On a Concatenation Problem

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Abstract: This article has been inspired by questions asked by Charles Ashbacher in the *Journal of Recreational Mathematics, vol. 29.2.* It concerns the Smarandache Deconstructive Sequence. This sequence is a special case of a more general concatenation and sequencing procedure which is the subject of this study. Answers are given to the above questions. The properties of this kind of sequences are studied with particular emphasis on the divisibility of their terms by primes.

1. Introduction

In this article the concatenation of a and b is expressed by a_b or simply ab when there can be no misunderstanding. Multiple concatenations like abcabcabc will be expressed by 3(abc).

We consider n different elements (or n objects) arranged (concatenated) one after the other in the following way to form:

$A=a_1a_2 \ldots a_n$.

Infinitely many objects A, which will be referred to as cycles, are concatenated to form the chain:

$B= a_1a_2 \ldots a_n\ a_1a_2 \ldots a_n\ a_1a_2 \ldots a_n\ldots$

B contains identical elements which are at equidistant positions in the chain. Let's write B as

$B=b_1b_2b_3, \ldots b_k\ldots..$ where $b_k=a_j$ when $j\equiv k \pmod n$, $1\le j\le n$.

An infinite sequence C_1, C_2, C_3, … C_k, …. is formed by sequentially selecting 1, 2, 3, …k, … elements from the chain B:

$C_1=b_1=a_1$

$C_2=b_2b_3=a_2a_3$

$C_3=b_4b_5b_6=a_4a_5a_6$ (if $n\le 6$, if n=5 we would have $C_3=a_4a_5a_1$)

The number of elements from the chain B used to form first k-1 terms of the sequence C is 1+2+3+ … +k-1=(k-1)k/2. Hence

$$C_k = b_{\frac{(k-1)k}{2}+1}\, b_{\frac{(k-1)k}{2}+2} \cdots b_{\frac{k(k+1)}{2}}$$

However, what is interesting to see is how C_k is expressed in terms of $a_1,\ldots,a_n$. For sufficiently large values of k C_k will be composed of three parts:

The first part $F(k)=a_u\ldots a_n$

The middle part $M(k)=AA\ldots A$ The number of concatenated As depends on k.

The last part $L(k)=a_1a_2\ldots a_w$

Hence

$C_k=F(k)M(k)L(k)$. (1)

The number of elements used to form $C_1, C_2, \ldots C_{k-1}$ is (k-1)k/2. Since the number of elements in A is finite there will be infinitely many terms C_k which have the same first element a_u. u can be determined from $\frac{(k-1)k}{2} + 1 \equiv u (\bmod n)$. There can be at most n^2 different combinations to form F(k) and L(k). Let C_j and C_i be two different terms for which F(i)=F(j) and L(i)=L(j). They will then be separated by a number m of complete cycles of length n, i.e.

$$\frac{(j-1)j}{2} - \frac{(i-1)i}{2} = mn$$

Let's write j=i+p and see if p exists so that there is a solution for p which is independent of i.

$$(i+p-1)(i+p)-(i-1)i=2mn$$
$$i^2+2ip+p^2-i-p-i^2+i=2mn$$
$$2ip+p^2-p=2mn$$
$$p^2+p(2i-1)=2mn$$

If n is odd we will put p=n to obtain n+2i-1=2m, or $m = \frac{n+2i-1}{2}$. If n is even we put p=2n to obtain m=2n+2i-1. From this we see that the terms C_k have a peculiar periodic behavior. The periodicity is p=n for odd n and p=2n for even n. Let's illustrate this for n=4 and n=5 for which the periodicity will be p=8 and p=5 respectively.

