Symmetrically reproducing quark and lepton mass and charge

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It is shown that a particle set possessing electric charges and masses that coincide with those of the quarks and leptons can be produced with the aid of the symmetry of the cuboctahedron. Specifically, it is shown that small powers of 4.1 are useful in economically reproducing the quark and lepton masses, and that these powers—and thereby the masses they represent—can be joined automatically with their correct values for electric charge with the aid of cuboctahedral symmetry.

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I. THE QUARK AND LEPTON MASS RATIOS

It is possible, with the aid of the symmetry of the cuboctahedron, to generate a particle set possessing electric charges and masses that coincide with those of the quarks and leptons. We begin by noting that the twelve vertices of a cuboctahedron form a surface comprised of four hexagonal rings, six squares, and eight triangles. In Fig. 1 we label these vertices so as to assign values for Q and R to each quark and lepton, where:

- $Q = \{\ 0, -1/3, +2/3, -1\ \}$, the values for electric charge (correctly assigned), and,
- $R = \{\ 0, 1, 1, 2, 3, 5\ \}$, values that will serve as parameters to a mass formula used to generate eight key quark and lepton mass ratios.

Now notice that the cuboctahedron of Fig. 1 exhibits the following symmetry:

- Each of its four hexagonal rings contains three quarks and three leptons.
- Each of its six squares has absolute values for Q that sum to 2.
- Each of its eight triangles has values of R that sum to 6.

The above observations demonstrate that the assignments of Fig. 1 are significantly constrained by symmetry. These assignments will now be used to generate a set of twelve particles whose masses closely approximate those of the quarks and leptons, where it is the symmetrical interplay of the values for charge Q and the mass formula parameter R that will assure that all charges and masses are correctly paired.

But before we generate the quark and lepton masses (or, more precisely, eight mass ratios), we need first to assign values for an additional parameter T. We let $T = 1$ for the u-, d-, c-, and b-quarks (which in the cuboctahedron of Fig. 1 reside in a single plane), while letting $T = 0$ for all other particles. We also let $m = 1$ for all heavy particles, and $m = 2$ for all light particles. These assignments, along with values for R taken from Fig. 1, allow the empirical mass formula

$$M(\Delta R, \Delta T, m) = 4.1^{\Delta R/m} \times 0.1^{\Delta T} \times 3^{1/m} \qquad (1)$$

to produce these symmetrically-related mass ratios for heavy quarks and leptons

$$\begin{aligned} m_\tau/m_e &= M(R_\tau - R_e, T_\tau - T_e, 1) \\ &= 4.1^5 \times 3\ , \end{aligned}$$

$$\begin{aligned} m_\mu/m_e &= M(R_\mu - R_e, T_\mu - T_e, 1) \\ &= 4.1^3 \times 3\ , \end{aligned}$$

$$\begin{aligned} m_t/m_c &= M(R_t - R_c, T_t - T_c, 1) \\ &= 4.1^1 \times 3 \times 10\ , \end{aligned}$$

$$\begin{aligned} m_b/m_c &= M(R_b - R_c, T_b - T_c, 1) \\ &= 4.1^0 \times 3\ , \end{aligned}$$

as well as their light particle counterparts

$$\begin{aligned} m_{\nu_3}/m_{\nu_1} &= M(R_{\nu_3} - R_{\nu_1}, R_{\nu_3} - R_{\nu_1}, 2) \\ &= 4.1^{5/2} \times 3^{1/2}\ , \end{aligned}$$

$$\begin{aligned} m_{\nu_2}/m_{\nu_1} &= M(R_{\nu_2} - R_{\nu_1}, R_{\nu_2} - R_{\nu_1}, 2) \\ &= 4.1^{3/2} \times 3^{1/2}\ , \end{aligned}$$

$$\begin{aligned} m_s/m_u &= M(R_s - R_u, T_s - T_u, 2) \\ &= 4.1^{1/2} \times 3^{1/2} \times 10\ , \end{aligned}$$

$$\begin{aligned} m_d/m_u &= M(R_d - R_u, T_d - T_u, 2) \\ &= 4.1^0 \times 3^{1/2}\ , \end{aligned}$$

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which are readily summarized as follows

$$\left(\frac{m(\nu_3)}{m(\nu_1)}\right)^2 = \frac{m(\tau)}{m(e)} = 4.1^5 \times 3 \ , \quad (2)$$

$$\left(\frac{m(\nu_2)}{m(\nu_1)}\right)^2 = \frac{m(\mu)}{m(e)} = 4.1^3 \times 3 \ , \quad (3)$$

$$\left(\frac{m(s)}{m(u)} \times 0.1\right)^2 = \frac{m(t)}{m(c)} \times 0.1 = 4.1^1 \times 3 \ , \quad (4)$$

$$\left(\frac{m(d)}{m(u)}\right)^2 = \frac{m(b)}{m(c)} = 4.1^0 \times 3 \ . \quad (5)$$

