

CAN A MASSIVE GRAVITON BE A STABLE PARTICLE ?

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This document is due to a question asked in the Dark Side of the Universe conference, 2010, in Leon, Mexico, when a researcher from India asked the author about how to obtain a stability analysis of massive gravitons. The answer to this question involves an extension of the usual Pauli-Fierz Lagrangian, with non zero graviton mass contributing to a relationship between the trace of a reduced GR stress-energy tensor (assuming non zero graviton mass), and the trace of a reduced symmetric tensor, times a tiny mass for a 4 dimensional graviton. The resulting analysis makes use of Visser's treatment of a stress energy tensor, with experimental applications discussed in the resulting analysis. If the square of frequency of a massive graviton is real valued and greater than zero, stability can be possibly confirmed experimentally.

1 Introduction

The supposition advanced in this article is that relic energy flux initially is central to making predictions as to $S_{entropy} \sim n_f^{1,2}$, where n_f is a 'particle count' per phase space 'volume' in the beginning of inflation. Having said that, is n_f due to gravitons in near relic conditions? If so, can the gravitons carry information? The author in a previous manuscript² identified criteria as to $S_{entropy} \sim n_f \Big|_{start-of-inf} \propto 10^7 \Leftrightarrow$ initial information $\propto 10^{10}$ bits of 'information' in line with G. Smoot's³ Paris (2007) talk as presented in the Paris observatory. Having said that, the relevant issue as raised in DSU 2010, if gravitons with a small mass are part of the bridge between $S_{entropy} \sim n_f \Big|_{start-of-inf} \propto 10^7 \Leftrightarrow$ initial information $\propto 10^{10}$ bits of 'information', can one make a statement about necessary conditions for 'massive' graviton stability? The conclusion is that stability of a massive graviton needs the square of frequency to be positive real valued, for reasons the author will bring up in this manuscript.

1.1 What can be said about massive graviton stability? Necessary conditions

We look at work presented by Maggiore,⁴ which specifically delineated for non zero graviton mass, where $h \equiv \eta^{uv} h_{uv} = Trace \cdot (h_{uv})$ and $T = Trace \cdot (T^{uv})$ that

$$-3m_{graviton}^2 h = \frac{\kappa}{2} \cdot T \quad (1)$$

Our work uses Visser's⁵ 1998 analysis of non zero graviton mass for both T and h. We will use the above equation with a use of particle count n_f for a way to present initial GW relic inflation density using the definition given by Maggiore⁴ as a way to state that a particle count

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}} \right] \cdot \left(\frac{f}{1kHz} \right)^4 \quad (2)$$

where n_f is the frequency-based numerical count of gravitons per unit phase space. To do so, let us give the reasons for using Visser's⁵ values for T and h above, in Eqn. (1).

While Maggiore's explanation⁴, and his treatment of gravitational wave density is very good, the problem we have is that any relic conditions for GW involve stochastic background, and also that many theorists have relied upon either turbulence/ and or other forms of plasma induced generation of shock waves, as stated by Duerrer, et. al.⁶ and others looking at the electro weak transition as a GW generator. If relic conditions can also yield GW / graviton production, and the consequences exist up to the present era, as Beckwith presented, then the question of stability of gravitons is even more essential Beckwith write up an early energy flux for GW/ gravitons which he wrote as²

$$E_{initial-flux} \cong \left[\frac{r^2}{64\pi} \right] \cdot \left| \partial_r^2 h^+ \right|^2 \cdot [\tilde{n} \cdot t_{Planck}]^3 \cdot \Omega_{effective} \quad (3)$$

The n_f value obtained, was used to make a relationship, using Y. J. Ng's entropy⁷ counting algorithm of roughly $S_{entropy} \sim n_f$. We assert that in order to obtain $S_{entropy} \sim n_f$ from initial graviton production, as a way to quantify n_f , that a small mass of the graviton can be assumed. A small mass graviton in four dimensions only makes sense if it is a stable construct. The remainder of this article will be in giving specific cases as to criteria for stability for the low mass 4 dimensional graviton assumed by the author in obtaining his value of $S_{entropy} \sim n_f$ ^{1,2,7} and resultant information content present in the early universe. In doing so, the author will address if the correspondence principle and the closeness of the links to massless formalism of the graviton as will be brought up is due to T'Hoof's^{1,2,8} idea of an embedding of QM within what he calls deterministic quantum theory, involving an embedding of quantum physics within a slightly 'larger' highly non linear structure.

