Notion of Neutrosophic Risk and Financial Markets Prediction

Sukanto Bhattacharya
Bond University
Queensland, Australia

1 Introduction

The *efficient market hypothesis* based primarily on the statistical principle of *Bayesian inference* has been proved to be only a special-case scenario. The generalized financial market, modeled as a *binary, stochastic system* capable of attaining one of two possible states (High $\rightarrow$ 1, Low $\rightarrow$ 0) with finite probabilities, is shown to reach *efficient equilibrium* with $\mathbf{p} \cdot \mathbf{M} = \mathbf{p}$ if and only if the transition probability matrix $\mathbf{M}_{2x2}$ obeys the additionally imposed condition $\{m_{11} = m_{22}, m_{12} = m_{21}\}$, where m_{ij} is an element of M (Bhattacharya, 2001). [1]

Efficient equilibrium is defined as the stationery condition $\mathbf{p} = [0.50, 0.50]$ i.e. the state in t + 1 is equi-probable between the two possible states given the market vector in time t. However, if this restriction $\{m_{11} = m_{22}, m_{12} = m_{21}\}$ is removed, we get inefficient *equilibrium* $\rho = [m_{21}/(1-v), m_{12}/(1-v)]$, where $v = m_{11} - m_{21}$ may be derived as the *eigenvalue of* **M** and ρ is a generalized version of **p** whereby the elements of the market vector are no longer restricted to their efficient equilibrium values. Though this proves that the generalized financial market cannot possibly get reduced to *pure random walk* if we do away with the *assumption of normality*, it does not necessarily rule out the possibility of *mean reversion* as M itself undergoes transition over time implying a probable re-establishment of the condition $\{m_{11} = m_{22}, m_{12} = m_{21}\}$ at some point of time in the foreseeable future. The temporal drift rate may be viewed as the *mean reversion parameter* k such that $\mathbf{k^j M_t \rightarrow M_{t+j}}$. In particular, the options market demonstrates a rather perplexing departure from efficiency. In a *Black-Scholes type world*, if stock price volatility is known *a priori*, the option prices are completely determined and any deviations are quickly arbitraged away.

Therefore, statistically significant mispricings in the options market are somewhat unique as the only non-deterministic variable in option pricing theory is volatility. Moreover, given the knowledge of implied volatility on the short-term options, the miscalibration in implied volatility on the longer term options seem odd as the parameters of the process driving volatility over time can simply be estimated by an AR1 model (Stein, 1993). [2]

Clearly, the process is not quite as straightforward as a simple parameter estimation routine from an autoregressive process. Something does seem to affect the market players' collective pricing of longer term options, which clearly overshadows the straightforward considerations of implied volatility on the short-term options. One clear reason for inefficiencies to exist is through *overreaction* of the market players to new information. Some inefficiency however may also be attributed to purely *random white noise* unrelated to any coherent market information. If the process driving volatility is indeed mean reverting then a low implied volatility on an option with a shorter time to expiration will be indicative of a higher implied volatility on an option with a longer time to expiration. Again, a high implied volatility on an option with a shorter time to expiration will be indicative of a lower implied volatility on an option with a longer time to expiration. However statistical evidence often contradicts this *rational expectations hypothesis* for the *implied volatility term structure*.

Denoted by σ'_t (t), (where the symbol ' indicates first derivative) the implied volatility at time t of an option expiring at time T is given in a Black-Scholes type world as follows:

$$\sigma'_t(t) = \int_{j=0}^{T} [\{\sigma_M + k^j(\sigma_t - \sigma_M)\}/T]\, dj$$
$$\sigma'_t(t) = \sigma_M + (k^T - 1)(\sigma_t - \sigma_M)/(T \ln k) \qquad (1)$$

Here σ_t evolves according to a *continuous-time, first-order Wiener process* as follows:

$$\mathbf{d\sigma_t = - \beta_0 (\sigma_t - \sigma_M) dt + \beta_1\sigma_t \varepsilon\sqrt{dt}} \qquad (2)$$

β_0 = - ln k, where k is the *mean reversion parameter*. Viewing this as a *mean reverting AR1 process* yields the expectation at time t, E_t (σ_{t+j}), of the instantaneous volatility at time t+j, in the required form as it appears under the integral sign in equation (1).

