The New Prime theorem（39）

On the $P, jP^4+k-j(j=1,\cdots,k-1)$

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Abstract

Using Jiang function we prove that if $J_2(\omega)\neq 0$ then there are infinitely many primes P such that each of jP^4+k-j is a prime, if $J_2(\omega)=0$ then there are finite primes P such that each of jP^4+k-j is a prime.

Theorem . Let k be a given prime.

$$P, jP^4+k-j(j=1,\cdots,k-1) \quad (1)$$

We have Jiang function [1,2]

$$J_2(\omega)=\prod_P[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^4+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

From (2) and (3) we have that if $J_2(\omega)\neq 0$ then there are infinitely many primes P such that each of jP^5+k-j is a prime, if $J_2(\omega)=0$ then it has only finite prime solutions. If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^4+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(4)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (4)$$

Example 1. Let $k=3$. From (1) we have

$$P, P^4+2, 2P^4+1 \quad (5)$$

From (3) we have $\chi(3)=2$. From (2) we have

$$J_2(3)=0, \quad J_2(\omega)=0 \quad (6)$$

We prove that (5) contain only a solution $P=3$, $P^4+2=83$, $2P^4+1=163$.

Example 2. Let $k=5$. From (1) we have

$$P, jP^4+5-j(j=1,2,3,4) \quad (7)$$

From (3) we have $\chi(5)=4$. From (2) we have

$$J_2(5)=0, \quad J_2(\omega)=0 \quad (8)$$

We prove that (7) contain no prime a solutions.

Example 3. Let $k=7$. From (1) we have

$$P,\ jP^4+7-j(j=1,2,3,4,5,6) \qquad (9)$$

From (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (10)$$

We prove that (9) contain infinitely many prime solutions. If $k\geq 7$ we have

$$J_2(\omega)\neq 0 \qquad (11)$$

We prove that (1) contain infinitely many prime solutions.

Remark. The prime number theory is basically to count the Jiang function $J_{n+1}(\omega)$ and Jiang prime k-tuple singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1,2], which can count the number of prime numbers. The prime distribution is not random. But Hardy-Littlewood prime k-tuple singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [3-8], which cannot count the number of prime numbers[3].

References

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Szemerédi's theorem does not directly to the primes, because it cannot count the number of primes. Cramér's random model cannot prove any prime problems. The probability of $1/\log N$ of being prime is false. Assuming that the events "P is prime", "$P+2$ is prime" and "$P+4$ is prime" are independent, we conclude that P, $P+2$, $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ primes less than N. Letting $N \to \infty$ we obtain the prime conjecture, which is false. The tool of additive prime number theory is basically the Hardy-Littlewood prime tuples conjecture, but cannot prove and count any prime problems[6].

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler(1707-1783)

It will be another million years, at least, before we understand the primes.

Paul Erdos(1913-1996)

Of course, the primes are a deterministic set of integers, not a random one, so the predictions given by random models are not rigorous (Terence Tao, Structure and randomness in the prime numbers, preprint). 陶哲轩认为素数不是随机的改变他过去看法。

Erdos and Turán(1936) contributed to probabilistic number theory, where the primes are treated as if they were random, which generates Szemerédi's theorem (1975) and Green-Tao theorem(2004). But they cannot actually prove and count any simplest prime examples: twin primes and Goldbach's conjecture. They don't know what prime theory means, only conjectures.