Author name
Giuliano Bettini

Title

An interpretation of the laws of gravity and inertia

Abstract

Ideas on a source of inertia from fixed stars have crossed Physics least Mach onwards. Equations "Maxwell-like" for gravitation and inertia were obtained by several authors as a subspecies of the simplified theory of General Relativity. A precursor was Dennis Sciama.
Introducing a four-potential, I submit here a tentative interpretation of our laws about gravity and inertia, in complete analogy with electromagnetism.
In classical mechanics is not introduced, usually, a four potential.
The field produced by this four-potential describes both gravitational and inertial forces.
Admit the gauge transformation on potential is equivalent to enunciate the equivalence principle, and vice versa.
Inertial forces (ex. Coriolis force) are interpreted as a field action.
All the inertia is interpreted as a field action.
The physical presence of this field seems to be a fact, even more concrete the usual admission of inertial "fictitious forces".

An interpretation of the laws of gravity and inertia

Introduction

Understanding the laws of gravity and inertia, or at least the interpretation of our laws on gravity and inertia, include primarily the abandonment of a longstanding mental position which, as often happens in these cases, conceals a subtle assumptions. It covers the acceleration. Faced with the expression

$$(1) \quad \frac{d^2\vec{x}}{dt^2} = 2{,}5g(along\_\hat{x})$$

I believe that most of us "feel" on his body a push of 2.5 g, as opposed to the motion. This means an unjustified a priori identification of the kinematics with force fields. In (1) in fact the first member contains a kinematic quantity, that is part of all those quantities, trajectory, speed etc, which describe the motion of a point irrespective of the causes that produce it. I have gained from time the conviction that it is essential to make a clear distinction between the kinematics on one hand and field strengths on the other hand, they are either electromagnetic or gravitational (or gravitational / inertial, but use the term "gravity" for simplicity covering in total).
We can imagine the reasons for this confusion, making the following attempt statements, but no impact at all on the theory that comes after, and then can be read or skipped, as desired.
Let’s say that the confusion arose from the fact that, given the presence of the fixed stars or space as a dominant mass anywhere, any kind of motion, so kinematic situation, can not ignore, in fact is directly connected, to a field action. You can not do an experiment, at least for now, when you try to move (some kinematic) by removing this field. This gave a dangerous identification between kinematic (d**2x/dt**2) and the forces of inertia, as if d**2x/dt**2 was synonymous with that feeling. Actually the fact is, so to say, purely coincidental and is due to the presence of the dominant mass that creates the field.
It’s the field that gives the feeling, not the kinematics.
This is evident in electromagnetism where:
a) - I move, I feel something, because there is a charge;
b) - take off the charge, I move as before, and feel nothing.
The feeling is not given by kinematics, but the fact that there is a field.
The distinction between kinematics and fields carries my conviction, or if you want my doubt, that the properties of gravitation should not be dealt with rulers and clocks, or at least, not always pay. The motion must be examined with rulers and clocks ; fields are examined with a test particle. This places on an equal footing electromagnetic and gravitational field, and give to kinematics a common role in both disciplines.

To make a statement less demanding we say, if I step on a carousel, is more practical to consider the inertial and gravitational phenomena that take place there through the concept of field and test particles, not with a change in geometry, examined with rulers and clocks.
Closed the parentheses on this strong conceptual distinction between kinematic and field I go to describe a formula of Hestenes and how it has aroused a strong impression on me, mainly because some of my previous ideas and beliefs .

The field structure

The formula in question is the "rotor equation of motion" [1]:

$$(2)\quad \dot{R} = \frac{\Omega}{2} R$$

Without going into details now not relevant formula (2) is, to put it in simpler terms, the "equation of motion for a material point", using the methods and symbolism of the "Space Time Algebra" or Clifford algebra whatever. In (2) R is a kinematic quantity that describes the motion, whereas Omega is a quantity of a mechanical nature, in particular a space time bivector, ie an entity with some mathematical structure with a part such as "electric field vector" and a part such as "magnetic field vector", whose role in the formula is ultimately to specify the external forces. Assigned Omega, R is obtained "integrating the equations of motion", the (2) separating the kinematics by force fields.
Hestenes over time called Omega variously, "proper angular velocity", "generalized rotational velocity", etc. However, while I took a different message to Omega:

"external forces can be assigned with a space time bivector".

