# Where Does Universal Expansion Equal Gravitational Attraction?

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Abstract

A comparison of the attractive motion experienced by masses due to gravitational interaction over relatively short distances with the recessional motion of masses at relatively large distances (that adhere to the velocity increases described by Hubble's $v = Hr$ relation) is presented to demonstrate the similarities between the two motions. Based on the similarities of the two motions, and the observation that gravitational acceleration *decreases* as distance *increases* while recessional acceleration *decreases* as distance *decreases* the distance at which the two accelerations are equal in magnitude but in opposite directions resulting in zero net acceleration is calculated and compared to similar results provided by Chernin et al. [1]. The summation of the attractive gravitational acceleration and the recessional acceleration is presented and plotted depicting a smooth, continuous transition from gravitational attraction to universal expansion. The underlying cause of these accelerations is not addressed.

## 1. INTRODUCTION

This paper examines the similarities between two types of spontaneous motion: gravitational acceleration and the recessional acceleration associated with universal expansion. Based on the similarities between these two types of motion the interaction between the two motions is considered. Specifically, since gravitational acceleration *decreases* as distance *increases* and recessional acceleration *decreases* as distance *decreases* the distance at which the two accelerations are equal in magnitude but in opposite directions resulting in zero net acceleration is calculated and compared to similar results provided by Chernin et al. [1]. A single equation is presented and plotted that depicts the summation of the attractive gravitational acceleration as well as the recessional acceleration.

## 2. BACKGROUND

Edwin Hubble showed us that when examining a group of distant heavenly bodies at some instant in time such bodies are receding away from us and that the rate of recession of the individual bodies increases in proportion to their distances from us. Does this mean that, for a given body, as that body moves away from us over a period of time the recessional velocity of that individual body increases? In other words, are distant bodies accelerating away from us? The recessional acceleration consistent with Hubble's $v = Hr$ relation is derived below and is assumed to be in effect for the remainder of this paper.

But first, consideration of the spontaneous recessional acceleration of distant bodies away from us as described above calls to mind another type of spontaneous acceleration – one characterized by accelerated motion towards us – namely, gravitational acceleration. The two types of motion – gravitational and recessional – are in opposite directions but they share a number of similarities so, while the underlying causes of these motions are not a subject of this paper, a comparison of the observable characteristics of the two motions is in order.

## 3. SIMILARITIES

The two types of motion discussed above – gravitational and recessional - share the following characteristics:

1.) The motions are accelerations. 2.) The accelerations change as a function of distance. 3.) The accelerations are inertial.

3.1 The Motions are Accelerations

Clearly, gravitational free fall motion towards the Earth is an acceleration. As far as the distant motion away from Earth goes, in order to adhere to the relationship discovered by Hubble, since velocity increases as a function of distance, and since distant bodies are increasing their distance from us over time, the velocity of these distant bodies will increase over time. That is, for constant H, in order to adhere to Hubble's law the distant bodies must accelerate. The following mathematical derivation will illustrate further.

The well known Hubble's Law

$$v = Hr \qquad (1)$$

results in an acceleration $A_h$ of

$$A_h = \frac{dv}{dt} = H\frac{dr}{dt} + r\frac{dH}{dt} \qquad (2)$$

where for constant H we have

$$A_h = H\frac{dr}{dt} = Hv \qquad (3)$$

and from Equation (1) above

$$A_h = H^2 r \qquad (4)$$

which describes the acceleration corresponding to Hubble's Law for constant H. Thus both attractive gravitational motion and recessional motion are described as accelerations. (Note: The discovery of non-constant universal expansion [2] is acknowledged but does not materially contribute to the main observations in this article.)

3.2. The Accelerations Change as a Function of Distance

As Equation (4) above shows, the recessional motion described by Hubble is an acceleration which varies as a function of distance *r*. The well known equation describing the gravitational acceleration, *Ag*, of some small test mass near some much larger attracting mass M is

$$A_g = \frac{(GM)}{r^2} \qquad (5)$$

Thus both attractive gravitational acceleration and recessional acceleration are shown to vary as a function of distance.

3.3. The Acceleration is Inertial

Observers in an imaginary “opaque box” (an imaginary container with walls through which no information of any kind can pass) reference frame in free fall towards the Earth cannot tell they are accelerating towards the Earth (Subtleties, such as the fact that point masses in a free falling box will move towards each other as they move towards the same center of mass point of the Earth are noted, but do not contribute to this discussion.). Similarly, observers in an “opaque box” reference frame encompassing the entire planet Earth cannot tell they are accelerating away from distant heavenly bodies. Put another way, inhabitants of the Earth or of some distant celestial body don’t “feel” any accelerations away from each other just as occupants of a free falling reference frame cannot “feel” an acceleration towards Earth. Thus both attractive gravitational acceleration and recessional acceleration are shown to inertial.

