The New Prime theorem（2）

$P_1 = P + 2$ and $P_2 = 2P + 1$

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Abstrat

Using Jiang function we prove that there exist infinitely many primes P such that P_1 and P_2 are all prime.

Theorem

$$P_1 = P + 2 \quad \text{and} \quad P_2 = 2P + 1 \qquad (1)$$

There exist infinitely many priems P such that P_1 and P_2 are all prime.

Proof. We have Jiang function [1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad (2)$$

where

$$\omega = \prod_P P.$$

$\chi(P)$ is the number of solutions of congruence

$$(q+2)(2q+1) \equiv 0 \pmod{P} \qquad (3)$$

where $q = 1, \cdots, P-1$.

From (3) we have $\chi(2) = 0$, $\chi(3) = 1$, $\chi(P) = 2$ otherwise.

From (3) and (2) we have

$$J_2(\omega) = \prod_{5 \le P} (P - 3) \ne 0 \qquad (4)$$

We prove that there exist infinitely many primes P such that P_1 and P_2 are all prime.

we have the best asymptotic formula

$$\pi_3(N, 2) = \left|\{P \le N : P + 2 = prime, 2P + 1 = prime\}\right| \sim \frac{J_2(\omega)\omega^2}{\phi^3(\omega)} \frac{N}{\log^3 N} \qquad (5)$$

where $\phi(\omega) = \prod_P (P - 1)$.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution, http://www. wbabin. net/math/xuan2.pdf.