On the Origins *of* the Stellar Initial Mass Function

Azimuthally Symmetric Theory *of* Gravitation (III)

G. G. Nyambuya[*],

ABSTRACT

In this reading, a new theoretical model of star and cluster formation is posited. This model seeks to set a mathematical framework to understand the origins of the stellar Initial Mass Function and within this framework, explain star and cluster formation from a unified perspective by tieing together into a single garment three important observational facts: (1) that the most massive stars of most observed clusters of stars are preferentially found in their centers; (2) Larson's 1982 empirical observation that the maximum stellar mass is related to the total mass of the parent cloud; (3) that clump masses in giant molecular clouds exhibit a power mass spectrum law akin to that found in star clusters and this behavior is also true for molecular clouds as well. Key to this model is the way the cloud fragments to form cores from which the new stars are born. We show that the recently proposed azimuthally symmetric theory of gravitation has two scale of fragmentation where one is the scale that leads to cloud collapse and the other is the scale on which the cloud fragments. The collapse and fragmentation takes place simultaneously. If the proposed model is anything to go by, then, one can safely posit that the slope of the IMF can be explained from two things: the star formation rate of the cores from which these stars form and the density index describing the density profile. Additionally and more importantly, if the present is anything to by, then, fragmentation of molecular clouds is posited as being a result of them possessing some spin angular momentum.

Key words: stars: evolution — stars: formation — stars: general — stars: luminosity function, mass function — *(Galaxy:)* open clusters and associations: general.
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1 INTRODUCTION

It is clear and goes without saying, that a complete comprehension and resolution of the fundamental origins of the stellar Initial Mass Function (IMF), is at present, one of the "*holy grails*" of astrophysical research. Its complete understanding would lead to a better understanding of the way stars and star clusters come into being. The IMF is an important physical attribute of stellar systems because it determines the baryonic content, chemical enrichment and evolution of galaxies – in-turn it determines the Universe's light and baryonic content and thus an in-depth knowledge of it, is of paramount importance. Reviews on the subject have been carried out over the years (see *e.g.* Miller & Scalo 1979; Scalo 1986; Scalo 1998; Meyer *et al.* 2000; Kroupa 2001; Kroupa & Boily 2002; Larson 2003; amongst others).

Today, it is fifty five years ago since Salpeter (1955) published the

first, important and most famous paper of all on stellar IMF and since then countless efforts have been made to try and decipher and understand its fundamental origins and implications on the evolution of stellar systems. The significant observational work carried out to date (*e.g.* Scalo 1986; Kennicutt 1998, amongst others) confirm the Salpeter Law is linear on a logarithmic scale for the mass range $0.4 - 10.0 \mathcal{M}_\odot$ (see *e.g.* Larson 1982) and at the sametime universal (see *e.g.* Kennicutt 1998; Leitherer 1998; Scalo 1998). Further, for the mass range $10 - 100 \mathcal{M}_\odot$, the IMF exhibits a Salpeter IMF (see review Zinnecker & Yorke 2007) thus the IMF, on the average, has the same Salpeter slope for the mass range $0.4 - 100 \mathcal{M}_\odot$.

Despite all the above cited works, we still lack a proper theory to explain it and this is intrinsically a product of our knowledge paucity regarding the star formation process itself. The question – what is the physics behind such an orderly distribution of matter that ends up locked as stars has never stopped to bother the agile & curious minds. It is unfortunate (or maybe fortunate) that there are many theories of clustered star formation that have been proposed to date each with the ability to explain the IMF. Most of these theories are able

[*] **Electronic mail address:** *gadzirai@gmail.com*.

to explain the IMF after some fine tuning. This fine tuning is made possible because the IMF does not constrain the dominant physics that leads to its raise (see *e.g.* Clarke 1998). These theories of the IMF fall into three main categories:

(i) Fragmentation (see *e.g.* Vazquez-Semadeni *et al.* 1997)
(ii) Negative Feedback Processes (see *e.g.* Silk 1995; Adams & Fatuzzo 1996)
(iii) Competitive Accretion (see *e.g.* Zinnecker 1982; Clarke 1998; Bonnell et al. 2001a,b; Bate & Bonnell 2004)

In reality all these processes may play an important and decisive role in shaping the IMF but the elusive answer to the question – which of these processes dominates giving rise to the IMF remains unanswered. Because of this, a pandemonium arises since none of these theories can be ruled out unless sufficient observations are available to act up as the arbiter thus pressure is put on the need for more refined observations. Should these observations became available, there is hope that only the correct theory will stand the test as the fine tuning of these theories requires that stars and star clusters form under meticulous and specific conditions in-order to reproduce the observed IMF. Clearly, is a need to seek a completely new theoretical avenue of thought (if at all possible there) within the realm of present observational evidence and known Physical Laws which requires little if any fine turning at all, so as to make it possible to easily falsify the theory. It is our hope that the ideas propagated in present satisfy the just said.

