the total combinations about them as follows;
① both values of f^c and g^c are even numbers
② both values of f^c and g^c are odd numbers
and cases where
(a) $2M > f^c = (M+n) \geqq M,\quad M \geqq g^c = (M-n) > 0$
(b) $2M > g^c = (M+n) \geqq M,\quad M \geqq f^c = (M-n) > 0$.

In the case of ① and (a), as $f^c = g^c + 2n$, if we double the both sides of the expression and add 1, then the following expression comes.

$2 \times f^c + 1 = 2 \times g^c + 1 + 4n = 2 \times g^c - 1 + 4n + 2$, which means $(4k + 1) = (4k' - 1) + 4n + 2$. Therefore we get $k - k' = n$.

In the case of ① and (b), as $g^c = f^c + 2n$, if we double the both sides of the expression, subtracting 1, then the following expression comes.

$2 \times g^c - 1 = 2 \times f^c - 1 + 4n = 2 \times f^c + 1 + 4n - 2$, which means $(4k - 1) = (4k' + 1) + 4n - 2$. Therefore we get $k - k' = n$.

In the case of ② and (a), as $f^c = g^c + 2n$, if we double the both sides of the expression, adding 1, then the following expression comes.

$2 \times f^c + 1 = 2 \times g^c + 1 + 4n = 2 \times g^c - 1 + 4n + 2$, which means $(4k - 1) = (4k' + 1) + 4n + 2$. Therefore we get $k - k' = n + 1$.

In the case of ② and (b), as $g^c = f^c + 2n$, if we double the both sides of the expression, subtracting 1, then the following expression comes.

$2 \times g^c - 1 = 2 \times f^c - 1 + 4n = 2 \times f^c + 1 + 4n - 2$, which means $(4k + 1) = (4k' - 1) + 4n - 2$. Therefore we get $k - k' = n - 1$.

In short when we take the increasing prime numbers of type $(4k + 1)$ and type $(4k - 1)$ between the range from $2M$ to $4M$, while we take the increasing prime numbers type $(4k' + 1)$ and type $(4k' - 1)$ between the range from 0 to $2M$ when the value of M increases, the values of n necessarily exist increasingly in number.

Furthermore, even when $k \to \infty$, the values of n necessarily exist to infinity in number in the above mentioned style.

Accordingly, the procedures of 'the decomposition into complementary numbers' can be also guaranteed as the numbers of them(= proportional to G(2N)) and values of n are sufficiently supplied. 《See Note 6.》

We can 'choose' the value of n on the basis of the above.

Note 6

According to the Japanese statistics in the on-line blogs, the decomposed pairs of primes by the ‘Goldbach's decomposition’ of the same even number (also ‘the decomposition of the same even number into complementary numbers’) increase in number as the value of M increases.

This number of the pairs of primes is often described as G(2N) or G(n) (n; an even number) which is closely related to PNT(=the prime number theory) increases, for the ratio of the prime numbers included in the range from M to 2M (=range(A)) compared with those numbers from 0 to M (=range(B)) is surely small , but the absolute number of the former primes must increase in the case where $M \to \infty$ (This fact comes from the nature of the distribution of the prime numbers.), and accordingly the number of the combinations of the decomposed primes (=G(2N)) must increase to the maximum. This is the reason the ‘Goldbachs’ decomposition’ will continue to infinity.

(The reason why the above total combinations(=G(2N)) increase as the value of M increases is that a combination of one pair of primes consists of one prime p_A from the range (A) and another prime p_B from the range(B). If the total primes included in the both range (A) and range(B) increase as the value of M increases, then the number of the total combinations of the primes which generate the same even number 2M will increase at the same time. That is to say, to form the same even number 2M, the expression $(p_A - M) = (M - p_B)$ must be satisfied, and such combinations of p_A and p_B will become great in number as the result of the great enlargement of the value of M, which is confirmed by the statistical data.)

G(2N) (or G(n)) is thought to be the important number of the increasing function of M in which the logarithmic function is included, and it is also believed to be a significant key that leads us from the ’Goldbach's Conjecture’ to ’The Riemann Hypothesis’ according to the articles in the American on-line blogs.

《Postscript》

The complementary numbers were discovered in August 2009 when the author was pursuing the mathematical expressions that cannot be factorized while engaging in the method of the demonstration of the 'Goldbach's Conjecture' when studying Bertrand-Chebyshev's existence theorem of a prime between the range of M and 2M which was rewritten by Mr. Paul Ｅｒdöｓ' at the same time.

We can also easily apply this new method of generating prime numbers to demonstrating the infinity of 'Sophie Germain prime number series.'

Above episodes and so on are described in the author's book with copy rights in both English and Japanese languages published in Japan.