Smarandache Φ-Theorem

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Abstract.
Fermat's and Euler's theorem on congruencies are generalized to the case when the integers a and m are not necessarily co-prime.

Smarandache Φ-Theorem.

If $a, m \in Z$ and $m \neq 0$, then

$$a^{\varphi(m_s)+s} \equiv a^s \left(\bmod m\right)$$

where φ is Euler's totient function, and m_s and s are obtained from the below

Smarandache Φ-Algorithm:

Step 1. $A := a,\ M := m,\ i := 0$

Step 2. Calculate $d = \left(A, M\right)$ and $M' = \dfrac{M}{d}$

Step 3. If $d = 1$ take $s = i,\ m_s = M'$, and stop.

If $d \neq 1$ take $A := d,\ m = M',\ i := i + 1$ and go to Step 2.

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This is a generalization of Euler's Theorem: if $a, m \in Z$ and $\left(a, m\right) = 1$, then $a^{\varphi(m)} \equiv 1\left(\bmod m\right)$.

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Smarandache totient function is defined as:

$$S\Phi : Z^2 \to Z^2 .$$

For $m \neq 0,\ S\Phi\left(a, m\right) = \left(m_s, s\right)$ such that $a^{\varphi(m_s)+s} \equiv a^s \left(\bmod m\right)$.

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Study the Smarandache Φ-Theorem, Smarandache Φ-Algorithm, and Smarandache totient function.