Before we lay down our idea(s) on the origins of the IMF, we need to make one thing clear to our reader, that, this reading is meant to lay down the mathematical foundations of the IMF model we are going to propose. It is not a comparative study of this IMF model with those currently in existence. We believe we have to put thrust on lieing down these ideas and only worry about their plausibility, *i.e.* whether or not they correspond with experience and only thereafter make a literature wide comparative study. This literature wide comparative study is expected to be done once a mathematical model of our proposed IMF model is in full swing. This mathematical model is expected to form part of the future works where only-after that, it would make more sense to embark on this literature wide comparative study. The next reading is expected be one such where our IMF model is going to be full swing and a literature wide comparative study is conducted. We hope the reader concurs with us that this is perhaps the best way to set into motion a new idea amid a plethora of ideas that champion a similar if not the same endeavor.

Now, let us begin our expedition (*i.e.*, seeking this theory of the IMF) by looking into the natural world as it lay hind for us to marvel at. So doing, we see that a mass distribution or segregation similar to the IMF is not only exhibited by a population of stars but larger systems such as cores inside clumps and; clumps inside MCs. Clumps are found residing in MCs; while cores are found residing in Clumps. Clumps have masses of the order of $10^3 \mathcal{M}_\odot$ and size of about 1.0 pc and average H_2 density of 10^3 cm^{-3} and these are the sites where clusters are born. Studies of clumps have revealed that their masses are distributed in an orderly fashion much in the same manner as what is found in a population of stars, *i.e.*, they exhibit a mass function and the observed slope lies in a surprisingly narrow range ($1.40 \leqslant \Gamma \leqslant 1.70$) and this is for a clump mass range [$2.00 \leqslant \log_{10}(\mathcal{M}/\mathcal{M}_\odot) \leqslant 6.00$] (see *e.g.* Lada 1990; Blitz 1991;

Kramer *et al.* 1998; Motte *et al.* 1998; Kroupa & Boily 2002). Let us call this the Clump Mass Function (C*l*MF).

As already stated the story does not end here because when MCs are inspected in their large proportions or numbers, once again the number of MCs within a certain mass range are seen exhibiting a similar mass distribution as the IMF (Solomon *et al.* 1987; Motte *et al.* 1998; Heuithausen & Corneliussen 1995; Miyazaki *et al.* 2000) and in-passing, it is important to note that gravitational fragmentation (Fiege & Pudritz 2000) and turbulent compression and fragmentation models are capable of explaining the MCMF (see *e.g.* Vazquez-Semadeni *et al.* 1997). Let us call this the Molecular Cloud Mass Function (MCMF). This is the same for cores that reside in clumps that later go on to form the stars and star clusters (see *e.g.* Motte *et al.* 1998) – let us call the mass function exhibited by cores the Core Mass Function (C*o*MF). Sure, something universal and subtle must be at work giving rise to such important prominent characteristic of stellar systems on the different scales.

It is only natural to ask the important and perdurable question; why is it so that *Nature* appears to distribute, segregate or partition matter in such an orderly fashion on the different scales? Further, is the mechanism that gives rise to the MCMF, C*l*MF, C*o*MF and the IMF universal? That is to say, do these arise from the same physical mechanism? Given that *Nature* almost always manifest Herself in the simplest imaginable, universal and unified manner as is evident when one looks at the broad spectrum of physics, it is natural to conceive that the mechanism that gives rise to the mass function on any scale, *i.e.*, cosmological, galactic, molecular cloud or clump scale ought or might be universal. This leads one to the idea of a possible Universal Mass Function (UMF) whose shape and characteristic is dependent on the initial conditions and scale. The self similar structures observed in *Nature e.g.* in the Orion Nebula maps observed by Maddalena *et al.* (1986) at $8.7'$ (65 pc) resolution in CO (Dutrey *et al.* 1991), $1.7'$ (3.5 pc) resolution in C^{18}O and Wiseman (1995) at $8''$ (0.6 pc) resolution in NH$_3$ of the same region but different scales strongly suggests that *Nature* may behave in a self-similar manner on different scales somewhat in support of a universal mechanism giving raise the mass functions (MCMF, C*l*MF, C*o*MF & the IMF) at the different scales.

