The New Prime theorem（11）

$2\times a^2 \pm 1$

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Abstract

Using Jiang function we prove that $2\times a^2 \pm 1$ has infinitely many prime solutions

Theorem. We define the prime equation

$$P_1 = 2\times(P-1)^2 - 1 \qquad (1)$$

There exist infinitely many primes P such that P_2 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$2\times(q-1)^2 - 1 \equiv 0 \pmod P, \quad q = 1, \cdots, P-1 \qquad (3)$$

From (3) we have $(\frac{2}{P}) = (-)^{\frac{P^2-1}{8}}$, if $\left(\frac{2}{P}\right) = 1$ then $\chi(P) = 2$, if $\left(\frac{2}{P}\right) = -1$ then $\chi(P) = 0$.

Substituting it into (2) we have

$$J_2(\omega) = \prod_{3\le P}\left[P - 2 - (-1)^{\frac{P^2-1}{8}}\right] \ne 0 \qquad (4)$$

We prove there exist infinitely many primes P such that P_2 is a prime.

We have asymptotic formula [1]

$$\pi_2(N,2) = \left|\left\{P \le N : 2\times(P-1)^2 - 1 = prime\right\}\right| \sim \frac{J_2(\omega)\omega}{2\phi^2(\omega)}\frac{N}{\log^2 N} \qquad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

In the same way we are able to prove that $2\times a^2 + 1$ has infinitely many prime solutions.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.