New prime K-tuple theorem (5)

$$P_1, P_2, \; jP_1+(j+1)P_2 (j=1,\cdots,k)$$

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Abstract

Using Jiang function we prove that for every positive integer k there exist infinitely many primes P_1 and P_2 such that each of $jP_1+(j+1)P_2$ is prime.

Theorem

$$P_1, P_2, \; jP_1+(j+1)P_2 (j=1,\cdots,k). \quad (1)$$

For every psitive integer k there exist infinitely many primes P_1 and P_2 such that each of $jP_1+(j+1)P_2$ is prime.

Proof. We have Jiang function [1, 2]

$$J_3(\omega)=\prod_P[(P-1)^2-\chi(P)], \quad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k}[jq_1+(j+1)q_2]\equiv 0 \pmod P, \quad (3)$$

$q_i=1,\cdots,P-1, i=1,2.$.

From (3) we have

If $P\le k+1$ then $\chi(P)=(P-1)(P-2)$, if $k+1<P$ then $\chi(P)=k(P-1)$.

From (3)and (2) we have

$$J_3(\omega)=\prod_{P=3}^{P\le k+1}(P-1)\prod_{k+1<P}[(P-1)^2-k(P-1)]\neq 0. \quad (4)$$

We prove that for every positive integer k there exist infinitely many primes P_1 and P_2 such that each of $jP_1+(j+1)P_2$ is primie.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N,3)=\left|\{P_1,P_2\le N: jP_1+(j+1)P_2=prime\}\right|\sim\frac{J_3(\omega)\omega^k}{\phi^{k+2}(\omega)}\frac{N^2}{\log^{k+2}}, \quad (5)$$

where $\phi(\omega)=\prod_P(P-1)$.

References

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