Table 1. n=4. A=abcd. B= abcdabcdabcdabcdabcd……

i	C_i	Period #	F(i)	M(i)	L(i)
1	a		a		
2	bc		bc		
3	dab	1	d		ab
4	cdab	1	cd		ab
5	cdabc	1	cd		abc
6	dabcda	1	d	abcd	a
7	bcdabcd	1	bcd	abcd	
8	abcdabcd	1		2(abcd)	
9	abcdabcda	1		2(abcd)	a
10	bcdabcdabc	1	bcd	abcd	abc
11	dabcdabcdab	2	d	2(abcd)	ab
12	cdabcdabcdab	2	cd	2(abcd)	ab
13	cdabcdabcdabc	2	cd	2(abcd)	abc
14	dabcdabcdabcda	2	d	3(abcd)	a
15	bcdabcdabcdabcd	2	bcd	3(abcd)	
16	abcdabcdabcdabcd	2		4(abcd)	
17	abcdabcdabcdabcda	2		4(abcd)	a
18	bcdabcdabcdabcdabc	2	bcd	3(abcd)	abc
19	dabcdabcdabcdabcdab	3	d	4(abcd)	ab
20	cdabcdabcdabcdabcdab	3	cd	4(abcd)	ab

It is seen from table 1 that the periodicity starts for i=3.

Numerals are chosen as elements to illustrate the case n=5. Let's write i=s+k+pj , where s is the index of the term preceding the first periodical term, k=1,2,…,p is the index of members of the period and j is the number of the period (for convenience the first period is numbered 0). The first part of C_i is denoted B(k) and the last part E(k). C_i is now given by the expression below where q is the number of cycles concatenated between the first part B(k) and the last part E(k).

$$C_i=B(k)\_qA\_E(k), \text{ where k is determined from } i-s\equiv k \pmod p \qquad (2)$$

Table 2. n=5. A=12345. B= 123451234512345………

i		C_i	k	q	F(i)/B(k)	M(i)	L(i)/E(k)
1		1			1		
s=2		23			23		
	j=0						
3		451	1	0	45		1
4		2345	2	0	2345		
5		12345	3	1		12345	
6		123451	4	1		12345	1
7		2345123	5	0	2345		123
	j=1						
3+5j		45123451	1	j	45	12345	1
4+5j		234512345	2	j	2345	12345	
5+5j		1234512345	3	j+1		2(12345)	
6+5j		12345123451	4	j+1		2(12345)	1
7+5j		234512345123	5	j	2345	12345	123
	j=2						
3+5j		4512345123451	1	j	45	2(12345)	1
4+5j		23451234512345	2	j	2345	2(12345)	
……							

2. The Smarandache Deconstructive Sequence

The Smarandache Deconstructive Sequence of integers [1] is constructed by sequentially repeating the digits 1-9 in the following way:

1,23,456,789123,4567891,23456789,123456789,1234567891, …

The sequence was studied in a booklet by Kashihara [2] and a number of questions on this sequence were posed by Ashbacher [3]. In thinking about these questions two observations lead to this study.

1. Why did Smarandache exclude 0 from the integers used to create the sequence? After all 0 is indispensable in all arithmetics most of which can be done using 0 and 1 only.
2. The process used to create the Deconstructive Sequence is a process that applies to any set of objects as has been shown in the introduction.

The periodicity and the general expression for terms in the "generalized deconstructive sequence" shown in the introduction may be the most important results of this study. These results will now be used to examine the questions raised by Ashbacher. It is worth noting that these divisibility questions are dealt with in base 10 although only the nine digits 1,2,3,4,5,6,7,8,9 are used to express numbers. In the last part of this article questions on divisibility will be posed for a deconstructive sequence generated from A="0123456789".

For $i>5$ ($s=5$) any term C_i in the sequence is composed by concatenating a first part B(k), a number q of cycles A="123456789" and a last part E(k), where $i=5+k+9j$, $k=1,2,\dots 9$, $j\geq 0$, as expressed in (2) and $q=j$ or $j+1$ as shown in table 3.

Members of the Smarandache Deconstructive Sequence are now interpreted as decimal integers. The factorization of B(k) and E(k) is shown in table 3. The last two columns of this table will be useful later in this article.