This theorizes that volatility is rationally expected to gravitate geometrically back towards its long-term mean level of σ_M. That is, when instantaneous volatility is above its mean level ($\sigma_t > \sigma_M$), the implied volatility on an option should be decreasing as $t \to T$. Again, when instantaneous volatility is below the long-term mean, it should be rationally expected to be increasing as $t \to T$. That this theorization does not satisfactorily reflect reality is attributable to some kind combined effect of overreaction of the market players to *excursions in implied volatility of short-term options* and their *corresponding underreaction to the historical propensity of these excursions to be rather short-lived.*

2 A Cognitive Dissonance Model of Behavioral Market Dynamics

Whenever a group of people starts acting in unison guided by their hearts rather than their heads, two things are seen to happen. Their individual suggestibilities decrease rapidly while the suggestibility of the group as a whole increases even more rapidly. The 'leader', who may be no more than just the most vociferous agitator, then primarily shapes the groupthink. He ultimately becomes the focus of the group opinion. In any financial market, it is the gurus and the experts who often play this role. The crowd hangs on their every word and makes them the uncontested Oracles of the marketplace.

If figures and formulae continue to speak against the prevailing groupthink, this could result into a *mass cognitive dissonance* calling for reinforcing self-rationalizations to be strenuously developed to suppress this dissonance. As individual suggestibilities are at a lower level compared to the group suggestibility, these self-rationalizations can actually further fuel the prevailing groupthink. This groupthink can even crystallize into something stronger if there is also a simultaneous *vigilance depression effect* caused by a tendency to filter out the dissonance-causing information. The non-linear feedback process keeps blowing up the bubble until a critical point is reached and the bubble bursts ending the prevailing groupthink with a recalibration of the position by the experts.

Our proposed model has two distinct components – a *linear feedback process* containing no looping and a *non-linear feedback process* fuelled by an *unstable rationalization loop*. It is due to this loop that perceived true value of an option might be pushed away from its theoretical true value. The market price of an option will follow its *perceived true value* rather than its *theoretical true value* and hence the inefficiencies arise. This does not mean that the market as a whole has to be inefficient – the market can very well be close to strong efficiency! Only it is the perceived true value that determines the actual price-path meaning that all market information (as well as some of the random white noise) gets automatically *anchored* to this perceived true value. This would also explain why excursions in short-term implied volatilities tend to dominate the historical considerations of mean reversion – the perceived term structure simply becomes anchored to the prevailing groupthink about the nature of the implied volatility.

Our conceptual model is based on two primary assumptions:

The *unstable rationalization loop* comes into effect if and only if the group is a reasonably well-bonded one i.e. if the initial group suggestibility has already attained a certain minimum level as, for example, in cases of strong cartel formations and;

The *unstable rationalization loop* stays in force till some critical point in time t* is reached in the life of the option. Obviously t* will tend to be quite close to T – the time of expiration. At that critical point any further divergence becomes unsustainable due to the extreme pressure exerted by real economic forces 'gone out of sync' and the gap between perceived and theoretical true values close very rapidly.

2.1 The Classical Cognitive Dissonance Paradigm

Since Leon Festinger presented it well over four decades ago, cognitive dissonance theory has continued to generate a lot of interest as well as controversy. [3] [4] This was mainly due to the fact that the theory was originally stated in very generalized, abstract terms. As a consequence, it presented possible areas of application covering a number of psychological issues involving the interaction of cognitive, motivational, and emotional factors. Festinger's dissonance theory began by postulating that pairs of cognitions (elements of knowledge), given that they are relevant to one another, can either be in agreement with each other or otherwise. If they are in agreement they are said to be *consonant*, otherwise they are termed *dissonant*. The mental condition that forms out of a pair of dissonant cognitions is what Festinger calls *cognitive dissonance*.

The existence of dissonance, being psychologically uncomfortable, motivates the person to reduce the dissonance by a process of *filtering out* information that are likely to increase the dissonance. The greater the degree of the dissonance, the greater is the pressure to reduce dissonance and change a particular cognition. The likelihood that a particular cognition will change is determined by the *resistance to change* of the cognition. Again, resistance to change is based on the *responsiveness* of the cognition to reality and on the extent to which the particular cognition is in line with various other cognitions. Resistance to change of cognition depends on the extent of loss or suffering that must be endured and the satisfaction or pleasure obtained from the behavior. [5] [6] [7] [8] [9] [10] [11] [12]

We propose the conjecture that cognitive dissonance is one possible (indeed highly likely) *critical behavioral trigger* [13] that sets off the rationalization loop and subsequently feeds it.