There's more. In the case of a particle of charge q and mass m subjected to an electromagnetic field F (F in this case is the "electromagnetic field bivector", consisting of E and H) the dynamics of the particle is specified by [2]:

$$(3)\quad \Omega = \frac{q}{m} F$$

With some step mathematical formula (2) and (3) provides the law of motion expressed by the Lorentz force, the law with which the kinematic parameter "acceleration" is matched to the action of the external field.
Hestenes has been quite struck by this simple relationship (3) which essentially says, apart dimensional facts, which in this case Omega is "identical" to F, coincides with the electromagnetic field F. Since Omega coincides with F and F can be derived from a scalar potential and vector potential, also Omega "identical" will be.
Hestenes says [1]:

“This role of the electromagnetic field F as a rotational velocità is so simple and natural that it deserves a name. I propose to dub the relation Omega=(q/m)F the *Lorentz Torque*”.

But I have picked up a different message:
"External forces act in this case through a bivector field derived from a scalar potential and vector potential, a four-vector, with the typical formulas of electromagnetism”.
This fact there is no doubt why Omega coincides with F and F can be derived from a scalar potential and vector potential. However I, polarized by my previous ideas, I made the following conjecture:
"External forces act always (ie also in the case of gravity) through a bivector field derived from a scalar potential and vector potential, with the typical formulas of electromagnetism”.
With this working hypothesis in mind I took old ideas, however, focusing attention not just on the field but on this hypothetical potential.

The necessary link with the classical mechanics

Ideas on a source of inertia on the part of the fixed stars have crossed Physics least Mach onwards. Equations "Maxwell-like" for gravitation and inertia were obtained by several authors as a subspecies of the simplified theory of General Relativity, ex. [3]. A precursor was Dennis Sciama [4]. For some reason unknown to me these things rarely discussed, whereas even read on the Internet that some people would be taught in school. It is probably theories on which not all agree.
But two things struck me:
1. for those who places (not all) the origin of inertia in the distant stars seems to remain an open question: why you feel the inertia without delay? In fact, with a potential radiated from the fixed stars action be felt much later, that of retarded potentials, say 14 billion years .... How is it that instead the inertia forces are felt at once?
2. why to deduce the equations should be necessary the general relativity? For weak gravitational fields should suffice classical mechanics and/or no more than a bit of relativistic mechanics.
In my opinion all questions must be answered within confines of classical mechanics, and 3D vector calculus. In fact, as if there is an analogy of the formulas with those of electromagnetism, and since the inertia is still felt at low speeds and weak gravitational fields, and because electromagnetism can be explained also without recourse to the theory relativity, it follows that mathematics of the ordinary vector calculus must be sufficient and classical mechanics should already contain all the ingredients and explanations necessary.
In other words, the identification of the potential, scalar potential plus vector potential, must be possible within the framework of classical mechanics.

I therefore promised to consider the classical mechanics (nonrelativistic) to see whether it can be rephrased as electromagnetism, and I repeated, as they do in basic courses of electromagnetism, not to have recourse to methods calculation more or less sophisticated but ordinary vector calculus. This I will in the next section with the risk of not being strict at times, but certainly with the advantage of better physical understanding of phenomena.

Forces

Any good book of classical mechanics, ex. Sommerfeld [5], "Mechanics", it tends to be very fussy in clarify definitions. Sommerfeld distinguishes forces in "real" and "apparent", placing them between the forces of inertia. Sommerfeld says: :”all bodies have the tendency to remain in a state of rest or of uniform rectilinear motion. We can think of this tendency as a resistance to changes in the motion (…) or, for brevity, as an *inertial force*. The definition is therefore….”:

(4) $$\vec{F}_{app} = -m\frac{d\vec{v}}{dt}$$

Sommerfeld puts also the centrifugal force between them. Although in (4) appears under the symbol "v" the velocity of mass m, we note that the “apparent” force (4) acts on all points of the space in question, as it is found going into a car and speeding up, so it is more convenient to indicate with "V". the speed of points in space in question and write:

