A Model of Charmonium

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Abstract

A geometrical/mechanical model of charmonium has been developed, based on the logarithmic confinement potential. The quark and antiquark pair orbit around the centre of mass, with their gluon and colour fields contained within a torus structure of characteristic radius.

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1. Introduction

A model of charmonium structure has been developed, based on Einstein's equations of general relativity, as was done for previous models of the electron and proton, (Wayte, Papers 1 and 2). Essentially, the model is compatible with measured spectra of charmonium and bottomium. Although different types of quark/antiquark potentials have been invoked in the literature, our application of Einstein's equations is quite selective. The Coulomb + linear potential presented by Eichten *et al* (1978), (1980) appears to cover the data in many ways, but it is not as straightforward as the logarithmic potential described by Quigg and Rosner (1977), (1979). Whereas the former is effectively two superimposed fields, involving a *variable* strength factor, the latter describes a single field extending smoothly from a source radius to infinity, with a familiar *constant* field strength factor. Furthermore, the corresponding theoretical leptonic decay widths of vector mesons appear to fit the observations better.

2. The Potential

Previous work on the Coulomb and Yukawa potentials (Papers 1 and 2) indicated that those potential functions were inherently relativistic and very simple in form. We shall now find that a logarithmic confinement potential, based on the work of Quigg and Rosner (1979, pp217-223), can be very successful when introduced into Einstein's field equations, namely:

$$V(r) = C \ln(r / r_q) \quad , \tag{2.1}$$

where C = 0.733GeV, r_q = 0.89GeV^{-1} (0.18fm) is the separation of quark and antiquark. These two semi-empirical values may be used to calculate the quark-antiquark field strength relative to the electromagnetic field. Given that the electron classical radius is usually defined as $r_{oe} = e^2/m_e c^2$= 2.818fm, and the Quigg-Rosner fundamental charmed quark mass is m_c = 1.08GeV/c^2, then the field strength ratio could be of the order:

$$\frac{q^2}{e^2} = \frac{(r_q / 2)(2m_c)c^2}{r_{oe} m_e c^2} \approx 135 \quad . \tag{2.2}$$

Intuitively this ratio should be 137.036 (the inverse fine structure constant α^{-1}), comparable with the strong force. So the analysis of Quigg and Rosner, using the nonrelativistic Schrodinger equation, appears reasonably accurate for further application. They determined the empirical charmonium masses from their expression:

$$M_{nl} = CE_{nl} + E_o \quad , \tag{2.3}$$

where E_{nl} is the eigenvalue, C = 0.733GeV, and E_o is a reference mass constant given by:

$$E_o = 2.329\,\text{GeV} = 2m_c - \tfrac{1}{2} C \ln\left(Cm_c r_q^{\,2}\right) \quad . \tag{2.4}$$

It is not clear how to adjust these parameters, in order to get the inferred 137 relativistic field strength ratio in (2.2).

Compatibility of proton and charmonium as electromagnetic structures is confirmed since the proton has spin radius ($r_p = \hbar/m_p c$ = 1.066GeV^{-1} = 0.2103fm), and mass m_p = 0.938GeV, so

$$r_p m_p = \hbar / c \approx (r_q / 2)(2m_c) \quad . \tag{2.5}$$

Quigg and Rosner (1979) also give the fundamental mass of the upsilon family as m_Q = 4.52GeV, while the field strength q^2 and the radius r_q remain the same as for charmonium. Consequently, extra mass is incorporated *passively* in the quark and the antiquark mechanisms, as was found for baryons (unpublished). In this case we have:

$$m_Q = m_c(37.7/9) \quad , \tag{2.6}$$

which will be interpreted by postulating that a charmonium quark is made of 3 pearls, each of 3 grains, whereas an upsilon quark consists of 37 such grains, but the overall charge must be saturated. The three pearls allow for the concept of 3 colours to be introduced, if they differ in some feature or just in relative phase. Design of a charmonium quark is then something like a proton's design; however, r_q could be a characteristic radius, rather than being a structural constant like the proton radius r_p.

3. Application of Einstein' Equations:

In order to interpret the attractive inter-quark force in a way compatible with the electromagnetic and hadronic forces, the metric tensor component will be given a similar form:

$$\gamma = \left[1 + V(r) / M_C c^2\right] \quad , \tag{3.1}$$

For this, *C* in (2.1) will be set equal to $M_C c^2 / 2 \times 2^{1/2}$, where M_C = $2m_c$ is the fundamental charmonium mass. Then the potential energy, for an antiquark in the field of a quark, may be written as:

$$V(r) = \frac{q^2}{2 \times 2^{1/2}(r_q / 2)} \ln\left(\frac{r}{r_q}\right) = \frac{M_C c^2}{2 \times 2^{1/2}} \ln\left(\frac{r}{r_q}\right) \quad , \tag{3.2a}$$

which means that:

$$\gamma = \left\{1 + \frac{1}{2 \times 2^{1/2}} \ln\left(\frac{r}{r_q}\right)\right\} \quad . \tag{3.2b}$$

We shall now discover that the potential (3.2a) cannot be spherically symmetric, but is linear as for a tube of gluons.