In this reading, we shall put forward the new ideas of the IMF and these fall under Fragmentation Theories – *i.e.* Jeans-collapse and subsequent-fragmentation leading to the formation of Jeans-cores that in-turn collapse to form the stars. Basically, the greater vision of these ideas is the endeavor to tie together in a single garment the origins of the all the mass functions, that is, the MCMF, C*l*MF, C*o*MF and the IMF. In our endeavor to derive the IMF, we also seek to answer to the following questions:

Pertinent Questions:

(i) Why in stellar populations there exist well ordered systematic radial mass segregation with a pronounced concentration of massive stars in the cluster center.

(ii) When, where, how and why does mass segregation occur? By mass segregation it must be understood to mean not only the spatially varying mass distribution found in stars clusters, that is, concentration of more massive stars in the cluster center but also the well ordering of the mass across a given mass range.

(iii) Is the IMF universal? By this what is meant is do all the IMF's observed in *Nature* originate from the same physical mechanism and if so what does it depend on?

Observational Clues:

What observational clues – *if any*; do we have to guide us in the search for the origins of the IMF? The following is a catalogue of paramount and vital clues:

(1) Any theory of star formation must be able to explain the observational fact why massive stars or the most massive member of any stellar population is found in the center.

(2) The IMF exhibits a cutoff in the mass $\mathcal{M}_{ch} = 0.4 - 1.0\mathcal{M}_\odot$ and this behavior seems to be a universal character of the IMF, the suggestion of which is that the mechanism that gives rise to it also ought to be universal. Two basic type of hypothesis have been proposed to explain this characteristic mass (see *e.g.* Larson 2003 and references therein) and these are:

(i) Stars form from MC clumps whose masses are similar to those of the stars that form from them and in-turn they derive their typical masses from the clumps. The properties of the clumps presumably are determined by the cloud fragmentation process.

(ii) The other possibility is that stars continue to accrete mass indefinitely from an extended medium and they may determine their own masses by producing outflows that terminate the accretion process at some stage.

These clues shall guide us in the formulation of our new ideas on the IMF. It is important that we mention that the chief idea leading to our proposal for the IMF, surprisingly comes from the recent ideas set out in Nyambuya (2010a,b). The azimuthally symmetric gravitational field (see Nyambuya 2010a,b) sets up two scales on which the cloud must collapse, the first scale is one of the order of the cloud size; collapse on this scale leads to the entire cloud to collapse. The second is the fragmentation scale which is much smaller than the cloud size. If this scale where to lead to fragmentation, then, it must be of the order of the molecular cores from which the stars from. Thus collapse and fragmentation takes place simultaneously. No theory proposed to date is capable of simultaneous collapse and fragmentation, thus if our survey of the literature has been good, the ideas in the present reading are appearing for the first time.

The synopsis of this reading is as follows: in the subsequent section, we lay down the Jeans collapse process as it is commonly understood. In §(3) we look at the Jeans collapse process under the azimuthally symmetric gravitational field and show that in the first order approximation, the azimuthally symmetric gravitational field leads to a two scale collapse scenario. We show there-in how this twin collapse scale leads to cloud fragmentation. In §(4) we argue and show that the Jeans collapse should lead to the formation of massive stars in the centre. In §(5) we derived the IMF. In §(6) we argue that the present ideas should lead to the observed King profiles. In the penultimate, we give an overall discussion and lay down our conclusions.

2 SPHERICALLY SYMMETRIC GRAVITATIONAL JEANS COLLAPSE

It is generally believed that the star formation process begins kick started by the collapse of a MC in which process when the MC collapses, it contracts causing portions thereof to also further collapse and this causes the cloud to fragment into smaller parcels of matter (cores) and these collapse further to form stars. This collapse process is generally believed to be kicked started probably by local density inhomogeneities in the MC caused by random fluctuations due to cloud turbulence which causes the density in some parts of the cloud to be high enough for gravity to overcome the local cloud pressure. Also, it is believed that the collapse may be triggered by compression caused by outside forces such as a shock wave from a nearby supernova event.

