The New Prime theorem（14）

$$P_j = (j)^2 P + (k-j)^2,\ j = 1, \cdots, k-1$$

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Abstract

Using Jiang function we prove that there exist infinitely many primes P such that each of $(j)^2 P + (k-j)^2$ is a prime.

Theorem. Let k be a given prime.

$$P_j = (j)^2 P + (k-j)^2 (j = 1, \cdots, k-1) \tag{1}$$

There exist infinitely many prime P such that each of $(j)^2 P + (k-j)^2$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [(j)^2 q + (h-j)^2] \equiv 0 \pmod P, q = 1, \cdots, P-1. \tag{3}$$

From (3) we have $\chi(2) = 0$ if $P < k$ then $\chi(P) \le P-2, \chi(k) = 1$, if $k < P$ then $\chi(P) \le k-1$. Jiang functions a subset of Euler function: $J_2(\omega) \subset \phi(\omega)$. From (3) we have

$$J_2(\omega) \ne 0. \tag{4}$$

We prove that there exist inifinitely many primes P such that each of $(j)^2 P + (k-j)^2$ is a prime.

We have asymptotic formula

$$\pi_k(N, 2) = \left| \left\{ P \le N : (j)^2 P + (k-j)^2 = prime \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}, \tag{5}$$

where $\phi(\omega) = \prod_P (P-1)$.

We have [2]

$$\left| \left\{ P \le N : jP + k - j = prime \right\} \right| \le \left| \left\{ P \le N : (j)^2 P + (k-j)^2 = prime \right\} \right| \tag{6}$$

Example 1. Let $K = 3$. From (1) we have

$$P_1 = P + 4, P_2 = 4P + 1 \tag{7}$$

We have Jiang function

$$J_2(\omega)=\prod_{5\le P}(P-3)\neq 0 \quad (8)$$

There exist infinitely many primes P such that P_1 and P_2 are all prime.
We have asymptotic formula

$$\pi_3(N,2)=\left|\{P\le N: P_1=prime, P_2=prime\}\right|\sim\frac{J_2(\omega)\omega^2}{\phi^3(\omega)}\frac{N}{\log^3 N} \quad (9)$$

Example 2. Let $k=5$, from (1) we have

$$P_j=(j)^2P+(5-j)^2(j=1,2,3,4) \quad (10)$$

We have jiang function

$$J_2(\omega)=\prod_P[P-1-\chi(P)]. \quad (11)$$

We have $\chi(3)=1$, $\chi(5)=1$, $\chi(7)=2$, $\chi(11)=2$, $\chi(13)=3$, $\chi(17)=3$, $\chi(P)=4$ otherwise.
Substituting it into (11) we have

$$J_2(\omega)=11232\prod_{19\le P}(P-5)\neq 0 \quad (12)$$

There exist infinitely many primes P such that P_1,P_2,P_3 and P_4 are all prime.
We have asymptotic formula

$$\pi_5(N,2)=\left|\{P\le N: P_1,P_2,P_3,P_4=prime\}\right|\sim\frac{J_2(\omega)\omega^4}{\phi^5(\omega)}\frac{N}{\log^5 N} \quad (13)$$

Example 3. Let $k=7$. From (1) we have

$$P_j=(j)^2P+(7-j)^2(j=1,2,3,4,5,6) \quad (14)$$

We have jiang function

$$J_2(\omega)=\prod_P[P-1-\chi(P)]. \quad (15)$$

Where $\chi(2)=0$, $\chi(3)=1$, $\chi(5)=2$, $\chi(7)=1$, $\chi(11)=5$, $\chi(13)=5$, $\chi(17)=4$,

$\chi(29)=5$, $\chi(37)=5$, $\chi(P)=6$ otherwise.

From (15) we have

$$J_2(\omega)\neq 0 \quad (16)$$

We prove that there exist infinitely many primes P such that each of $(j)^2P+(7-j)^2$ is a prime.

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1-3], which can count the number of prime numbers. The prime number is not random. But Hardy singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [4-6], which can not count the number of prime numbers.

References

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