3.1 Spherically symmetric field

The energy-momentum tensor components, for a conserved spherically symmetric radial field, are from the adapted Einstein's Equations:

$$8\pi\left(\frac{Q}{c^4}\right)T_1^1 = 8\pi\left(\frac{Q}{c^4}\right)T_4^4 = \left(\frac{1}{r^2}\right)\frac{d}{dr}\left[r(1-\gamma^2)\right] \quad , \tag{3.3}$$

$$8\pi\left(\frac{Q}{c^4}\right)T_2^2 = 8\pi\left(\frac{Q}{c^4}\right)T_3^3 = -\gamma\left(\frac{1}{r^2}\right)\frac{d}{dr}\left(r^2\frac{d\gamma}{dr}\right) - \left(\frac{d\gamma}{dr}\right)^2 \quad , \tag{3.4}$$

where Q is a quark constant. As previously, this expression for T_2^2 is most like the classical Poisson's Equation and will be developed by keeping the charge component on the right side separate from the field component. After introducing (3.2b), we have:

$$8\pi\left(\frac{Q}{c^4}\right)T_2^2 = -\left[\frac{\gamma}{2^{1/2}r^2}\right] - \left[\frac{1}{2^{1/2}r}\right]^2 \quad . \tag{3.5}$$

The first term on the right represents momentum density of space-charge gluons, and the second term gives the momentum density of the 'colour' field emitted by these gluons. This field strength decreases linearly with radius.

Energy density T_4^4 may likewise be derived from (3.3) and arbitrarily expressed:

$$8\pi\left(\frac{Q}{c^4}\right)T_4^4 = -\left[\frac{\gamma}{2^{1/2}r^2}\right] - \left[\frac{1}{r^2}\left(\gamma^2 - 1\right) + \frac{\gamma}{2^{1/2}r^2}\right] \quad . \tag{3.6}$$

Here the first term on the right represents energy density of space-charge gluons, for an attractive force. If a spherically symmetric integration is carried out on (3.6), as was done for the proton nuclear force field, then total field energy could approach infinity. It is concluded that the empirical logarithmic potential (2.1) is not due to an actual spherical field, so (3.6) and (3.5) could only describe a field of limited extent.

3.2 Linear field

We will let the charmonium binding field of gluons and colour quanta from the quark and antiquark be *confined* to a torus structure, analogous to the spin-loop of an electron or proton, see figure 1. The electron's spin-loop energy was calculated on the assumption of a *spherical field*, so let the quark's *effective* area of emission be $4\pi r_{oq}^2$. We can then determine the total energy within a tube of effective cross-sectional area $4\pi r_{oq}^2$, by integrating the energy density T_4^4.

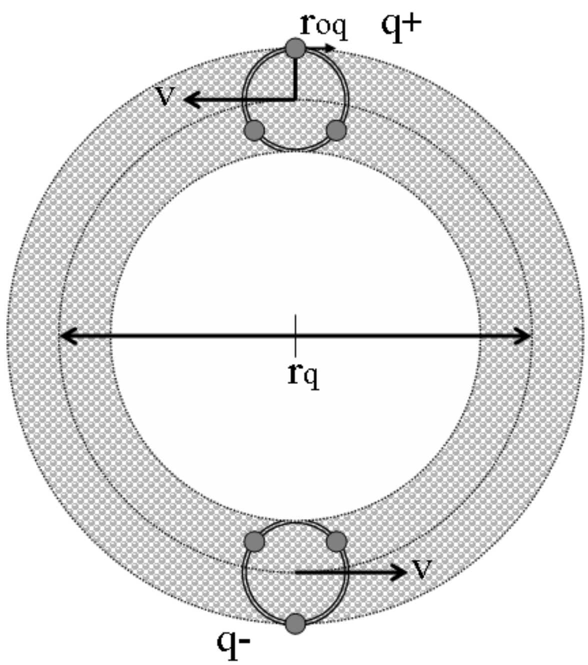

Figure 1 Schematic diagram of charmonium in which a quark and antiquark orbit around a centre of mass, while emitting a field of gluons with their colour field, all confined in a torus.

