New prime K-tuple theorem (4)

$$P, P+(2j)^2 (j=1,\cdots,k)$$

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that for every positive integer k there exist infinitely many primes P such that each of $P+(2j)^2$ is prime.

Theorem

$$P, P+(2j)^2 (j=1,\cdots,k). \quad (1)$$

For every psitive integer k there exist infinitely many primes P such that each of $P+(2j)^2$ is prime.

Proof. We have Jiang function [1]

$$J_2(\omega)=\prod_P (P-1-\chi(P)), \quad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k}[q+(2j)^2]\equiv 0 \pmod P, \quad (3)$$

where $q=1,\cdots,P-1$.

From (3) we have

If $P<2k$ then $\chi(P)=(P-1)/2$, if $2k<P$ then $\chi(P)=k$.

Frome (3)and (2) we have

$$J_2(\omega)=\prod_{P=3}^{P<2k}\frac{P-1}{2}\prod_{2k<P}(P-1-k)\neq 0. \quad (4)$$

We prove that for every positive integer k there exist infinitely many primes P such that each of $P+(2j)^2$ is prime.

We have the best asymptotic formula [1]

$$\pi_{k+1}(N,2)=\left|\left\{P\le N: P+(2j)^2=prime\right\}\right|\sim\frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}. \quad (5)$$

The auther takes a day to write this paper.

References

[1] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. Inter. Acad. Press, 2002,MR2004c:110011, (http://www.i-b-r.org/docs/jiang.pdf) (http://www. wbabin. net/math /xuan13.pdf)