II. COMPARISON AGAINST EXPERIMENT

Tables I–III reveal that these equations closely reproduce their corresponding experimental values, where for the tau- and muon-electron mass ratios the fit is especially precise: to roughly 1 part in 2,000 and 1 part in 40,000, respectively. However, for the neutrinos the following ratio between squared-mass splittings should also hold

$$\frac{m(\nu_3)^2 - m(\nu_2)^2}{m(\nu_2)^2 - m(\nu_1)^2} = \frac{4.1^5 \times 3 - 4.1^3 \times 3}{4.1^3 \times 3 - 1} \approx 15.8 \ . \quad (6)$$

Data exist for both of these splittings, namely [1]

$$|\Delta m^2_{32}| = 2.74^{+0.44}_{-0.26} \times 10^{-3} \ \Delta \mathrm{eV}^2 \quad (7)$$

and [2]

$$|\Delta m^2_{21}| = 8.0^{+0.4}_{-0.3} \times 10^{-5} \ \Delta \mathrm{eV}^2 \ , \quad (8)$$

which combine to yield

$$\frac{|\Delta m^2_{32}|}{|\Delta m^2_{21}|} = \frac{2.74 \times 10^{-3} \ \Delta \mathrm{eV}^2}{8.0 \times 10^{-5} \ \Delta \mathrm{eV}^2} \approx 34.2 \ . \quad (9)$$

This ratio is about twice its calculated value and constitutes the sole major discrepancy.

III. SUMMARY AND CONCLUSION

It has been shown that the quark and lepton masses can be reproduced economically with the aid of an empirical mass formula that relies especially on small powers of 4.1. It has also been shown that these powers, and thereby the masses they represent, can be paired automatically with their correct charges with the aid of the symmetry of cuboctahedron, which also helps determine the powers themselves. All this supports the general conclusion that the mass formula's effectiveness derives from hidden symmetry among the quark and lepton masses and charges. It is logical to ask what physics might underlie this mass formula and the symmetry it exploits.

TABLE I: The heavy lepton mass ratios of Eqs. (2) and (3) compared against experiment.

Ratio	Calculated	Experimental
m_τ/m_e	$4.1^5 \times 3 = 3475.6\ldots$	$3,477.48^{+0.57}_{-0.51}$ [a]
m_μ/m_e	$4.1^3 \times 3 = 206.763$	206.7682823 [b]

[a] Derived from the tau and electron masses taken from Ref. [2]. Fit to roughly 1 part in 2,000.
[b] Ref. [3]. Fit to roughly 1 part in 40,000.

TABLE II: Equations (4) and (5) allow the b- and c-quark masses to be calculated from the top quark's experimental mass of $170,900 \pm 1,800$ MeV [4]; below these calculated masses are compared against experiment.

Mass	Calculated	Experimental [a]
m_b	4,168 ± 44 MeV [b]	$4,200 \pm 70$ MeV
m_c	1,389 ± 15 MeV [c]	$1,250 \pm 90$ MeV

[a] Ref. [2]. It is important to recognize, however, that the experimental values for m_b and m_c are the "running" masses in the $\overline{\mathrm{MS}}$ scheme at $\mu = m_b$ and $\mu = m_c$, respectively.
[b] $170,900 \pm 1,800$ MeV$/4.1^1 \times 3^0 \times 10 = 4,168 \pm 44$ MeV.
[c] $170,900 \pm 1,800$ MeV$/4.1^1 \times 3^1 \times 10 = 1,389 \pm 15$ MeV.

TABLE III: The light quark mass ratios of Eqs. (4) and (5) compared against experiment.

Ratio	Calculated	Experimental [a]
m_s/m_d	$4.1^{1/2} \times 10 = 20.2\ldots$	17 to 22
m_u/m_d	$1/3^{1/2} = 0.577\ldots$	0.3 to 0.6
$\frac{m_s - (m_d + m_u)/2}{m_d - m_u}$	$46.0\ldots$	30 to 50

[a] Ref. [2].