(5) $$\vec{F}_{app} = -m\frac{d\vec{V}}{dt}$$

More V, and the resulting acceleration can be varied between different points in space in question, as the example of centrifugal force and how it can be seen going on a carousel. Sommerfeld always says: "It, too, is a fictitious force. It corresponds to (...) *centripetal* acceleration, ie, directed toward the centre of curvature”.
Besides these forces are the "real" forces that is, if we exclude the impact or the pressure, the forces that come from a "physical situation" as gravitation, whose expression is:

(6) $$\vec{F}_{real} = -mgrad\varphi$$

Summing up the total force in the presence of gravitational forces plus inertial is:

(7) $$\vec{F}_{tot} = \vec{F}_{real} + \vec{F}_{app}$$

ie the grouping (5) and (6):

(8) $$\frac{\vec{F}_{tot}}{m} = -grad\varphi - \frac{d\vec{V}}{dt}$$

We can now make the following consideration. The latter expression is visibly similar to the expression of the force exerted on a charge q, force referred to the charge (electric field E) written as a function of potential:

(9) $$\vec{E} = -grad\varphi - \frac{\partial\vec{A}}{\partial ct}$$

and is even more visible as a force field, arising from a scalar + vector potential if written instead of (8), in a form with V=U/c ie:

(10) $$\vec{g} = -grad\varphi - \frac{\partial\vec{U}}{\partial ct}$$

Summarize:
mechanics, see Sommerfeld, teaches us that for a point mass m in a stationary reference S'(nb: we think of ourselves on the car seat with reference axes attached to the car) the total acceleration "g" or total strength by mass, perceived, is generally the sum of the forces of gravity, or "real forces", and forces of inertia, forces due to acceleration of the reference S' with respect to an inertial reference S (apparent forces) . The formula (10) as written allows us to connect with the same formula (9) of electromagnetism, with a scalar potential plus vector potential who makes its appearance in the explicit "electromagnetic form"

(11) $$U = (\varphi, \vec{U})$$

with $\vec{U} = \vec{V}c$ and therefore:

(12) $$U = (\varphi, \vec{U}) = \left(\varphi, \vec{V}c\right)$$

"V" being the speed of points of reference S' with respect to inertial space. Remains to be seen whether this is merely a coincidence or whether it has a physical meaning.

The four potential

In classical mechanics is not introduced, usually, a four potential.
In other words the mechanical gravitational potential φ and velocity V of the points of S' with respect to inertial space are in no way connected to each other and even accepted as part of a single entity "four-vector".

Mechanics however makes a statement connecting the forces or accelerations, when you say that you can exchange a real force (-grad φ) with an equal amount of apparent force (-dV/dt) ("equivalence between gravitational forces and inertial forces"). The principle of equivalence expressed in terms of forces passes first through the mechanical relationship (7), which then is completed in writing that I can add and remove a force:

$$(13)\quad \vec{F}_{tot} = (\vec{F}_{vera} + \vec{f}\,) + (\vec{F}_{app} - \vec{f}\,)$$

Now it is interesting, and probably indicates a link between the scalar and vector potential, note that:

the principle of equivalence leads to a gauge transformation on the potentials:

$$(14)\quad \vec{U} \rightarrow \vec{U} + grad\chi \qquad\qquad \varphi \rightarrow \varphi - \frac{\partial \chi}{\partial ct}$$

In fact, just define a "**χ**" appropriate and, with some calculation, it is noted that it may completely disappear in (10) the gravitational field and it is replaced by an equivalent inertial action, or you may disappear inertia and appear a gravitational field the same amount, all to your liking, using the standard formulas (14) of a gauge transformation.

To give a concrete example refer to Appendix 1 after we have established the following final formula (19) for the potential.

We can therefore say that

admit the gauge transformation (14) on potential (11) is equivalent to enunciate the principle of equivalence, and vice versa!

This is suspect, but this could still be seen as a mathematical artifice with the use of an auxiliary "four potential" U scalar + vector . A step further would be provided by the identification of a meaning assigning to the four potential the rank of a physical quantity: a real "field" of something. (Note: U is dimensionally related to the ratio energy/mass).

To do this we make a brief digression concerning the relativistic formulation of electromagnetism.

It tells us that the potential A similar to U, if you want the relativistic invariance of the equations, need to be a four-vector.

In expression (11) of U a 3D velocity vector V appears.