2. Defining the Graviton problem and using Visser's (1998) inputs into T_{uv}

We begin our inquiry by initially looking at a modification of what was presented by R. Maartens⁹, as done by Beckwith^{1,2}

$$m_n(\text{Graviton}) = \frac{n}{L} + 10^{-65} \text{ grams} \quad (4)$$

On the face of it, this assignment of a mass of about 10^{-65} grams for a 4 dimensional graviton, allowing for $m_0(\text{Graviton} - 4D) \sim 10^{-65}$ grams^{1,2} violates all known quantum mechanics, and is to be avoided. Numerous authors, including Maggiore⁴ have richly demonstrated how adding a term to the Fierz Lagrangian for gravitons, and assuming massive gravitons leads to results which appear to violate field theory, as we can call it. Turning to the problem, we can examine what inputs to the Eqn. (1) above can tell us about if there are grounds for $m_0(\text{Graviton} - 4D) \sim 10^{-65}$ grams^{1,2}, and what this says about measurement protocol for both GW and gravitons as given in Eqn. (2) above. Visser⁵, in 1998 came up with inputs into the GR stress tensor and also, for the perturbing term h_{uv} , which will be given below. We will use them in conjunction with Eqn (1) to perform a stability analysis of the consequences of setting the value of $m_0(\text{Graviton} - 4D) \sim 10^{-65}$ grams^{1,2,5}, and from there discuss how T'Hooft's^{1,2,8} supposition of deterministic QM, as an embedding of QFT, and more could play a role if there are conditions for stability of $m_0(\text{Graviton} - 4D) \sim 10^{-65}$ grams^{1,2,5}.

Visser's treatment of the stress energy tensor of GR, and its applications

Visser⁵ in 1998, stated a stress energy treatment of gravitons along the lines of

$$T_{uv}|_{m \neq 0} = \left[\left(\frac{\hbar}{l_p^2 \lambda_g^2} \right) \cdot \left(\frac{GM}{r} \right) \cdot \exp\left(\frac{r}{\lambda_g} \right) + \left(\frac{GM}{r} \right)^2 \right] \times \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

Furthermore, his version of $g_{uv} = \eta_{uv} + h_{uv}$ can be written as setting

$$h_{uv} \equiv 2 \frac{GM}{r} \cdot \left[\exp\left(\frac{-m_g r}{\hbar} \right) \right] \cdot (2 \cdot V_\mu V_\nu + \eta_{uv}) \quad (6)$$

If one adds in velocity ‘reduction’ put in with regards to speed propagation of gravitons⁵

$$v_g = c \cdot \sqrt{1 - \frac{m_g^2 \cdot c^4}{\hbar^2 \omega_g^2}} \quad (7)$$

As well as often setting $(MG/r) \approx 1/5$ for reasons which Visser⁵ outlined, one can insert all this into Eqn. (1) to obtain a real value for the square of frequency > 0 , i.e.

$$\hbar^2 \omega^2 \cong m_g^2 c^4 \cdot [1/(1 - \tilde{A})] > 0 \quad (8)$$

$$\tilde{A} = \left\{ 1 - \frac{1}{6m_g c^2} \left(\frac{\hbar^2}{l_p^2 \lambda_g^2} \cdot \exp\left[-\frac{r}{\lambda_g} + \frac{m_g \cdot r}{\hbar}\right] + \left(\frac{MG}{r}\right) \cdot \exp\left(\frac{m_g r}{\hbar}\right) \right) \right\}^2 \quad (9)$$

According to Jin Young Kim¹⁰, if the square of the frequency of a graviton, with mass, is > 0 , and real valued, it is likely that the graviton is stable, at least with regards to perturbations. Kim’s article¹⁰ is with regards to Gravitons in brane / string theory, but it is likely that the same dynamic for semi classical representations of a graviton with mass.

