New prime k-tuple theorem（2）

$$P, P+j(j+1)(j=1,\cdots,k)$$

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that for every positive integer k there exist infinitely many primes P such that each of $P+j(j+1)$ is prime.

Theorem.

$$P, P+j(j+1)(j=1,\cdots,k). \qquad (1)$$

For every positive integer k there exist infinitely many primes P such that each of $P+j(j+1)$ is prime.

Proof. We have Jiang function [1, 2, 3]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \qquad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k}[q+j(j+1)]\equiv 0 \pmod P, \qquad (3)$$

where $q=1,\cdots,P-1$.

From (3) we have

If $P<2k$ then $\chi(P)=\frac{P-1}{2}$, If $2k<P$ then $\chi(P)=k$.

From (3) and (2) we have.

$$J_2(\omega)=\prod_{P=3}^{P<2k}\frac{P-1}{2}\prod_{2k<P}(P-1-k)\neq 0 \qquad (4)$$

We prove that for every positive integer k there exist infinitely many primes P such that each of $P+j(j+1)$ is prime.

We have the asymptotic formula [1, 2, 3]

$$\pi_{k+1}(N,2)=\left|\left\{P\le N: P+j(j+1)=prime\right\}\right|\sim\frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}, \qquad (5)$$

where $\phi(\omega)=\prod_P(P-1)$.

Note Let $P=11$, $11+j(j+1)(j=1,\cdots,9)$ are all prime.

Let $P=41$, $41+j(j+1)(j=1,\cdots,39)$ are all prime.

Example 1. Let $k=1, P, P+2$, twin primes theorem.

From (4) we have

$$J_2(\omega)=\prod_{3\le P}(P-2)\ne 0. \qquad (6)$$

We prove twin primes theorem. There exist infinitely many primes P such that $P+2$ is prime.

From (5) we have the best asymptotic formula

$$\pi_2(N,2)\sim 2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}. \qquad (7)$$

Exampe 2. Let $k=2, P, P+2, P+6$.

From (4) we have

$$J_2(\omega)=\prod_{5\le P}(P-3)\ne 0. \qquad (8)$$

We prove that there exist intinitely many primes P such that $P+2$ and $P+6$ are all prime.

From (5) we have the best asymptotic formula

$$\pi_3(N,2)\sim \frac{9}{2}\prod_{5\le P}\frac{P^2(P-3)}{(P-1)^3}\frac{N}{\log^3 N}. \qquad (9)$$

Example 3. Let $k=6, P, P+j(j+1)(j=1,\cdots,6)$

From (4) we have

$$J_2(\omega)=30\prod_{13\le P}(P-7)\ne 0. \qquad (10)$$

We prove that there exist infinitely many primes P such that each of $P+j(j+1)$ is prime.

From (5) we have the best asymptotic formula

$$\pi_7(N,2)\sim \frac{1}{16}\left(\frac{231}{48}\right)^6\prod_{13\le P}\frac{(P-7)P^6}{(P-1)^7}\frac{N}{\log^7 N}. \qquad (11)$$

The author takes a day to write this paper.

References

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http://www wbabin.net/math/xuan2.pdf)(http://vixra.org/pdf/0812.0004v2.pdf).

[2] Chun-Xuan Jiang, The Hardy-Littlewood prime *k*-tuple conjecture is false. http:// www.wbabin.net/math/xuan77.pdf. This conjecture is generally believed to be true, but has not been proven (Odlyzko et al. 1999).

[3] Chun-Xuan Jiang, New prime *k*-tuple theorem (1), http://www.wbabin.net/math/ xuan78.pdf

http://wbabin.net/xuan.htm#chun-xuan

Remark. Cramér's random model of prime theory is false.

Example. Assming that the events " P is prime" and " $P+2$ and $P+4$ are primes" are independent, we conclude that $P, P+2$ and $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ 3-tuple prime less than N. Letting $N \to \infty$ we obtain the 3-tuple conjecture which is false.