The New Prime theorem（15）

$$P_j = (j)^3 P + (k-j)^3,\ j = 1, \cdots, k-1$$

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Abstract

Using Jiang function we prove that there exist infinitely many primes P such that each of $(j)^3 P + (k-j)^3$ is a prime.

Theorem. Let k be a given prime.

$$P_j = (j)^3 P + (k-j)^3 (j = 1, \cdots, k-1) \quad (1)$$

There exist infinitely many prime P such that each of $(j)^3 P + (k-j)^3$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [(j)^3 q + (k-j)^3] \equiv 0 \pmod{P}, q = 1, \cdots, P-1. \quad (3)$$

From (3) we have $\chi(2) = 0$, if $P < k$ then $\chi(P) \le P-2, \chi(k) = 1$, if $k < P$ then $\chi(P) \le k-1$. From (3) we have

$$J_2(\omega) \ne 0. \quad (4)$$

We prove that there exist inifinitely many primes P such that each of $(j)^3 P + (k-j)^3$ is a prime. Jiang function is a subset of Euler function: $J_2(\omega) \subset \phi(\omega)$.

We have asymptotic formula [1]

$$\pi_k(N, 2) = \left|\left\{P \le N : (j)^3 P + (k-j)^3 = prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}. \quad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Example 1. Let $k = 3$. From (1) we have

$$P_1 = P + 8, \quad P_2 = 8P + 1 \quad (6)$$

We have Jiang function

$$J_2(\omega) = \prod_{5 \le P} (P - 3) \ne 0 \quad (7)$$

There exist infinitely many primes P such that P_1 and P_2 are all prime.
We have asymptotic formula

$$\pi_3(N,2)=\left|\{P\le N: P_1=prime, P_2=prime\}\right|\sim\frac{J_2(\omega)\omega^2}{\phi^3(\omega)}\frac{N}{\log^3 N} \quad (8)$$

Example 2. Let $k=5$, from (1) we have

$$P_j=(j)^3P+(k-j)^3 (j=1,2,3,4) \quad (9)$$

We have jiang function

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \quad (10)$$

where $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{4}[(j)^3q+(k-j)^3]\equiv 0 \pmod P \quad (11)$$

From (11) we have $\chi(2)=0$, $\chi(3)=1$, $\chi(5)=1$, $\chi(7)=2$, $\chi(11)=4$, $\chi(13)=3$,

$\chi(P)=4$ otherwise.

Substituting it into (10) we have.

$$J_2(\omega)=648\prod_{17\le P}(P-5)\ne 0 \quad (12)$$

We prove that there exist infinitely many primes P such that each of $(j)^3P+(k-j)^3$ is prime.

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1-2], which can count the number of prime number. The prime number is not random. But Hardy singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false. [2-5], which can not count the number of prime numbers.

References

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