4. THE BREAK EVEN POINT

The comparisons above reveal several similarities that lead one to wonder how the two accelerations interact. Firstly, the rates at which the two types of acceleration vary as a function of distance differ in that the gravitational acceleration varies as $1/r^2$ while the recessional acceleration varies linearly with r. However, as a result of the shared similarities described above, there is a smooth transition from one acceleration to the other as distance increases without any discontinuity. In fact, the only significant difference between the two types of motion is the direction in which the acceleration occurs. Consider that when looking out into space from our vantage point on planet Earth we see the Hubble recessional acceleration *decrease* as the distance from Earth *decreases* while the gravitational acceleration *decreases* as distance from Earth *increases*. That is, if we imagine moving in from some relatively large distance towards some point in space closer to Earth the spontaneous (recessional) acceleration decreases towards zero while if we imagine moving out from Earth to the same point in space the spontaneous (gravitational) acceleration also decreases towards zero. The decreasing acceleration towards the same point in space, and the overall similarities of the two accelerations described above leads one to wonder: Will the two accelerations “cancel one another out” at some point in space?

Viewed another way, one must wonder - Are there any remnants of the recessional tendencies present at relatively short distances? Are there remnants of the gravitation tug we observe locally in operation over relatively large distances? There must be *some* relationship between gravitational and recessional acceleration – What is that relationship?

In order to find where the two accelerations "cancel each other out" consider again Equation (4)

$$A_h = H^2 r \qquad (4)$$

and Equation (5)

$$A_g = \frac{(GM)}{r^2} \qquad (5)$$

We can now set $A_h$ equal to $A_g$ and solve for the distance $r = R_c$, the critical distance at which the two accelerations would have to be equal.

$$A_g = A_h \qquad (6)$$

From (4) and (5)

$$\frac{GM}{R_c{}^2} = H^2 R_c \qquad (7)$$

Solving for *Rc*

$$\left[\frac{GM}{H^2}\right]^{1/3} = R_c \qquad (8)$$

This is the distance beyond which gravitational acceleration is exceeded by recessional acceleration. (As an aside, it is interesting step back and consider how such fundamental constants as G and H can be combined with mass as in Equation (8) to produce a parameter with units of length, that is, a distance. Also, if the exponent 1/3 is left out of Equation (8) a volume is produced.)

To put Equation (8) into some kind of perspective consider *Rc* for a mass the size of the Milky Way galaxy. For

$G = 6.672 \text{x} 10^{-11}\ Nm^2/kg^2$

M = Mass of the Milky Way galaxy, $1.2 \text{x} 10^{42}$ kg

H = 72 km $s^{-1}$ $\text{megaparsec}^{-1}$

we have

$$R_c \approx 2.6 \text{ Million light years} \qquad (9)$$

As mentioned above, Chernin et al [1] calculate an equivalent to Equation (8), but they approach the matter from a general relativistic consideration of dark energy. Specifically, they describe a "zero-gravity surface" of radius $R_v$ where

$$\left[\frac{3M}{8\pi\rho_v}\right]^{1/3} = R_v \qquad (10)$$

and where $\rho_v$ is the local dark energy density. So, how does Equation (8), which is derived from the assumption that Hubble's Law arises from a recessional acceleration, compare to Equation (10), which is derived from a consideration of the local dark energy density? Specifically - How do Equations (8) and (10) compare numerically for some arbitrary mass M? This boils down to determining whether the following relation is true:

$$\frac{G}{H^2} = \frac{3}{8\pi\rho_v} \qquad (11)$$

For G and H defined as above we have

$$\frac{G}{H^2} = 1.2254x10^{25} \qquad (12)$$

On a global scale current estimates give $\rho_v = 10^{-29}$, which yields

$$\frac{3}{8\pi\rho_v} = 1.1937\text{x}10^{25} \qquad (13)$$

The close agreement (less than 3% difference) of Equations (12) and (13) above demonstrates that the zero-gravity surface can be computed based on G and H without resorting to any general relativistic considerations.