Table 3. Factorization of Smarandache Deconstructive Sequence

i	k	B(k)	q	E(k)	Digit sum	$3\mid C_i$?
6+9j	1	$789=3\cdot 263$	j	$123=3\cdot 41$	30+j45	3
7+9j	2	$456789=3\cdot 43\cdot 3541$	j	1	40+j45	No
8+9j	3	23456789	j		44+j45	No
9+9j	4		j+1		$(j+1)\cdot 45$	$9\cdot 3^z$ *
10+9j	5		j+1	1	$1+(j+1)\cdot 45$	No
11+9j	6	23456789	j	123=341	50+j45	No
12+9j	7	$456789=3\cdot 43\cdot 3541$	j	$123456=2^6\cdot 3\cdot 643$	60+j45	3
13+9j	8	$789=3\cdot 263$	j+1	1	$25+(j+1)\cdot 45$	No
14+9j	9	23456789	j	$123456=2^6\cdot 3\cdot 643$	65+j45	No

*) where z depends on j.

Together with the factorization of the cycle $A=123456789=3^2\cdot 3607\cdot 3803$ it is now possible to study some divisibility properties of the sequence. We will first find expressions for C_i for each of the 9 values of k. In cases where E(k) exists let's introduce $u=1+[\log_{10}E(k)]$. We also define the function $\delta(j)$ so that $\delta(j)=0$ for $j=0$ and $\delta(j)=1$ for $j>0$. It is possible to construct one algorithm to cover all the nine cases but more functions like $\delta(j)$ would have to be introduced to distinguish between the numerical values of the strings "" (empty string) and "0" which are both evaluated as 0 in computer applications. In order to avoid this four formulas are used.

For k=1, 2, 6, 7 and 9:

$$C_{5+k+9j} = E(k) + \delta(j)\cdot A\cdot 10^{u}\cdot \sum_{r=0}^{j-1} 10^{9r} + B(k)\cdot 10^{9j+u} \quad (3)$$

For k=3:

$$C_{5+k+9j} = \delta(j)\cdot A\cdot \sum_{r=0}^{j-1} 10^{9r} + B(k)\cdot 10^{9j} \quad (4)$$

For k=4:

$$C_{5+k+9j} = A\cdot \sum_{r=0}^{j} 10^{9r} \quad (5)$$

For k=5 and 8:

$$C_{5+k+9j} = E(k) + A\cdot 10^{u}\cdot \sum_{r=0}^{j} 10^{9r} + B(k)\cdot 10^{9(j+1)+u} \quad (6)$$

Before dealing with the questions posed by Ashbacher we recall the familiar rules: An even number is divisible by 2; a number whose last two digit form a number which is divisible by 4 is divisible by 4. In general we have the following:

Theorem. Let N be an n-digit integer such that $N>2^{\alpha}$ then N is divisible by 2^{α} if and only if the number formed by the α last digits of N is divisible by 2^{α}.

Proof. To begin with we note that

If x divides a and x divides b then x divides (a+b)
If x divides one but not the other of a and b then x does not divide (a+b)
If x does not divides neither a nor b then x may or may not divide (a+b)

Let's write the n-digit number in the form $a\cdot 10^{\alpha}+b$. We then see from the following that $a\cdot 10^{\alpha}$ is divisible by 2^{α}.

$10 \equiv 0 \pmod 2$
$100 \equiv 0 \pmod 4$
$1000 = 2^3\cdot 5^3 \equiv 0 \pmod{2^3}$
…
$10^{\alpha} \equiv 0 \pmod{2^{\alpha}}$

and then

$a\cdot 10^{\alpha} \equiv 0 \pmod{2^{\alpha}}$ independent of a.

Now let b be the number formed by the α last digits of N we then see from the introductory remark that N is divisibe by 2^{α} if and only if the number formed by the α last digits is divisibele by 2^{α}.

Question 1. Does every even element of the Smarandache Deconstructive Sequence contain at least three instances of the prime 2 as a factor?

Question 2. If we form a sequence from the elements of the Smarandache Deconstructive Sequence that end in a 6, do the powers of 2 that divide them form a montonically increasing sequence?
These two questions are realated and are dealt with together. From the previous analysis we know that all even elements of the Smarandache Deconstructive Sequence end in a 6. For $i \leq 5$ they are:

$C_3=456=57\cdot 2^3$

$C_5=23456=733\cdot 2^5$

For i>5 they are of the forms:

C_{12+9j} and C_{14+9j} which both end in …789123456.

