New prime K-tuple theorem (6)

$$P, P+4^n (n=1,\cdots,k)$$

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Abstract

Using Jiang function we prove that for every positive integer k there exist infinitely many primes P such that each of $P+4^n$ is prime.

Theorem

$$P, P+4^n (n=1,\cdots,k). \qquad (1)$$

For every psitive integer k there exist infinitely many primes P such that each of $P+4^n$ is prime.

Proof. We have Jiang function [1]

$$J_2(\omega)=\prod_P [P-1-\chi(P)], \qquad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{n=1}^{k}[q+4^n]\equiv 0 \pmod{P}, \qquad (3)$$

where $q=1,\cdots,P-1$.

From (3) we have

If $P<2k$ then $\chi(P)=\frac{P-1}{2}$, if $2k<P$ then $\chi(P)=k$.

Frome (3) and (2) we have

$$J_3(\omega)=\prod_{P=3}^{P<2k}\frac{P-1}{2}\prod_{2k<P}(P-1-k)\neq 0. \qquad (4)$$

We prove that for every positive integer k there exist infinitely many primes P_1 and P_2 such that each of $P+4^n$ is prime.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N,2)=\left|\left\{P\le N: P+4^n=prime\right\}\right|\sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}. \qquad (5)$$

where $\phi(\omega)=\prod_P (P-1)$.

References

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http://www. wbabin. net/math/xuan2. pdf)