The New Prime theorem（16）

$$P_j = (j)^n P + (k-j)^n,\ j = 1, \cdots, k-1$$

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Abstract

Using Jiang function we prove that there exist infinitely many primes P such that each of $(j)^n P + (k-j)^n$ is a prime.

Theorem. Let k be a given prime.

$$P_j = (j)^n P + (k-j)^n (j = 1, \cdots, k-1, n = 1, 2, \cdots) \quad (1)$$

There exist infinitely many prime P such that each of $(j)^n P + (k-j)^n$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [(j)^n q + (k-j)^n] \equiv 0 \pmod P, q = 1, \cdots, P-1. \quad (3)$$

From (3) we have $\chi(2) = 0$, if $P < k$ then $\chi(P) \le P - 2, \chi(k) = 1$, if $k < P$ then $\chi(P) \le k - 1$. From (3) we have

$$J_2(\omega) \ne 0. \quad (4)$$

We prove that there exist inifinitely many primes P such that each of $(j)^n P + (k-j)^n$ is a prime.

Jiang function is a subset of Euler function: $J_2(\omega) \subset \phi(\omega)$.

We have asymptotic formula

$$\pi_k(N, 2) = \left|\left\{P \le N : (j)^n P + (k-j)^n = prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}. \quad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Example 1. Let $k = 3$. From (1) we have

$$P_1 = P + 2^n, \quad P_2 = 2^n P + 1 \quad (6)$$

We have Jiang function

$$J_2(\omega) = \prod_{5 \le P} (P - 3) \ne 0 \quad (7)$$

We prove that there exist infinitely many primes P such that P_1 and P_2 are all prime.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf. http://wbabin.net/xuan.htm#chun-xuan.