2.2 Non-linear Feedback Statistics Generating a Rationalization Loop

In a linear autoregressive model of order R, a time series y_n is modeled as a linear combination of N earlier values in the time series, with an added correction term x_n:

$$\mathbf{y_n = x_n - \Sigma a_j\, y_{n-j}} \qquad \textbf{(3)}$$

The autoregressive coefficients a_j (j = 1, ... N) are fitted by minimizing the mean-squared difference between the modeled time series y_n and the observed time series y_n. The minimization process results in a system of linear equations for the coefficients a_n, known as the ***Yule-Walker equations***. Conceptually, the time series y_n is considered to be the output of a discrete linear feedback circuit driven by a noise x_n, in which delay loops of lag j have *feedback strength* a_j. For Gaussian signals, an autoregressive model often provides a concise description of the time series y_n, and calculation of the coefficients a_j provides an indirect but highly efficient method of spectral estimation. In a full nonlinear autoregressive model, quadratic (or higher-order) terms are added to the linear autoregressive model. A constant term is also added, to counteract any net offset due to the quadratic terms:

$$\mathbf{y_n = x_n - a_0 - \Sigma a_j\, y_{n-j} - \Sigma b_{j,\,k}\, y_{n-j} y_{n-k}} \qquad \textbf{(4)}$$

The autoregressive coefficients a_j (j = 0, ... N) and $b_{j,\,k}$ (j, k = 1, ... N) are fit by minimizing the mean-squared difference between the modeled time series y_n and the observed time series y_n^*. The minimization process also results in a system of linear equations, which are generalizations of the *Yule-Walker equations* for the linear autoregressive model.

Conceptually, the time series y_n is considered to be the output of a circuit with nonlinear feedback, driven by a noise x_n. In principle, the coefficients $b_{j,\,k}$ describes dynamical features that are not evident in the power spectrum or related measures. Although the equations for the autoregressive coefficients a_j and $b_{j,\,k}$ are linear, the estimates of these parameters are often unstable, essentially because a large number of them must be estimated often resulting in significant estimation errors. This means that all *linear predictive systems* tend to break down once a rationalization loop has been generated. As parameters of the volatility driving process, which are used to extricate the implied volatility on the longer term options from the implied volatility on the short-term ones, are estimated by an AR1 model, which belongs to the class of regression models collectively referred to as the GLIM (General Linear Model), the parameter estimates go 'out of sync' with those predicted by a theoretical pricing model.

Unfortunately, there is no straightforward method to distinguish linear time series models (H_0) from non-linear alternatives (H_A). The approach generally taken is to test the H_0 of linearity against a pre-chosen particular non-linear H_A. Using the classical theory of statistical hypothesis testing, several test statistics have been developed for this purpose. They can be classified as Lagrange Multiplier (LM) tests, likelihood ratio (LR) tests and Wald (W) tests. The LR test requires estimation of the model parameters both under H_0 and H_A, whereas the LM test requires estimation only under H_0. Hence in case of a complicated, non-linear H_A containing many more parameters as compared to the model under H_0, the LM test is far more convenient to use. On the other hand, the LM test is designed to reveal specific types of non-linearities. The test may also have some power against inappropriate alternatives. However, there may at the same time exist alternative non-linear models against which an LM test is not powerful. Thus rejecting H_0 on the basis of such a test does not permit robust conclusions about the nature of the non-linearity. One possible solution to this problem is using a W test which estimates the model parameters under a well-specified non-linear H_A [14].

3 The Zadeh argument revisited

In the face of non-linear feedback processes generated by *dissonant information sources*, even mathematically sound rule-based reasoning schemes often tend to break down. As a pertinent illustration, we take Zadeh's argument against the well-known Dempster's rule [15] [16]. Let $\mathbf{\Theta = \{\theta_1, \theta_2 \ldots \theta_n\}}$ stand for a set of n mutually exhaustive, elementary events that cannot be precisely defined and classified making it impossible to construct a larger set Θ_{ref} of disjoint elementary hypotheses.

The assumption of exhaustiveness is not a strong one because whenever θ_j, j = 1, 2 … n does not constitute an exhaustive set of elementary events, one can always add an extra element θ_0 such that θ_j, j = 0, 1 … n describes an exhaustive set. Then, if Θ is considered to be a *general frame of discernment* of the problem under consideration, a map $\mathbf{m\ (.): D^{\Theta} \rightarrow [0, 1]}$ may be defined associated with a given body of evidence *B* that can support paradoxical information as follows:

$$\mathbf{m\ (\phi) = 0} \qquad (5)$$
$$\mathbf{\Sigma_{A \in D^{\Theta}}\ m\ (A) = 1} \qquad (6)$$