The only four-vector that can be formed with a 3D velocity V is the four-velocity, or a four-vector proportional to it. So U must be the four-velocity of points in space S' or a four-vector proportional to it. Let ourselves in an area without nearby masses (say, empty space) and let V the 3D speed of points of our space with respect to an inertial reference. Formable with V is the four-vector (about, for low speed):

(15) $(1,\frac{\vec{V}}{c})$

and therefore U, as in electromagnetism, should be <u>proportional</u> to that four-vector. The constant of proportionality is easy to find. To comply with the formula (12) for U must be mandatory:

(16) $U = c^2(1,\frac{\vec{V}}{c}) = (c^2,\vec{V}c)$

Apart from the meanings more or less of fantasy on who determines the presence of a scalar potential c ** 2 in (16) what emerges is that relativistic invariance requirements in order to match the ordinary laws of mechanics (10), the four-potential must have that form.

In particular, the scalar potential is c ** 2, from which it follows that the potential energy of mass m in a reference S' in which it is stationary with respect to inertial space (V = 0) and no other masses close , is mc ** 2. This result, although born in a non-intended, is consistent with what Physics states by other means.

The potential (16) is unable to determine on masses of empty space no force of gravity according to (10) because c ** 2 is constant.

If there is at a distance "r" a mass M which gives a scalar potential that we know is:

(17) $\Phi = -\frac{kM}{r}$

and if reasonably suppose that the potential to be additive as in electromagnetism, it follows that we must add to (16) the four-potential:

(18) $U_M = \Phi(1,\frac{\vec{V}}{c})$

The second term provides a contribution to the vector potential (ie inertia) absolutely lowest, although philosophically have an important meaning that is close to masses a modification of inertia. For the present considerations are practically classical mechanics can certainly ignore, while certainly be added in (16) to the scalar potential c ** 2 the potential of the present masses, (17).

The total four-potential which takes account of gravity and inertia is therefore for any practical use:

(19) $U = (c^2 + \Phi,\vec{V}c)$

then being given by (11), with the vector potential (12) and the scalar potential:

(20) $\varphi = c^2 + \Phi$

With this the calculation of the potential ended.
The potential generates a field (10), similar to the electric field E, which includes gravity and inertia. This field we expect to appear in the equation of motion of a mass m expressed by the formula of Lorentz force, providing the force that depends on the speed v of the mass m (not to be confused with V).
By analogy with electromagnetism due to the presence of four-potential U is also a field similar to the magnetic field H generated by:

(21) $\vec{\Omega} = rot\vec{U}$

This field is expected to appear in the Lorentz force as the field H, giving a force dependent on the speed v of the mass m. Indeed it is noted that this field is really, as you can see from the case of the rotating frame.

The rotation field

We apply the formulas to the case of a reference S' rotating counterclockwise with respect to inertial space.
Following the "instructions" and in the absence of forces due to the masses we calculate four-potential. The calculation is reduced in this case to calculate the (12), "V" being the velocity field of points of S' with respect to an inertial reference frame S. Take a common origin O, a common z-axis and x-axis of S' coincides with the x-axis of S for t = o. Let P be a point S' of coordinates (r, phi) in S'. Omega is the angular velocity of S' around the z-axis. The speed V of generic point P of S' is:

(22) $\vec{V} = \hat{i}\exp(i\varphi)i\varpi r\exp(i\varpi t)$

This completely determines the four-potential U.
U uniquely determines the field (force related to the mass) that is present in the space of S'. We calculate this field by following the instructions. From (10) we have in S'a field "type E" which holds:

(23) $\vec{g} = -\dfrac{\partial \vec{V}c}{\partial ct} = \hat{i}\exp(i\varphi)\varpi^2 r\exp(i\varpi t)$

The formula shows the unit vector of x-axis of S'

(24) $\hat{i}' = \hat{i}\exp(i\varpi t)$

so that the field in S' is:

(25) $\vec{g} = \hat{i}'\exp(i\varphi)\varpi^2 r$

Since (r, phi) are the coordinates of P in S’ we can introduce explicitly the unit vector:

(26) $\hat{r} = \hat{i}' \exp(i\varphi)$

directed "outward" from O to P. Overall, therefore, S’ has the force field:

(27) $\vec{g} = \varpi^2 r\hat{r}$

Always following the instructions there is an additional field "type H" given by (21), (12) and (22). It’s better to perform the calculation in cylindrical coordinates of S’. First express V in S’. From (22) with (24) and (26) we have:

(28) $\vec{V} = i\hat{r}\varpi r$

from which, having V the only component along phi, is obtained:

(29) $\vec{\Omega} = rot\vec{V}c = \frac{1}{r}\frac{\partial rV_{phi}}{\partial r}c\hat{z} = 2\varpi c\hat{z}$

and ultimately, with component along z-axis:

(30) $\vec{\Omega} = 2\vec{\varpi}c$

Summing up: we finally arrived albeit on an example to calculate a field similar to E, the field (27) which expresses the action of gravity and inertia, and also appears in more than a field type H who visibly from (30) is connected to rotations with respect to inertial space.
Given the field, continuing in the analogy the force should be calculated as the Lorentz force.
Based on the foregoing we should check whether it's a law of motion for mass m expressed by the formula of Lorentz force:

(31) $m\frac{d\vec{v}}{dt} = m(\vec{g} + \frac{1}{c}\vec{v}\times\vec{\Omega})$

Substituting (27) and (30) give up the law of motion:

(32) $m\frac{d\vec{v}}{dt} = m\varpi^2 r\hat{r} + m2\hat{v}\times\vec{\varpi}$

The second term on the right gives the expression of the Coriolis force (and the Coriolis force *“comes under the heading of inertial forces”*, [5]).
So (32) is exactly the usual law of motion in a rotating frame written in the books of mechanics, however, calculated here for a completely different way and with a completely different philosophical approach.

One might object that that exposure is concrete, while this it is fictional. Paradoxically, the opposite is true.
Imagine a skeptical physicist that says:
"This theory of fields" Type E "and" Type H "is interesting but is abstract, tendentious, is an alternative mathematical exposition. *I'm going on a rotating frame and I make measurements*".
It is easy to realize that organizing field measurements with standard methods, with a test particle we will find exactly and exclusively the (31) and fields (27) and (30). These are the experimental facts. Paradoxically, then any claim that states something different is a hypothesis, and as Newton said (Newton's "Principia"):

> *"I have not as yet been able to discover the reason for these properties of gravity from phenomena, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy".*

With this end this brief exposition of the theory of potential. Because of the close formal analogy with electromagnetism can investigate further by referring to formulas that are ready on the books of electromagnetism. Naturally, "cum grano salis" ie verifying whether there is a physical reason for taking them and not just because they are written there. For this purpose, I purposely chose a system of units, in the definition of the fields, that made them with same dimension. The dimensions are those of an acceleration. Reference electromagnetic formulas are those written in Gauss system of units, in which E and H are precisely with same dimension. The identity of the basic formulas (9) and (10), as well as the (21) with the corresponding electromagnetic, allows the direct translation of the formulas.
So you can write just for copying, but you can also get the first pair of Maxwell equations:

$$(33) \quad rot\vec{g} = -\frac{1}{c}\frac{\partial\vec{\Omega}}{\partial t} \qquad div\vec{\Omega} = 0$$

And so on.
So this is a way of examining the relations of electromagnetism then decide whether to have a meaning and if acceptable.

Appendix 1: gauge transformation

Here's an example of gauge transformation on the potential starting from its final form (19). Carryover for convenience the formulas of interest.
The potential:

(19) $U = (c^2 + \Phi, \vec{V}c)$

(20) $\varphi = c^2 + \Phi$

(12) $\vec{U} = \vec{V}c$

The gauge transformation:

(14) $\vec{U} \to \vec{U} + grad\chi \qquad \varphi \to \varphi - \frac{\partial \chi}{\partial ct}$

The field:

(10) $\vec{g} = -grad\varphi - \frac{\partial \vec{U}}{\partial ct}$

(21) $\vec{\Omega} = rot\vec{U}$

First of all we remember, as is immediate to verify, that the gauge transformation leaves completely unaffected the fields. Let be now a gravitational field due to a mass M. Let the mass M at the centre of reference S'. The points of reference are fixed with respect to inertial space. Potential:

(34) $\varphi = c^2 - \frac{kM}{r} \qquad \vec{U} = 0$

Choose "χ" as made:

(35) $\chi = -\frac{kMct}{r}$

Applying the gauge transformation (14) potential (34) turns into:

(35) $\varphi = c^2 \qquad \vec{U} = (\frac{kM}{r^2})t\hat{r}c$

The fields are unchanged. Examination of the transformed potential (35) he describes what physically happened.