From a dynamical gravitational view-point, consider a gravitationally bound system of matter. For such a system, the viral theorem applies and as first realized since the subject was first studied by Sir James Jeans (1929), a MC or portion thereof can collapse gravitationally under its one own weight if its mass, density and temperature satisfy the criterion known as the Jeans Criterion where the kinetic energy ($\mathcal{K} = \rho k_B \mathcal{T}_{av}/\mu m_H$) of the particle in the gravitational field must be less than the absolute magnitude of the gravitational potential energy ($\mathcal{U} = -G\mathcal{M}^2/r$) at any given point ($\mathcal{K} < |\mathcal{U}|$). The condition ($\mathcal{K} < |\mathcal{U}|$) leads to Jeans collapse which can be defined as ($\mathcal{M}_{cl} < \mathcal{M}_J$). That is to say, for a uniform density and temperature (we neglect magnetic fields and rotation) MC of mass of $\mathcal{M}_{cl}$ (or portion thereof) to collapse, the cloud's mass (or portion thereof), must be greater than its Jeans mass *i.e.*, $\mathcal{M}_{cl} > \mathcal{M}_J$ where:

$$\mathcal{M}_J = \left(\frac{5k_B \mathcal{T}_{av}}{\mu G m_H}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi \rho_{av}}\right)^{\frac{1}{2}}, \tag{1}$$

where $\mathcal{T}_{av}$ is the average cloud temperature, ρ_{av} the average cloud mass density, G is Newton's universal constant of gravitation, μ is the mean molecular mass, k_B is Boltzmann's constant and m_H is mass of atomic hydrogen. Another way to express the Jeans criterion is to express it in terms of the Jeans radius, that is to say:

$$\mathcal{R}_J = \left(\frac{\pi k_B \mathcal{T}_{av}}{\mu G m_H \rho_{av}}\right)^{1/2}, \tag{2}$$

which is the minimum size that a MC (or portion thereof) must attain in order for it to collapse, *i.e.*, $\mathcal{R}_{cl} < \mathcal{R}_J$.

It is instructive for our purpose here to demonstrate the wave nature of the Jeans collapse process hence we shall go through a proper derivation of the Jeans criterion from a fluid dynamics vantage-point. For the cloud configuration described above, we know from fluid mechanics, Newton's second law and the universal Law *of* Gravitation; and from the idea gas law; that the system of equations describing the global dynamics of the cloud are:

$$\frac{\partial \rho}{\partial t} - \nabla \cdot (\rho \mathbf{v}) = 0, \tag{3}$$

$$\rho \frac{\partial \vec{\mathbf{v}}}{\partial t} + \rho \mathbf{v} \cdot \nabla \vec{\mathbf{v}} = -\nabla \mathcal{P} - \rho \vec{\nabla} \Phi = 0, \tag{4}$$

$$\nabla^2 \Phi = 4\pi G \rho, \tag{5}$$

$$\mathcal{P} = \left(\frac{\rho}{\mu m_H}\right) k_B \mathcal{T}_{av} = c_s^2 \rho, \tag{6}$$

where $\mathcal{P}$ is the cloud pressure, c_s the speed of sound in the cloud medium, $\mathbf{v}$ the velocity of the gas at any point in the cloud and Φ the Newtonian gravitational constant. Equations (3), (4), (5) and (6) are the continuity equation, Newton's second law, Newton's law of universal gravitation, and the ideal gas equation respectively. Now, (1) taking the divergence of equation (4) and (2) then dividing equation (3) by c_s and taking the partial time derivative of this same equation and then subtracting the two resultant equations and the rearranging, we are lead to:

$$\vec{\nabla}^2 \rho - \frac{1}{c_s^2}\frac{\partial^2 \rho}{\partial t^2} = -4\pi G \rho^2 + \vec{\nabla}\cdot(\dot{\rho}\vec{v}) - \vec{\nabla}\cdot(\rho \mathbf{v}\cdot\vec{\nabla}\vec{v}) - \vec{\nabla}\cdot(\rho\vec{\nabla}\Phi), \tag{7}$$

and now if we introduce a small perturbation, *i.e.* $\rho \longmapsto \rho + \delta\rho$, $\Phi \longmapsto \Phi + \delta\Phi$ and $\vec{v} \longmapsto \vec{v} + \delta\vec{v}$, and the terms with δ are the perturbation terms; and substituting these into the above equation and then neglecting all second order perturbation terms (see *e.g.* Perkünlü *et al.* 2002), we are lead to the equation:

$$\nabla^2 \delta\rho - \frac{1}{c_s^2}\frac{\partial^2 \delta\rho}{\partial t^2} = -\left(\frac{4\pi G \rho}{c_s^2}\right)\delta\rho. \tag{8}$$