For convenience, radial parameter *r* is retained as the *diametrical* separation of the quark and antiquark, which orbit the centre of mass at radius *r/2*.

There is a solution of Einstein's Equations, specifically for a linear field. For example, consider an ideally static field produced between a quark placed at the origin and an antiquark placed on the *x*-axis, say. The field of gluons is to be confined by a tube of cross-sectional area $4\pi r_{oq}{}^2$ parallel to the axis. It will have components of the energy-momentum tensor as derived from Dingle's formulae (Tolman, 1934, p. 253), for the line element:

$$ds^2 = -\gamma^{-2}dx^2 - dy^2 - dz^2 + \gamma^2 dt^2 \quad . \tag{3.10}$$

These components are mathematically:

$$8\pi\left(\frac{Q}{c^4}\right)T_1^1 = 8\pi\left(\frac{Q}{c^4}\right)T_4^4 = 0 \tag{3.11a}$$

$$8\pi\left(\frac{Q}{c^4}\right)T_2^2 = 8\pi\left(\frac{Q}{c^4}\right)T_3^3 = -\frac{1}{2}\frac{d^2\gamma^2}{dx^2} = -\gamma\frac{d^2\gamma}{dx^2} - \left(\frac{d\gamma}{dx}\right)^2 \quad . \tag{3.11b}$$

Upon introducing γ from (3.2b), with ($r = x$) we get the separate gluon and colour field tangential momentum densities:

$$8\pi\left(\frac{Q}{c^4}\right)T_2^2 = -\left(\frac{-\gamma}{2\times 2^{1/2}r^2}\right) - \left(\frac{1}{2\times 2^{1/2}r}\right)^2 \quad . \tag{3.12a}$$

T_2^2 is the effective momentum/stress density of the field of the source quark, as seen at the position of the antiquark. Given the form of (3.5) and (3.6), we shall infer that T_4^4 in (3.11a) cannot really represent zero energy density, and should be made compatible with T_2^2 by taking the form:

$$8\pi\left(\frac{Q}{c^4}\right)T_4^4 = -\left(\frac{-\gamma}{2\times 2^{1/2}r^2}\right) - \left(\frac{\gamma}{2\times 2^{1/2}r^2}\right) \quad . \tag{3.12b}$$

Integration of T_4^4 from $r = 2r_{oq}$ (where $\gamma = 0$) to $r = \infty$ will then lead to the separate and equal gluon and colour field energy components:

$$\left(\frac{Q}{c^4}\right)\int_{2r_{oq}}^{\infty} T_4^4 (4\pi r_{oq}{}^2)dr = -\left[\frac{-r_{oq}}{32}\right] - \left[\frac{r_{oq}}{32}\right] \quad . \tag{3.13}$$

The modulus of the first term on the right will be attributed to the energy of the gluons because the negative sign could be due to the opposite signs of quark and antiquark. Upon setting [$r_{oq} = QM_C/c^2$], analogous to the electron, then the maximum *gluon plus colour* field energy is (1/16) of the charmonium total mass energy:

$$W = -\int_{2r_{oq}}^{\infty} T_4^4 (4\pi r_{oq}{}^2)dr = \frac{1}{16}M_C c^2 \quad . \tag{3.14}$$

When $2r_{oq} < r < \infty$, the gluon plus colour field energy is less than this, such that at $r = r_q$ (where $\gamma = 1$), it is $(M_C c^2/28)$ approximately. The actual quark radius value ($r_{oq} \sim (\sqrt{2}/24)(r_q/2)$), defined at ($\gamma = 0$), has not been interpreted further.

The tangential momentum density may be integrated to get a similar result:

$$\left(\frac{Q}{c^4}\right)\int_{2r_{oq}}^{\infty} T_2^2 (4\pi r_{oq}{}^2)dr = -\left[\frac{-r_{oq}}{32}\right] - \left[\frac{r_{oq}}{32}\right] \quad . \tag{3.15}$$

This means that the gluons plus colour field quanta have unitary helicity and propagate at the velocity of light.

Clearly, it has been essential to introduce some prior knowledge of the real gluon and colour field into these solutions of Einstein's Equations, instead of trivially solving mathematical terms.

4. Conclusion

A model for charmonium has been developed, based on applying the logarithmic confinement potential to Einstein's equations of general relativity. The quark and antiquark orbit around a common centre of mass, with their gluon and colour fields entirely confined to a torus. The fundamental torus radius is $r_q/2 \approx 0.09$fm, but the empirical charmonium excited states are larger and more massive, as derived by Quigg and Rosner. Half the charmonium total energy remains in the external radial pionic field.

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