The New Prime theorem（17）

$P_n = 2P_1P_2\cdots P_{n-1} \pm 1$

Chun-Xuan Jiang

P. O. Box 3924, Beijing 100854, P. R. China

jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that such that $P_n = 2P_1P_2\cdots P_{n-1} \pm 1$ has infinitely many prime solutions.

Theorem. The prime equation

$$P_n = 2P_1P_2\cdots P_{n-1} + 1 \quad (1)$$

has infinitely many prime soultions

Proof. We have Jiang function[1]

$$J_n(\omega) = \prod_P [(P-1)^{n-1} - \chi(P)], \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$2q_1q_2\cdots q_{n-1} + 1 \equiv 0 \pmod P, \quad q_i = 1,\cdots,P-1, i = 1,\cdots,n-1, \quad (3)$$

From (3) we have

$$\chi(P) = (P-1)^{n-2} \quad (4)$$

Substituting (4) into (2) we have

$$J_n(\omega) = \prod_{3\le P} [(P-1)^{n-2}(P-2)] \ne 0. \quad (5)$$

We prove that (1) has infinitely many prime soultions. $J_n(\omega) \subset \phi^{n-1}(\omega)$.

We have the best asymptotic formula [1]

$$\pi_2(N,n) = \left|\{P_1,\cdots,P_{n-1} \le N : P_n = prime\}\right| \sim \frac{J_n(\omega)\omega}{(n-1)!\phi^n(\omega)} \frac{N^{n-1}}{\log^n N}. \quad (6)$$

Example 1. Let $n = 2$. From (1) we have

$$P_2 = 2P_1 + 1 \quad (7)$$

From (5) we have

$$J_2(\omega) = \prod_{3\le P}(P-2) \ne 0 \quad (8)$$

Example 2. Let $n = 3$. From (1) we have

$$P_3 = 2P_1P_2 + 1. \quad (9)$$

From (5) we have

$$J_3(\omega) = \prod_{3\le P}[(P-1)(P-2)] \ne 0. \quad (10)$$

In the same way we are able to prove that

$$P_m = 2P_1P_2\cdots P_{n-1} - 1 \qquad (11)$$

has infinitely many prime solutions.

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1], which can count the number of prime number. The prime number is not random. But Hardy singular series $\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [2-5], which can not count the number of prime numbers.

References

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf. http:/wbabin.net/xuan.htm#chun0xuan.

[2] G. H. Hardy and J. E. Littlewood, Some problems of "Partitio Numerorum", III: On the expression of a number as a sum of primes. Acta Math., 44(1923)-70.

[4] B. Green and T. Tao, Linear equations in primes. To appear, Ann. Math.

[5] D. Goldston, J. Pintz and C. Y. Yildirim, Primes in tuples I. Ann. Math., 170(2009) 819-862.