Prime Theorem: $P_2 = aP_1 + b$, Polignac Theorem and Goldbach Theorem

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Abstract

Using Jiang function we prove prime theorem: $P_2 = aP_1 + b$, Polignac theorem and Goldbach theorem.

We read Ribenboim paper [2] and write this paper.

Prime theorem [1]. Prime equation is

$$P_2 = aP_1 + b, \quad 2 \mid ab, \quad (a,b) = 1. \tag{1}$$

There exist infinitely many primes P_1 such that P_2 is a prime.

Proof. We have Jiang function [1]

$$J_2(\omega) = \prod_{P>2} (P - 1 - \chi(P)), \tag{2}$$

$\omega = \prod_{P \ge 2} P, \quad \chi(P)$ denotes the number of solutions for the following congruence

$$aq + b \equiv 0 \ (\mathrm{mod}\ P), \tag{3}$$

where $q = 1, 2, ..., P-1$.

If $P \mid ab$ then $\chi(P) = 0; \quad \chi(P) = 1$ otherwise. From (2) and (3) we have

$$J_2(\omega) = \prod_{P>2} (P-2) \prod_{P \mid ab} \frac{P-1}{P-2} \to \infty \quad \text{as} \quad \omega \to \infty \tag{4}$$

We prove that there exist infinitely many primes P_1 such that P_2 is a prime.

We have the best asymptotic formula for the number of primes P_1 [1]

$$\pi_2(N,2) = \left| \{ P_1 \le N : aP_1 + b = prime \} \right| = \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N} (1 + o(1))$$

$$= 2 \prod_{P>2} \left(1 - \frac{1}{(P-1)^2}\right) \prod_{P \mid ab} \frac{P-1}{P-2} \frac{N}{\log^2 N} (1 + o(1)) \tag{5}$$

where $\phi(\omega) == \prod_{P \ge 2} (P-1)$.

Polignac theorem [2]. Let $a = 1$ and $b = 2n (n \ge 1)$. From (1) we have Polignac equation

$$P_2 = P_1 + 2n \tag{6}$$

From (4) we have

$$J_2(\omega) = \prod_{P>2} (P-2) \prod_{P \mid n} \frac{P-1}{P-2} \to \infty \quad \text{as} \quad \omega \to \infty \tag{7}$$

We prove that for every $2n$ there exist infinitely many primes P_1 such that P_2 is a prime.

From (5) we have

$$\pi_2(N,2) = \left|\{P_1 \le N : P_1 + 2n = prime\}\right| = 2\prod_{P>2}(1-\frac{1}{(P-1)^2})\prod_{P|n}\frac{P-1}{P-2}\frac{N}{\log^2 N}(1+o(1)) \quad (8)$$

Goldbach theorem [3]. Let $b = N \ge 6$ be an even number, $a = -1$.
From (1) we have Goldbach equation

$$P_2 = N - P_1 \quad (9)$$

From (4) we have

$$J_2(\omega) = \prod_{P>2}(P-2)\prod_{P|N}\frac{P-1}{P-2} \to \infty \text{ as } \omega \to \infty \quad (10)$$

We prove that every even number $N \ge 6$ is the sum of two primes.
From (5) we have

$$\pi_2(N,2) = \left|\{P_1 < N : N - P_1 = prime\}\right| = 2\prod_{P>2}(1-\frac{1}{(P-1)^2})\prod_{P|N}\frac{P-1}{P-2}\frac{N}{\log^2 N}(1+o(1))$$

(11)

Note. Prime equation P^2+2 has the only prime solution, $3^2+2=11$, because $J_2(\omega)=0$.
Prime equation $(P+2)^2+2$ has infinitely many prime solutions, because

$$J_2(\omega) \to \infty \quad \text{as} \quad \omega \to \infty$$

References

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