Then m (A) is called A's *basic probability number*. In line with the *Dempster-Shafer Theory*, the *belief* and *plausibility functions* are defined as follows:

$$\mathbf{Bel\ (A) = \Sigma_{B \in D^{\Theta}, B \subseteq A}\ m\ (B)} \qquad (7)$$
$$\mathbf{Pl\ (A) = \Sigma_{B \in D^{\Theta}, B \cap A \neq \phi}\ m\ (B)} \qquad (8)$$

Now let Bel_1 (.) and Bel_2 (.) be two belief functions over the same frame of discernment Θ and their corresponding *information granules* m_1 (.) and m_2 (.). Then the combined *global belief function* is obtained as $\mathbf{Bel_1\ (.) = Bel_1\ (.) \oplus Bel_2\ (.)}$ by combining the information granules m_1 (.) and m_2 (.) as follows for m (ϕ) = 0 and for any $C \neq 0$ and $C \subseteq \Theta$;

$$\mathbf{[m_1 \oplus m_2]\ (C) = [\Sigma_{A \cap B = C}\ m_1\ (A)\ m_2\ (B)] / [1 - \Sigma_{A \cap B = \phi}\ m_1\ (A)\ m_2\ (B)]} \qquad (9)$$

The summation notation $\Sigma_{A \cap B = C}$ is necessarily interpreted as the sum over all A, B ⊆ Θ such that A ∩ B = C. The orthogonal sum m (.) is considered a basic probability assignment if and only if the denominator in equation (5) is non-zero. Otherwise the orthogonal sum m (.) does not exist and the bodies of evidences B_1 and B_2 are said to be in *full contradiction*.

Such a case can arise when there exists A ⊂ Θ such that Bel_1 (A) =1 and Bel_2 (A^c) = 1 – a problem associated with *optimal Bayesian information fusion rule* (Dezert, 2001). Extending Zadeh's argument to option market anomalies, if we now assume that under conditions of *asymmetric market information*, two market players with *homogeneous expectations* view implied volatility on the long-term options. One of them sees it as either arising out of (A) current excursion in implied volatility on short-term options with probability 0.99 or out of (C) random white noise with probability of 0.01. The other sees it as either arising out of (B) historical pattern of implied volatility on short-run options with probability 0.99 or out of (C) random white noise with probability of 0.01.

Using Dempster's rule of combination, the unexpected final conclusion boils down to the expression m (C) = [m1 ⊕ m2] (C) = 0.0001/(1 – 0.0099 – 0.0099 – 0.9801) = 1 i.e. the determinant of implied volatility on long-run options is random white noise with absolute certainty!

To deal with this information fusion problem a new combination rule has been proposed under the name of *Dezert-Smarandache combination rule of paradoxical sources of evidence*, which looks for the optimal combination i.e. the basic probability assignment **m (.) = m1 (.) ⊕ m2 (.)** that *maximizes the joint entropy* of the two information sources [17].

The Zadeh illustration originally sought to bring out the fallacy of automated reasoning based on the Dempster's rule and showed that some form of the *degree of conflict* between the sources must be considered before applying the rule. However, in the context of financial markets this assumes a great amount of practical significance in terms of how it might explain some of the recurrent anomalies in rule-based information processing by inter-related market players in the face of apparently conflicting knowledge sources. The traditional conflict between the *fundamental analysts* and the *technical analysts* over the credibility of their respective knowledge sources is of course all too well known!

4 Market Information Reconciliation Based on the Concept of *Neutrosophic Risk*

Neutrosophy is a new branch of philosophy that is concerned with *neutralities* and their interaction with various ideational spectra. Let T, I, F be real subsets of the *non-standard interval* $]^-0, 1^+[$. If $\varepsilon > 0$ is an infinitesimal such that for all positive integers n and we have $|\varepsilon| < 1/n$, then the non-standard finite numbers $1^+ = 1+\varepsilon$ and $0^- = 0-\varepsilon$ form the boundaries of the non-standard interval $]^-0, 1^+[$. Statically, T, I, F are *subsets* while dynamically they may be viewed as *set-valued vector functions*. If a logical proposition is said to be t% true in T, i% indeterminate in I and f% false in F then T, I, F are referred to as the *neutrosophic components*. Neutrosophic probability is useful to events that are shrouded in a *veil of indeterminacy* like the *actual* implied volatility of long-term options. As this approach uses a *subset-approximation* for truth-values, indeterminacy and falsity-values it provides a better approximation than classical probability to uncertain events.