We are now in a reference S’ equivalent with respect to the fields. The reference is in empty space. No longer act gravitational fields. At any point of reference space accelerates outwards with an acceleration equal to:

(36) $$\vec{a} = \frac{kM}{r^2}\hat{r}$$

An observer is unable to say if it is in either reference, unless they have other evidence to decide, in addition to the fields. It is understandable therefore that the physical meaning of gauge transformation is to provide all references mathematically equivalent with respect to gravitation. If they are physically plausible references, the observer can be in one of these references without having the evidence to decide, based purely on gravity, which is really his "type of space”.

Appendix 2: Physical considerations

We have thus seen that really is possible to argue that:
"External forces act <u>always</u> (ie also in the case of gravity) through a bivector field derived from a scalar potential and vector potential, with the typical formulas of electromagnetism".
I said it was my intention to focus not so much on the field but on this hypothetical potential.
This is just a mathematical artifice?
I told that a step further would be provided by the identification of a meaning assigning to the four potential the rank of a physical quantity: a real "field" of something.
What is this something?
Since U is dimensionally related to the energy/ mass ratio, we simply think of being immersed in a thick medium, which corresponds to a parameter c ** 2 of energy referred to mass, and U represents also a possible flow around us.
Consider the often cited "cosmological coincidence" (Sciama and others):

$$(37)\quad \frac{k\mathrm{M}_0}{Rc^2} = 1$$

where mass and radius of the universe appear, or of the whole system of fixed stars. This formula can be read back and we say that c ** 2 is caused by the fixed stars, and any presence of nearby masses adds a gravitational potential according to the formula we know:

$$(17)\quad \Phi = -\frac{kM}{r}$$

Until we stop or we are in uniform motion in this medium, it does not give sensitive actions, but if suddenly accelerate, we "collision with" and feel a force.

Conclusions

I submitted a tentative interpretation of our laws about gravity and inertia.
The physical presence of the fields seems to be a fact, even more concrete the usual admission of "fictitious forces".
I am reminded of the Hestenes phrase about the interpretation of the imaginary unit "i". To borrow the phrase and modifying it for the parts in parentheses I would say:

"I want to emphasize that this interpretation (of forces) is by no means a radical speculation; it is a fact! The interpretation has been implicit in the (classical mechanics) all the time. All we have done is make it explicit".

I insisted on using 3D vector notation even if with the Clifford algebra, or Hestenes STA, many things are (would be) stronger and clearer.
I also insisted that the inertia is a fact of everyday life and can not be tied only to the study of cosmology, or some theory which applies in the "black holes".
A word on the question of "delay". It appears right in a “c ** 2”, scalar potential produced by the fixed stars and a vector potential, responsible for the inertia, produced by the motion of the fixed stars. With the formula of the retarded potentials effect arrives late say after 14 billion years. The reasoning is: I speed up, consider instead a relative acceleration of the fixed stars with respect to me, comes a vector potential with the formula of retarded potentials, I feel the inertia .... 14 billion years later. Clicks on an inextricable delay paradox. It is said that Ciufolini & Wheeler in a recent book has solved the paradox with the contemporary intervention of an advanced potential coming from the distant future, but the explanation seems more complicated than the question.
I did not understand why the question that I had blocked for years, is now disappeared to me.
Maybe (?) the reason is that the inertial action of the fixed stars, as a result of my motion, is simply given by the transformation of the scalar potential c ** 2 saw in a moving reference.
In fact (16) implicitly says that a scalar potential c ** 2 (which has had plenty of time to settle in our area) performs here now and now and, just calculated in a reference S’ moving with respect to inertial space, shows an attached vector potential. The formula says that the potential is in fact attributable to a four-velocity (a flow of something who is here now) so that applies to the four-potential, just as in electromagnetism [6], the same transformation law "snapshot" of four-velocity. Thus there is no delay.

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