Examining the numbers formed by the 6, 7 and 8 last digits for divisibility by 2^6, 2^7 and 2^8 respectively we have:

$123456=2^6\cdot 3\cdot 643$

$9123456=2^7\cdot 149.4673$

89123456 is not divisible by 2^8

From this we conclude that all even Smarandache Deconstructive Sequence elements for $i \geq 12$ are divisible by 2^7 and that no elements in the sequence are divisible by higher powers of 2 than 7.

Answer to Qn 1. Yes
Answer to Qn 2. The sequence is monotonically increasing for i£12. For i³12 the powers of 2 that divide even elements remain constant=2^7.

Question 3. Let x be the largest integer such that $3^x|i$ and y the largest integer such that $3^y|C_i$. Is it true that x is always equal to y?

From table 3 we se that the only elements C_i of the Smarandache Deconstructive Sequence which are divisible by powers of 3 correspond to i=6+9j, 9+9j, or 12+9j. Furthermore, we see that i=6+9j and C_{6+9j} are divisible by 3 no more no less. The same is true for i=12+9j and C_{12+9j}. So the sta tement holds in these cases.
From the conguences

$9+9j\equiv 0 \pmod{3^x}$ for the index of the element

and

$45(1+j)\equiv 0 \pmod{3^y}$ for the corresponding element

we conclude that x=y.

Answer: The statement is true. It is interesting to note that, for example the 729 digit number C_{729} is divisible by 729.

Question 4. Are there other patterns of divisibility in this sequence?

A search for other patterns would continue by examining divisibility by the next lower primes 5, 7, 11, … It is obvious from table 3 and the periodicity of the sequence that there are no elements divisible by 5. The algorithms will prove very useful. For each value of k the value of C_i depends on j only. The divisibility by a prime p is therefore determined by finding out for which values of j and k the congruence $C_i\equiv 0 \pmod{p}$

holds. We evaluate $\sum_{r=0}^{j-1} 10^{9r} = \frac{10^{9j}-1}{10^9-1}$ and introduce $G=10^9-1$. We note that $G=3^4\cdot 37\cdot 333667$. From formulas (3) to (6) we now obtain:

For k=1,2,6,7 and 9:

$$C_i\cdot G=10^u\cdot(\delta(j)\cdot A+B(k)\cdot G)\cdot 10^{9j}+E(k)\cdot G-10^u\cdot\delta(j)\cdot A \quad (3')$$

For k=3:

$$C_i\cdot G=((\delta(j)\cdot A+B(k)\cdot G)\cdot 10^{9j}-\delta(j)\cdot A \quad (4')$$

For k=4:

$$C_i\cdot G=A\cdot 10^{9j}-A \quad (5')$$

For k=5 and 8:

$$C_i\cdot G=10^{u+9}(A+B(k)\cdot G)\cdot 10^{9j}+E(k)\cdot G-10^u\cdot A \quad (6')$$

The divisibility of C_i by a prime p other than 3, 37 and 333667 is therefore determined by solutions for j to the congruences $C_iG\equiv 0 \pmod p$ which are of the form

$$a\cdot(10^9)^j+b\equiv 0 \pmod p \quad (7)$$

Table 4 shows the results from computer implementation of the congruences. The appearance of elements divisible by a prime p is periodic, the periodicity is given by $j=j_1+m\cdot d$, m=1, 2, 3, … . The first element divisible by p appears for i_1 corresponding to j_1. In general the terms C_i divisible by p are $C_{5+k+9(j_1+md)}$ where d is specific to the prime p and m=1, 2, 3,… .We note from table 4 that d is either equal to p-1 or a divisor of p-1 except for the case p=37 which as we have noted is a factor of A. Indeed this periodicity follows from Euler's extension of Fermat's little theorem because if we write (mod p):

$$a\cdot(10^9)^j+b= a\cdot(10^9)^{j_1+md}+b\equiv a\cdot(10^9)^{j_1}+b \text{ for } d=p-1 \text{ or a divisor of } p-1.$$

Finally we note that the periodicity for p=37 is d=37.

