The New Prime theorem（18）

Hardy-Littlewood Conjecture $E: x^2+1$

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Abstract

Using Jiang function we prove Hardy-Littlewood conjecture $E: x^2+1$[2].

Theorem. The prime equation

$$P_n=(2P_1P_2\cdots P_{n-1})^2+1 \quad (1)$$

has infinitely many prime soultions

Proof. We have Jiang function[1]

$$J_n(\omega)=\prod_P[(P-1)^{n-1}-\chi(P)], \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$(2q_1q_2\cdots q_{n-1})^2+1\equiv 0 \pmod P, \quad q_i=1,\cdots,P-1, i=1,\cdots,n-1, \quad (3)$$

From (3) we have

$$\left(\frac{-1}{P}\right)=(-1)^{\frac{P-1}{2}}, \text{ if } \left(\frac{-1}{P}\right)=1 \text{ then } \chi(P)=2(P-1)^{n-2}, \text{ if } \left(\frac{-1}{P}\right)=-1 \text{ then } \chi(P)=0.$$

Substituting it into (2) we have.

$$J_n(\omega)=\prod_{3\le P}[(P-1)^{n-2}(P-2-(-1)^{\frac{P-1}{2}}]\ne 0. \quad (4)$$

We prove that (1) has infinitely many prime soultions. $J_n(\omega)\subset\phi^{n-1}(\omega)$

We have the best asymptotic formula [1]

$$\pi_2(N,n)=\left|\{P_1,\cdots,P_{n-1}\le N: P_n=prime\}\right|\sim\frac{J_n(\omega)\omega}{2\times(n-1)!\phi^n(\omega)}\frac{N^{n-1}}{\log^n N}. \quad (5)$$

Example 1. Let $n=2$. From (1) we have

$$P_2=(2P_1)^2+1 \quad (6)$$

From (4) we have

$$J_2(\omega)=\prod_{3\le P}[P-2-(-1)^{\frac{P-1}{2}}]\ne 0 \quad (7)$$

Example 2. Let $n=3$. From (1) we have

$$P_3=(2P_1P_2)^2+1. \quad (8)$$

From (4) we have

$$J_3(\omega)=\prod_{3\le P}[(P-1)(P-2-(-1)^{\frac{P-1}{2}}]\neq 0\,. \qquad (9)$$

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1], which can count the number of prime number. The prime number is not random. But Hardy singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [2], which can not count the number of prime numbers.

References

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf. http://wbabin.net/xuan.htm#chun-xuan.

[2] G. H. Hardy and J. E. Littlewood, Some problems of "Partitio Numerorum", III: On the expression of a number as a sum of primes. Acta Math., 44(1923)-70.