# The New Prime theorem（4）

$$P,\ jP+7-j\,(j=1,2,3,4,5,6)$$

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Abstrat

Using Jiang function we prove that there exist infinitely many primes $P$ such that each $jP+7-j$ is a prime.

**Theorem.**

$$P,\ jP+7-j\,(j=1,2,3,4,5,6). \qquad (1)$$

There exist infinitely many primes $P$ such that each of $jP+7-j$ is a prime.

**Proof**. We have Jiang function [1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \qquad (2)$$

where

$$\omega=\prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{6}(jq+7-j)\equiv 0 \pmod P, \qquad (3)$$

$q=1,\cdots,P-1$.

From (3) we have $\chi(2)=0$, $\chi(3)=1$, $\chi(5)=3$, $\chi(7)=1$, $\chi(P)=6$ otherwise.

From (3) and (2) we have

$$J_2(\omega)=5\prod_{11\le P}(P-7)\ne 0 \qquad (4)$$

We prove that there exist infinitely many primes $P$ such that each of $jP+7-j$ is a prime.

We have the best asymptotic formula [1]

$$\pi_7(N,2)=\left|\{P\le N: jP+7-j=prime\}\right|\sim\frac{J_2(\omega)\omega^6}{\phi^7(\omega)}\frac{N}{\log^7 N} \qquad (5)$$

where $\phi(\omega)=\prod_P(P-1)$.

## Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www.wbabin.net/math/xuan2.pdf.