Question: Table 4 indicates some interesting patterns. For instance, the primes 19, 43 and 53 only divides elements corresponding to k=1, 4 or 7 for j<150 which was set as an upper limit for this study. Similarly, the primes 41, 73, 79 and 91 only divides elements corresponding to k=4. Is 5 the only prime that cannot divide an element of the Smarandache Deconstructive Sequence?

Table 4. Smarandache Deconstructive Sequence elements divisible by p:

p	k	i_1	j_1	d	p	k	i_1	j_1	d
7	4	18	1	2	47	1	150	16	46
11	4	18	1	2	47	2	250	27	46
13	4	18	1	2	47	3	368	40	46
13	8	22	1	2	47	4	414	45	46
13	9	14	0	2	47	5	46	4	46
17	1	6	0	16	47	6	164	17	46
17	2	43	4	16	47	7	264	28	46
17	3	44	4	16	47	8	400	43	46
17	4	144	15	16	47	9	14	0	46
17	5	100	10	16	53	1	24	2	13
17	6	101	10	16	53	4	117	12	13
17	7	138	14	16	53	7	93	9	13
17	8	49	4	16	59	1	267	29	58
17	9	95	9	16	59	3	413	45	58
19	1	15	1	2	59	5	109	11	58
19	4	18	1	2	59	6	11	0	58
19	7	21	1	2	59	7	255	27	58
23	1	186	20	22	59	8	256	27	58
23	2	196	21	22	59	9	266	28	58
23	3	80	8	22	61	2	79	8	20
23	4	198	21	22	61	4	180	19	20
23	5	118	12	22	61	6	101	10	20
23	6	200	21	22	67	4	99	10	11
23	7	12	0	22	67	8	67	6	11
23	8	184	19	22	67	9	32	2	11
23	9	14	0	22	71	1	114	12	35
29	1	24	2	28	71	3	53	5	35
29	2	115	12	28	71	4	315	34	35
29	3	197	21	28	71	5	262	28	35
29	4	252	27	28	71	7	201	21	35
29	5	55	5	28	73	4	72	7	8
29	6	137	14	28	79	4	117	12	13
29	7	228	24	28	83	1	348	38	41
29	8	139	14	28	83	2	133	14	41
29	9	113	11	28	83	4	369	40	41
31	3	26	2	5	83	6	236	25	41
31	4	45	4	5	83	7	21	1	41
31	5	19	1	5	83	8	112	11	41
37	1	222	24	37	83	9	257	27	41
37	2	124	13	37	89	2	97	10	44
37	3	98	10	37	89	4	396	43	44
37	4	333	36	37	89	6	299	32	44
37	5	235	25	37	97	1	87	9	32
37	6	209	22	37	97	2	115	12	32
37	7	111	11	37	97	3	107	11	32
37	8	13	0	37	97	4	288	31	32
37	9	320	34	37	97	5	181	19	32
41	4	45	4	5	97	6	173	18	32
43	1	33	3	7	97	7	201	21	32
43	4	63	6	7	97	8	202	21	32
43	7	30	2	7	97	9	86	8	32

3. A Deconstructive Sequence generated by the cycle A=0123456789.

Instead of sequentially repeating the digits 1-9 as in the case of the Smarandache Deconstructive Sequence we will use the digits 0-9 to form the corresponding sequence:

0,12,345,6789,01234,567890,1234567,89012345,678901234, 5678901234, 56789012345,678901234567, …

In this case the cycle has n=10 elements. As we have seen in the introduction the sequence then has a period =2n=20. The periodicity starts for i=8. Table 5 shows how for i>7 any term C_i in the sequence is composed by concatenating a first part B(k), a number q of cycles A="0123456789" and a last part E(k), where i=7+k+20j, k=1,2,…20, j≥0, as expressed in (2) and q=2j, 2j+1 or 2j+2. In the analysis of the sequence it is important to distinguish between the cases where E(k)=0, k=6,11,14,19 and cases where E(k) does not exist, i.e. k=8,12,13,14. In order to cope with this problem we introduce a function u(k) which will at the same time replace the functions δ(j) and $u=1+[\log_{10}E(k)]$ used previously. u(k) is defined as shown in table 5. It is now possible to express C_i in a single formula

$$C_i=C_{7+k+20j} = E(k) + (A\cdot\sum_{r=0}^{q(k)+2j-1}(10^{10})^r + B(k)\cdot(10^{10})^{q(k)+2j})\cdot 10^{u(k)} \quad (8)$$

The formula for C_i was implemented modulus prime numbers less then 100. The result is shown in table 6. Again we note that the divisibility by a prime p is periodic with a period d which is equal to p- 1 or a divisor of p-1, except of p=11 and p=41 which are factors of 10^{10}-1. The cases p=3 and 5 have very simple answers and are not included in table 6.

