The New Prime theorem（5）

$P,\ jP+k-j\ (j=1,\cdots,k-1)$

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@vip.sohu.com

Abstrat

Using Jiang function we prove that there exist infinitely many primes P such that each $jP+k-j$ is a prime.

Theorem. Let k be a given prime.

$$P,\ jP+k-j\ (j=1,\cdots,k-1) \qquad (1)$$

There exist infinitely many primes P such that each of $jP+k-j$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \qquad (2)$$

where

$$\omega=\prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq+k-j)\equiv 0 \pmod{P}, \qquad (3)$$

$q=1,\cdots,P-1$.

From (3) we have $\chi(2)=0$, if $P<k$ then $\chi(P)=P-2$, $\chi(k)=1$, if $k<P$ then $\chi(P)=k-1$.

From (3) and (2) we have

$$J_2(\omega)=(k-2)\prod_{k<P}(P-k)\neq 0. \qquad (4)$$

We prove that there exist infinitely many primes P such that each of $jP+k-j$ is a prime

We have the asymptotic formula [1]

$$\pi_k(N,2)=\left|\{P\le N: jP+k-j=prime\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}, \qquad (5)$$

where $\phi(\omega)=\prod_P(P-1)$.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.