The neutrosophic probability approach also makes a distinction between "relative sure event", event that is true only in certain world(s): NP (rse) = 1, and "absolute sure event", event that is true for all possible world(s): NP (ase) $=1^+$. Similar relations can be drawn for "relative impossible event" / "absolute impossible event" and "relative indeterminate event" / "absolute indeterminate event". In case where the truth- and falsity-components are complimentary i.e. they sum up to unity, and there is no indeterminacy and one is reduced to classical probability. Therefore, neutrosophic probability may be viewed as a generalization of classical and imprecise probabilities. [18]

When a long-term option priced by the collective action of the market players is observed to be deviating from the theoretical price, three possibilities must be considered:

(1) The theoretical price is obtained by an inadequate pricing model, which means that the market price may well *be* the true price,

(2) An unstable rationalization loop has taken shape that has pushed the market price of the option 'out of sync' with its true price, or

(3) The nature of the deviation is indeterminate and could be due to either (a) or (b) or a super-position of both (a) and (b) and/or due to some random white noise.

However, it is to be noted that in none of these three possible cases are we referring to the efficiency or otherwise of the market as a whole. The market can only be as efficient as the information it gets to process. We term the systematic risk associated with the efficient market as *resolvable risk*. Therefore, if the information about the true price of the option is misleading (perhaps due to an inadequate pricing model), the market cannot be expected to process it into something useful – after all, the markets can't be expected to pull jack-rabbits out of empty hats! The perceived risk resulting from the imprecision associated with how human psycho-cognitive factors subjectively interpret information and use the processed information in decision-making is what we term as *irresolvable* (or *neutrosophic*) *risk*.

With T, I, F as the neutrosophic components, let us now define the following events:

H = {p: p is the true option price determined by the theoretical pricing model} and

M = {p: p is the true option price determined by the prevailing market price} **(10)**

Then there is a t% chance that the event ($H \cap M^c$) is true, or corollarily, the corresponding complimentary event ($H^c \cap M$) is untrue, there is a f% chance that the event ($M^c \cap H$) is untrue, or corollarily, the complimentary event ($M \cap H^c$) is true and there is a i% chance that neither ($H \cap M^c$) nor ($M \cap H^c$) is true/untrue; i.e. the determinant of the true market price is indeterminate. This would fit in nicely with possibility (c) enumerated above – that the nature of the deviation could be due to either (a) or (b) or a super-position of both (a) and (b) and/or due to some random white noise.
Illustratively, a set of AR1 models used to extract the mean reversion parameter driving the volatility process over time have *coefficients of determination* in the range say between 50%-70%, then we can say that t varies in the set T (50% - 70%). If the subjective probability assessments of well-informed market players about the weight of the current excursions in implied volatility on short-term options lie in the range say between 40%-60%, then f varies in the set F (40% - 60%). Then unexplained variation in the temporal volatility driving process together with the subjective assessment by the market players will make the event indeterminate by either 30% or 40%. Then the neutrosophic probability of the true price of the option being determined by the theoretical pricing model is **NP ($H \cap M^c$) = [(50 – 70), (40 – 60), {30, 40}]**.

5 Conclusion

Finally, in terms of our behavioral conceptualization of the market anomaly primarily as manifestation of mass cognitive dissonance, the joint neutrosophic probability NP ($H \cap M^c$) will also be indicative of the extent to which an unstable rationalization loop has formed out of such mass cognitive dissonance that is causing the market price to deviate from the true price of the option. Obviously increasing strength of the non-linear feedback process fuelling the rationalization loop will tend to increase this deviation. As human psychology; and consequently a lot of subjectivity; is involved in the process of determining what drives the market prices, neutrosophic reasoning will tend to reconcile market information much more realistically than classical probability theory. Neutrosophic reasoning approach will also be an improvement over rule-based reasoning possibly avoiding pitfalls like that brought out by Zadeh's argument. This has particularly significant implications for the vast majority of market players who rely on signals generated by some automated trading system following simple rule-based logic.

However, the fact that there is inherent subjectivity in processing the price information coming out of financial markets, given that the way a particular piece of information is subjectively interpreted by an individual investor may not be the globally correct interpretation, there is always the matter of irresolvable risk that will tend to pre-dispose the investor in favour of some *safe* investment alternative that offers some protection against both resolvable as well as irresolvable risk. This highlights the rapidly increasing importance and popularity of *safe* investment options that are based on some form of *portfolio insurance* i.e. an investment mechanism where the investor has some kind of in-built downside protection against adverse price movements resulting from erroneous interpretation of market information e.g. constant proportion portfolio insurance (CPPI) and its generalized form – options based portfolio insurance (OBPI). Such portfolio insurance strategies offer protection against all possible downsides, whether resulting out of resolvable or irresolvable risk, thereby making the investors feel confident about the decisions they take.

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