Table 5. n=10, A=0123456789

i	k	B(k)	q	E(k)	u(k)
8+20j	1	89	2j	$012345=3\cdot5\cdot823$	6
9+20j	2	$6789=3\cdot31\cdot73$	2j	$01234=2\cdot617$	5
10+20j	3	$56789=109\cdot521$	2j	$01234=2\cdot617$	5
11+20j	4	$56789=109\cdot521$	2j	$012345=3\cdot5\cdot823$	6
12+20j	5	$6789=3\cdot31\cdot73$	2j	$01234567=127\cdot9721$	8
13+20j	6	89	2j+1	0	1
14+20j	7	$123456789=3^2\cdot3607\cdot3803$	2j	$01234=2\cdot617$	5
15+20j	8	$56789=109\cdot521$	2j+1		0
16+20j	9		2j+1	$012345=3\cdot5\cdot823$	6
17+20j	10	$6789=3\cdot31\cdot73$	2j+1	$012=2^2\cdot3$	3
18+20j	11	$3456789=3\cdot7\cdot97\cdot1697$	2j+1	0	1
19+20j	12	$123456789=3^2\cdot3607\cdot3803$	2j+1		0
20+20j	13		2j+2		0
21+20j	14		2j+2	0	1
22+20j	15	$123456789=3^2\cdot3607\cdot3803$	2j+1	$012=2^2\cdot3$	3
23+20j	16	$3456789=3\cdot7\cdot97\cdot1697$	2j+1	$012345=3\cdot5\cdot823$	6
24+20j	17	$6789=3\cdot31\cdot73$	2j+2		0
25+ 20j	18		2j+2	$01234=2\cdot617$	5
26+20j	19	$56789=109\cdot521$	2j+2	0	1
27+20j	20	$123456789=3^2\cdot3607\cdot3803$	2j+1	$01234567=127\cdot9721$	8

Table 6a. Divisibility of the 10-cycle destructive sequence by primes 7≤p≤37

p	k	i_1	j_1	d	p	k	i_1	j_1	d
7	3	30	1	3	19	1	128	6	9
7	6	13	0	3	19	2	149	7	9
7	7	14	0	3	19	3	90	4	9
7	8	15	0	3	19	4	31	1	9
7	11	38	1	3	19	5	52	2	9
7	12	59	2	3	19	10	117	5	9
7	13	60	2	3	19	12	179	8	9
7	14	61	2	3	19	13	180	8	9
7	15	22	0	3	19	14	181	8	9
7	18	45	1	3	19	16	63	2	9
7	19	46	1	3	23	1	168	8	11
7	20	47	1	3	23	2	149	7	11
11	1	88	4	11	23	3	110	5	11
11	2	9	0	11	23	4	71	3	11
11	3	110	5	11	23	5	52	2	11
11	4	211	10	11	23	10	217	10	11
11	5	132	6	11	23	12	219	10	11
11	6	133	6	11	23	13	220	10	11
11	7	74	3	11	23	14	221	10	11
11	8	35	1	11	23	16	223	10	11
11	9	176	8	11	29	2	129	6	7
11	10	137	6	11	29	4	11	0	7
11	11	18	0	11	29	10	97	4	7
11	12	219	10	11	29	12	139	6	7
11	13	220	10	11	29	13	140	6	7
11	14	221	10	11	29	14	141	6	7
11	15	202	9	11	29	16	43	1	7
11	16	83	3	11	31	3	30	1	3
11	17	44	1	11	31	9	56	2	3
11	18	185	8	11	31	12	59	2	3
11	19	146	6	11	31	13	60	2	3
11	20	87	3	11	31	14	61	2	3
13	2	49	2	3	31	17	64	2	3
13	3	30	1	3	37	2	9	0	3
13	4	11	0	3	37	3	30	1	3
13	12	59	2	3	37	4	51	2	3
13	13	60	2	3	37	12	59	2	3
13	14	61	2	3	37	13	60	2	3
17	1	48	2	4	37	14	61	2	3
17	5	32	1	4					
17	10	37	1	4					
17	12	79	3	4					
17	13	80	3	4					
17	14	81	3	4					
17	16	43	1	4					

