The New Prime theorem（1）

$$P_2 = aP_1 + b$$

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@vip.sohu.com

Abstrat

Using Jiang function we prove that there exist infinitely many primes P_1 such that $aP_1 + b$ is prime.

Theorem

$$P_2 = aP_1 + b.\ (a,b) = 1,\quad 2 \mid ab, \tag{1}$$

There exist infinitely many priems P_1 such that P_2 is prime.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \tag{2}$$

where

$$\omega = \prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$aq + b \equiv 0 \pmod P, \tag{3}$$

$q = 1, \cdots, P-1$.

From (3) we have if $P \mid ab$ then $\chi(P) = 0,\ \chi(P) = 1$ otherwise.

From (3) and (2) we have

$$J_2(\omega) = \prod_{3 \le P} (P-2) \prod_{P \mid ab} \frac{P-1}{P-2} \ne 0. \tag{4}$$

We prove that there exist infinitely many primes P_1 such that P_2 is prime.

We have the best asymptotic formula [1, 2]

$$\pi_2(N,2) = \left|\{P_1 \le N : aP_1 + b = prime\}\right| \sim \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}$$

$$= 2 \prod_{3 \le P} \left(1 - \frac{1}{(P-1)^2}\right) \prod_{P \mid ab} \frac{P-1}{P-2} \frac{N}{\log^2 N}. \tag{5}$$

where $\phi(\omega) = \prod_P (P-1)$.

Twin primes theorem [1]. Let $a = 1$ and $b = 2$. From (1) we have

$$P_2 = P_1 + 2 \qquad (6)$$

From (4) we have

$$J_2(\omega) = \prod_P (P-2) \neq 0 \qquad (7)$$

We prove that there exist infinitely many primes P_1 such that $P_1 + 2$ is prime.

From (5) we have

$$\pi_2(N,2) = \left|\{P_1 \le N : P_1 + 2 = prime\}\right| \sim 2\prod_{3\le P}\left(1 - \frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}. \qquad (8)$$

Goldbach theorem [1]. Let $a = -1$ and $b = N$. From (1) we have

$$N = P_1 + P_2 \qquad (9)$$

From (4) we have

$$J_2(\omega) = \prod_{3\le P}(P-2)\prod_{P|N}\frac{P-1}{P-2} \neq 0 \qquad (10)$$

We prove that every even number $N \ge 6$ is the sum of two primes.

From (5) we have

$$\pi_2(N,2) = \left|\{P_1 \le N : N - P = prime\}\right| \sim 2\prod_{3\le P}\left(1 - \frac{1}{(P-1)^2}\right)\prod_{P|N}\frac{P-1}{P-2}\frac{N}{\log^2 N} \qquad (11)$$

Reference

[1] Chun-Xuan Jiang, On the Yu-Goldbach prime theorem (Chinese), Guangxi Science, 3 (1996) 9-12.

[2] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http://www.wbabin.net/math/xuan2.pdf) (http://vixra.org/pdf/0812.0004v2.pdf)