The New Prime theorem（10）
There are finite Mersenne primes
and
There are finite repunits primes

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Abstract

Using Jiang function we prove the finite Mersenne primes and the finite repunits primes.

Theorem. Suppose the prime equation

$$P_1=\frac{(P-1)^{P_0}-1}{P-2}. \qquad (1)$$

where P_0 is a given prime.

There exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\frac{(q-1)^{P_0}-1}{q-2}\equiv 0 \pmod P, \qquad q=1,\cdots,P-1. \qquad (3)$$

$\chi(P_0)=1$, $\chi(P)=P_0-1$ if $P\equiv 1 \pmod{P_0}$, $\chi(P)=0$ otherwise.

Since $J_2(\omega)\neq 0$, there exist infinitely many primes P such that P_1 is a prime.

We have the asymptotic formula [1]

$$\pi_2(N,2)=\left|\{P\le N: P_1=prime\}\right|\sim\frac{1}{P_0-1}\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}. \qquad (4)$$

where $\phi(\omega)=\prod_P(P-1)$.

Let $P=3$. From (1) we have equation of Mersenne numbers [2]

$$P_1=2^{P_0}-1. \qquad (5)$$

From (4) we have

$$\pi_2(3,2)=\left|\left\{3\le N: 2^{P_0}-1=prime\right\}\right|\sim\frac{1}{P_0-1}\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{3}{\log^2 3}\to 0 \quad \text{as} \quad P_0\to\infty \qquad (6)$$

We prove the finite Mersenne primes.

Let $P=11$. From (1) we have equation of repunits numbers [2]

$$P_1=\frac{10^{P_0}-1}{9}. \qquad (7)$$

From (4) we have

$$\pi_{11}(11,2)=\left|\left\{11\le N: \frac{10^{P_0}-1}{9}=prime\right\}\right|\sim\frac{1}{P_0-1}\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{11}{\log^2 11}\to 0 \quad \text{as} \quad P_0\to\infty. \qquad (8)$$

We prove the finite repunits primes.

In the same way we are able to prove that $(a^{P_0}-1)/(a-1)$ with $a=4,6,10,12,\cdots,$ has the finite prime solutions.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.

[2] P. Ribenboim, The new book of prime number records, 3rd edition, spring-Verlag, New York, NY, 1995.