Table 6b. Divisibility of the 10-cycle destructive sequence by primes 41≤p≤67

p	k	i_1	j_1	d
41	1	788	39	41
41	2	589	29	41
41	3	410	20	41
41	4	231	11	41
41	5	32	1	41
41	6	353	17	41
41	7	614	30	41
41	8	615	30	41
41	9	436	21	41
41	10	117	5	41
41	11	678	33	41
41	12	819	40	41
41	13	820	40	41
41	14	821	40	41
41	15	142	6	41
41	16	703	34	41
41	17	384	18	41
41	18	205	9	41
41	19	206	9	41
41	20	467	22	41
43	2	109	5	21
43	3	210	10	21
43	4	311	15	21
43	6	173	8	21
43	10	217	10	21
43	12	419	20	21
43	13	420	20	21
43	14	421	20	21
43	16	203	9	21
43	20	247	11	21
47	1	28	1	23
47	2	69	3	23
47	3	230	11	23
47	4	391	19	23
47	5	432	21	23
47	6	113	5	23
47	7	214	10	23
47	8	15	0	23
47	9	376	18	23
47	12	459	22	23
47	13	460	22	23
47	14	461	22	23
47	17	84	3	23
47	18	445	21	23
47	19	246	11	23
47	20	347	16	23

p	k	i_1	j_1	d
53	3	130	6	13
53	12	259	12	13
53	13	260	12	13
53	14	261	12	13
59	2	269	13	29
59	3	290	14	29
59	4	311	15	29
59	7	474	23	29
59	8	395	19	29
59	9	496	24	29
59	10	297	14	29
59	11	78	3	29
59	12	579	28	29
59	13	580	28	29
59	14	581	28	29
59	15	502	24	29
59	16	283	13	29
59	17	84	3	29
59	18	185	8	29
59	19	106	4	29
61	12	59	2	3
61	13	60	2	3
61	14	61	2	3
67	1	328	16	33
67	2	509	25	33
67	3	330	16	33
67	4	151	7	33
67	5	332	16	33
67	6	273	13	33
67	7	234	11	33
67	8	95	4	33
67	9	56	2	33
67	10	557	27	33
67	11	378	18	33
67	12	659	32	33
67	13	660	32	33
67	14	661	32	33
67	15	282	13	33
67	16	103	4	33
67	17	604	29	33
67	18	565	27	33
67	19	426	20	33
67	20	387	18	33

Table 6c. Divisibility of the 10-cycle destructive sequence by primes $71 \le p \le 97$

p	k	i_1	j_1	d	p	k	i_1	j_1	d
71	1	8	0	7	79	1	228	11	13
71	3	70	3	7	79	3	130	6	13
71	5	132	6	7	79	5	32	1	13
71	7	114	5	7	79	12	259	12	13
71	8	95	4	7	79	13	260	12	13
71	12	139	6	7	79	14	261	12	13
71	13	140	6	7	83	3	410	20	41
71	14	141	6	7	83	9	476	23	41
71	18	45	1	7	83	12	819	40	41
71	19	26	0	7	83	13	820	40	41
73	7	14	0	2	83	14	821	40	41
73	9	36	1	2	83	17	344	16	41
73	12	39	1	2	89	12	219	10	11
73	13	40	1	2	89	13	220	10	11
73	14	41	1	2	89	14	221	10	11
73	17	44	1	2	97	8	455	22	24
73	19	26	0	2	97	12	479	23	24
					97	13	480	23	24
					97	14	481	23	24
					97	18	25	0	24

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