## 5. A SINGLE EQUATION FOR BOTH GRAVITATIONAL AND RECESSIONAL ACCELERATIONS

Consider two relatively nearby masses adrift in space such that they are free to spontaneously move towards each other due to gravitational interaction. Application of Equation (5) alone to this system would indicate that as the distance between the two masses was continuously increased the tendency of the masses to move towards each other would become weaker and weaker - eventually going to zero only as the distance between the two masses approached infinity. Similarly, two masses with a very large distance between them would tend to spontaneously accelerate away from each other and an application of Equation (4) alone would indicate that this tendency would go to zero only as the distance between the masses went to zero. In order to examine the interaction between the two accelerations consider a summation of Equation (4) and Equation (5) to produce a resultant total acceleration $A_{total}$ as follows:

$$A_{total} = (H^2 R) - \frac{GM}{R^2} \qquad (14)$$

An application of Equation (14) to the system of two masses that are initially relatively close to each other would indicate that as distance increases the tendency to move towards each other gets weaker and weaker only until the distance $R_c << \infty$ is reached, at which point the tendency goes to zero per Equation (7), and beyond which there is a tendency to accelerate again, but now in the opposite direction, i.e., away from each other.

A plot of the combined gravitational and recessional accelerations described by Equation (14) for various distances R where M equals the mass of the Milky Way galaxy is provided as Figure 1. A plot of the corresponding gravitational acceleration only is included for reference.

The representation of Equation (14) by Figure 1 shows that for small R, that is for relatively small distances, the total acceleration is dominated by the gravitational term $GM/R^2$ while for relatively large distances the total acceleration is dominated by the recessional term $H^2R$. Of particular note is the lack of dependence of the recessional acceleration term on mass. Mass only comes into play at relatively short distances to generate a local deviation from the more generally observed recessional motion.

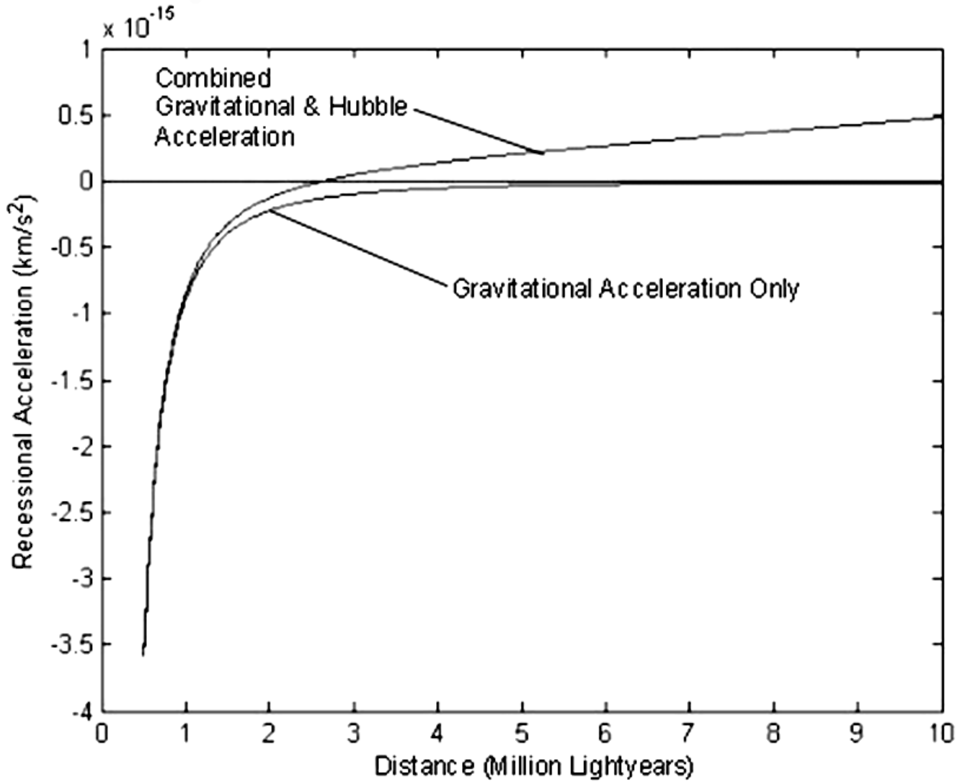


Figure 1.  Recessional Acceleration vs. Distance

6.  CONCLUSION

The similarities between gravitational acceleration and recessional acceleration are used to illustrate a continuous transition from one type of motion to the other as shown in Figure 1.  The results presented compare favorably to the "zero-gravity surface" proposed by Chernin et al. [1].

References:

[1]  Chernin et al., arXiv.org, Cornell Univeristy Library, “Detection of dark energy near the Local Group with the Hubble Space Telescope”, http://arxiv.org/abs/0706.4068v1  (2007)  (Accessed 15 November 2009)

[2]  Riess et al, Astronomical Journal, September 1998,  volume 116, page 1009