Five Properties of the Smarandache Double Factorial Function

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Abstract

In this paper some properties of the Smarandache double factorial function have been analyzed.

In [1], [2], [3] and [4] the Smarandache double factorial Sdf(n) function is defined as the smallest number such that Sdf(n)!! is divisible by n, where the double factorial by definition is given by [6]:

m!! = 1x3x5x...m, if m is odd;
m!! = 2x4x6x...m, if m is even.

In [2] several properties of that function have been analyzed. In this paper five new properties are reported.

1. $Sdf(p^{k+2}) = p^2$ *where* $p = 2 \cdot k + 1$ is any prime and k any integer

Let's consider the prime $p = 2k + 1$. Then:

$1 \cdot 3 \cdot 5 \cdot 7 \cdot .... \cdot p \cdot .... \cdot 3p \cdot .... \cdot 5p \cdot .... \cdot p^2 = m \cdot p^{k+2}$ where m is any integer.

This because the number of terms multiples of p up to p^2 are k+1 and the last term contains two times p.
Then p^2 is the least value such that $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot ..... \cdot p^2$ is divisible by p^{k+2}.

2. $Sdf(p^2) = 3 \cdot p$ where p is any odd prime.

In fact for any odd p we have:

$1 \cdot 3 \cdot 5 \cdot 7 \cdot .... \cdot p \cdot ...... \cdot 3p = m \cdot p^2$ where m is any integer.

3. $Sdf\left(k \cdot \left(\frac{10^n - 1}{9}\right)\right) = Sdf\left(\frac{10^n - 1}{9}\right)$ where n is any integer >1 and k=3,5,7,9

Let's suppose that $Sdf\left(\frac{10^n - 1}{9}\right) = m$ then:

$1 \cdot 3 \cdot 5 \cdot 7 \cdot .... \cdot m = a \cdot \left(\frac{10^n - 1}{9}\right)$ where a is any integer. But in the previous multiplication there are factors multiple of 3,5,7 and 9 and then:

$1 \cdot 3 \cdot 5 \cdot 7 \cdot ..... \cdot m = a' \cdot k \cdot \left(\frac{10^n - 1}{9}\right)$ where a' is any integer and k=3,5,7,9. Then:

$$Sdf\left(k \cdot \left(\frac{10^n - 1}{9}\right)\right) = m = Sdf\left(\frac{10^n - 1}{9}\right)$$

4. $Sdf\left(k \cdot \left(\frac{10^n - 1}{9}\right)\right) = Sdf\left(2 \cdot \left(\frac{10^n - 1}{9}\right)\right)$ where n is any integer >1 and k=2,4,6,8

Let's suppose that $Sdf\left(2 \cdot \left(\frac{10^n - 1}{9}\right)\right) = m$ then:

$2 \cdot 4 \cdot 6 \cdot 8 \cdot ..... \cdot m = a \cdot 2 \cdot \left(\frac{10^n - 1}{9}\right)$ where a is any integer. But in the previous multiplication there are factors multiple of 4, 6 and 8 and then:

$2\cdot 4\cdot 6\cdot 8\cdot .....\cdot m = a'\cdot 2\cdot k\cdot \left(\frac{10^n - 1}{9}\right)$ where a' is any integer and k=4,6,8.

Then:

$$Sdf\left(k\cdot\left(\frac{10^n-1}{9}\right)\right) = m = Sdf\left(2\cdot\left(\frac{10^n-1}{9}\right)\right)$$

5. $Sdf\left(p^m\right) = (2\cdot m - 1)\cdot p \quad for \;\; p \geq (2m-1)$. Here m is any integer and p any odd prime.

This is a generalization of property number 2 reported above.

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