3. Conditions permitting Eqn (8) to have positive values

Looking at Eqn. (8) is the same as looking at the following, analyzing how

$$\tilde{A} = \left\{ 1 - \frac{1}{6m_g c^2} \left(\frac{\hbar^2}{l_p^2 \lambda_g^2} \cdot \exp\left[-\frac{r}{\lambda_g} + \frac{m_g \cdot r}{\hbar}\right] + \left(\frac{MG}{r}\right) \cdot \exp\left(\frac{m_g r}{\hbar}\right) \right) \right\} < 1 \quad (10)$$

I.e. setting

$$0 < \frac{1}{6m_g c^2} \left(\frac{\hbar^2}{l_p^2 \lambda_g^2} \cdot \exp\left[-\frac{r}{\lambda_g} + \frac{m_g \cdot r}{\hbar}\right] + \left(\frac{MG}{r}\right) \cdot \exp\left(\frac{m_g r}{\hbar}\right) \right) < 1 \quad (11)$$

Note that Visser⁵ (1998) writes $m_g < 2 \times 10^{-29} eV \sim 2 \times 10^{-38} m_{nucleon}$, and a wave length $\lambda_g \sim 6 \times 10^{22}$ meters. The two values, as well as ascertaining when one can use

$\frac{MG}{r} \sim 1/5$, with r the usual distance from a graviton generating source, and M the

mass’ of an object which would be a graviton emitter put severe restrictions as to the volume of space time values for which r could be ascertained. If, however, Eqn. (10) had, in most cases, a setting for which, then in many cases, Eqn. (8) would hold.

$$0 \leq \exp\left[-\frac{r}{\lambda_g} + \frac{m_g \cdot r}{\hbar}\right] \ll 1 \quad (12)$$

The author believes that such a configuration would be naturally occurring in most generation of gravitons at, or before the Electro Weak transition point in early cosmology evolution.

The author, Beckwith, believes, that satisfying Eqn. (12) would allow to predict a particle count behavior along the lines where Beckwith² obtained $n_f \approx 10^6 \text{ to } 10^7$. This value of $n_f \approx 10^6 \text{ to } 10^7$ as given by Beckwith² would be put into Eqn. (2) above, which would have implications for what to look for in stochastic GW generation.

4. Revisiting Ng's counting algorithm for entropy, and Graviton mass

The wave length for a graviton as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be put below, namely: Argue that when taking the log, that the $1/N$ term drops out. As used by Ng⁷

$$Z_N \sim (1/N!) \cdot (V/\lambda^3)^N \quad (13)$$

This, according to Ng,⁷ leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith^{1,2}

$$S \approx N \cdot (\log[V/\lambda^3] + 5/2) \approx N \quad (14)$$

Eventually, the author hopes to put on a sound foundation what 'tHooft¹⁴ is doing with respect to 'tHooft⁸ deterministic quantum mechanics and equivalence classes embedding quantum particle structures.. Furthermore, making a count of gravitons with $S \approx N \sim 10^7$ gravitons⁷, with Seth Lloyd's^{2,11}

$$I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} \sim 10^7 \quad (15)$$

as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced graviton⁷. **Note**, Smoot³ gave initial values of the operations as

$$[\#operations]_{initially} \sim 10^{10} \quad (16)$$

The author's work tends to support this value, and if gravitons are indeed stable in initial conditions, information exchange between a prior to a present universe may become a topic of experimental investigation.

5. Conclusion, giant graviton stability possible, and may allow for survival of gravitons with mass in early universe conditions.

The author pursued this question, partly due to wishing to determine if a non brane theory way to identify graviton stability existed. The author was particularly impressed with Vissor's treatment of gravitons in the context of both an alleged graviton wave length, and the net slow down of gravitons, as referenced in Eqn. (7). Note, that the treatment of Eqn. (6) above heavily depends upon a small mass to the graviton very slightly lowering the speed of graviton to just below the speed of light. As the graviton mass is slight, the velocity of a spin two graviton is ALMOST the speed of light. If Eqn. (12) can be verified experimentally, and there is a search done as to permissible regions of space time for graviton production, then the author hopes for a refinement and vetting experimentally as to Duerrer's et. al's ⁶ supposition of turbulence in the electro weak transition being the major source for GW/ graviton production in early universe cosmology. In addition, it may give experimental evidence for the use by Beckwith ², of Alcubierre's expression of energy density as associated with gravitational waves, in Beckwith's ^{1,2} work with gravitons.

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