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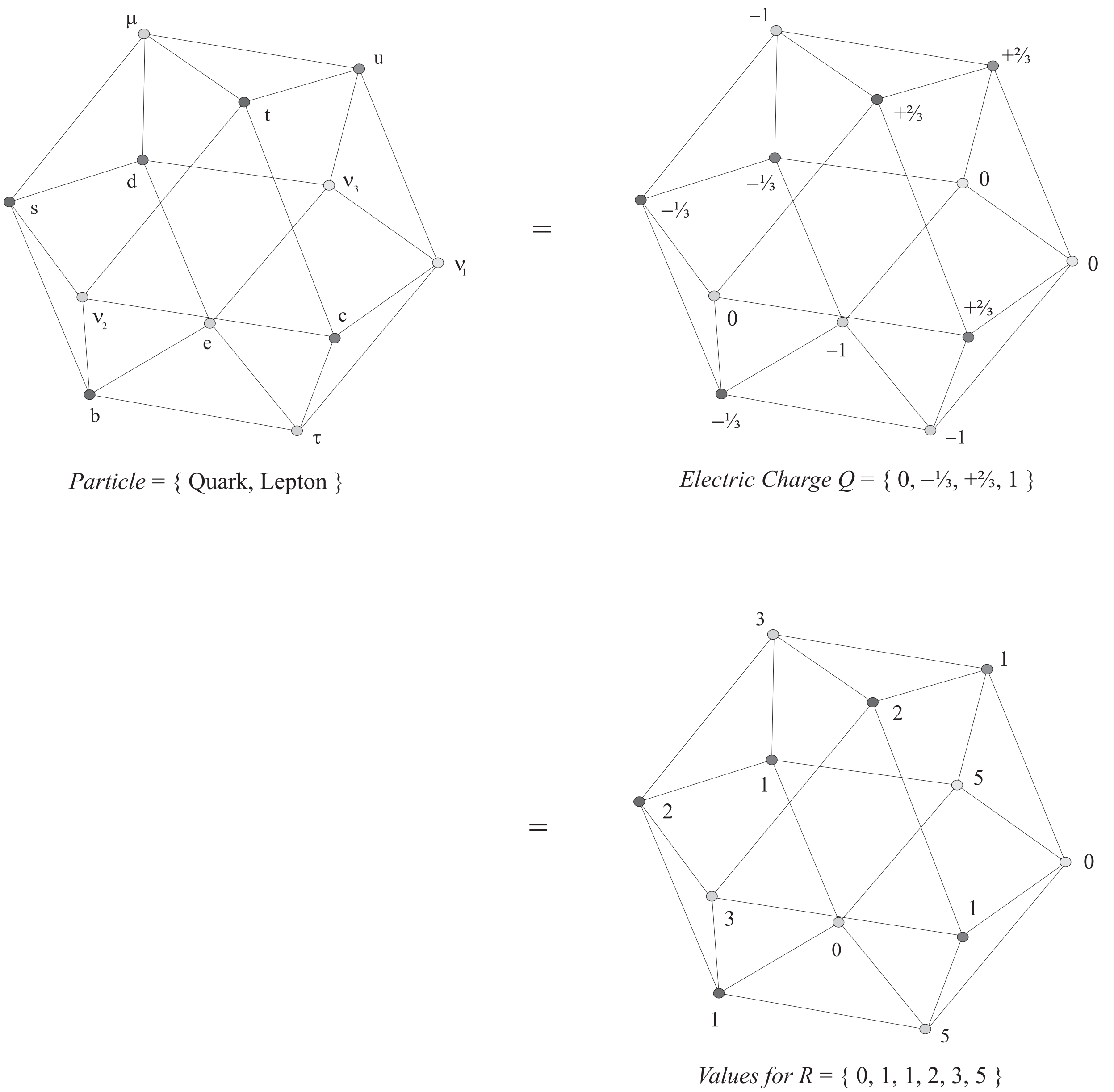

FIG. 1: A cuboctahedron is used to assign *en masse* values for electric charge Q and the mass formula parameter R to all quarks and leptons, where the particles at the upper left get their values for Q and R from the corresponding positions at the right. Note that the cuboctahedron's vertices define four hexagonal rings, each containing three quarks and three leptons. Its surface is also comprised of six squares, each containing absolute values of Q that sum to 2 (at the upper right), and eight triangles, each containing values of R that sum to 6 (at the lower right). It is this symmetry that automatically pairs the correct charges with the correct masses. But the above symmetry does not merely correctly align the values for Q and R: it also helps determine their values. Specifically, it is this cuboctahedral symmetry that helps fix the values of R in each of the above four hexagonal rings to be 0, 1, 1, 2, 3, and 5, the first six Fibonacci numbers, which are precisely those values required by Eq. (1) to reproduce the mass ratios of Eqs. (2)–(5).