In the derivation of the above equation, the assumptions made in the perturbation, namely that we are considering variables on a local scale, lead to vanishing derivatives of the terms ρ, Φ and $\vec{v}$, because locally these terms are constant. Now, assuming a general solution of the form $\delta\rho = \rho e^{-i\omega t}[A\sin(\mathbf{k}\cdot\mathbf{x}) + B\cos(\mathbf{k}\cdot\mathbf{x})]$ where A,B are normalization constants, ω is the density wave frequency and $|\mathbf{k}| = 2\pi/\lambda$ the density wave number and λ is the wavelength of this perturbation, we get the dispersion relation:

$$\omega^2 = k^2 c_s^2 - 4\pi G \rho. \tag{9}$$

Proceeding – now for self gravity to bring about the collapse of the cloud system, the density ρ must grow in time. A necessary condition for this is $\omega^2 < 0$. This is however not sufficient condition. A sufficient and necessary condition is that ω be purely an imaginary number, *i.e.*, $\omega = i\omega_*$ where ω_* is a real number, in which case:

$$\delta\rho = \rho e^{\omega_* t}[A\sin(\mathbf{k}\cdot\mathbf{x}) + B\cos(\mathbf{k}\cdot\mathbf{x})], \tag{10}$$

and as one can clearly see for themselves, the condition that ω must be imaginary brings an exponential time dependent growth into the amplitude of the perturbation. This condition ($\omega^2 < 0$) ensures growth of the density with time and is equivalent to $k^2 < 4\pi G \rho/c_s^2$ thus the critical size of the perturbation is:

$$\lambda_J = \left(\frac{\pi k_B \mathcal{T}_{av}}{\mu G m_H \rho_{av}}\right)^{1/2}, \tag{11}$$

which is the well known Jeans length given in equation (2). At this point, we shall digress a little from the main theme so that we solve a small but *very important* problem with the density wavefunction.

What we have to do is to impose boundary conditions to this equation. As one can recalled from quantum mechanics, that, when solving the problem of a particle in a box using Schröndiger's equation, one has to make the wavefunction to vanish at the edges of the infinite potential well. This is done because the wave is trapped between these wall. In a similar manner, we have to confine the density waves here so that they vanish at the center and at the edge of the cloud, that is $\delta\rho(r = 0) = 0$ and $\delta\rho(r = \mathcal{R}_{cl}) = 0 \Longrightarrow B = 0$ and clearly the perturbed density wavefunction will have to be given by:

$$\delta\rho = A\rho e^{\omega_* t}\sin(\mathbf{k}\cdot\mathbf{x}). \tag{12}$$

Once the cloud collapse has began, the density inhomogeneities and turbulence will, as already said before, cause the cloud to fragment into smaller aggregates of matter and become cores from which a star will be born. As the Jeans collapse theory stands, it can account for the collapse of the MC but not the fragmentation thereof. This is set to change as we shall see that the azimuthally symmetric gravitational field provides not just for the collapse of the entire cloud but the fragmentation of the cloud.

Assuming that by some unknown mechanism, the cloud fragments into core, these cores will undergo free-fall and assuming a star formation efficiency rate $\xi_{SFR} = 100\%$, meaning to say all the mass in the collapsing core forms the star, then, this would occur over a time scale:

$$t_{ff} = \left(\frac{3\pi}{32G\rho_{av}(0)}\right)^{1/2}, \tag{13}$$

where $\rho_{av}(0)$ is the average density of the core at time $t = 0$. This time scale is known as the free-fall time scale. This applies for low mass stars in their early stages of formation before radiation is thought[1] to begin to significantly affect the collapse. For intermediate and high mass stars more process other than the free-fall come into play during the gestation period of these stars thus resulting in the star, forming in a time significantly less than t_{ff}.

To give one an intuitive grasp of the Jeans process at work, suppose we are given the Jeans Criterion and we trace matter back to the time of matter-radiation decoupling era according to the currently accepted Big Bang Theory where the temperature of matter was about $300\,\mathrm{K}$ and its density about $6 \times 10^3\,\mathrm{cm}^{-3}$ and inserting these into equation (1) yields $\mathcal{M}_J \sim 10^5\mathcal{M}_\odot$. It therefore follows that the smallest possible mass capable of collapsing at the time of decoupling was about $10^5\mathcal{M}_\odot$. That is about the mass of a present day globular cluster. By contrast in the interstellar medium of our Galaxy where $T \sim 50\,\mathrm{K}$ and $n \sim 10^3\,\mathrm{cm}^{-3}$ the Jeans mass is about $500\mathcal{M}_\odot$. It is clear that today, far smaller parcels of gas can collapse and that is why MCs do and are home to individual and groups of stars. The question how they collapse or fragment to form stars in such an orderly fashion to give raise to the IMF is a question which we are shall make an attempt to address.

It is important to note that once the material has collapsed it becomes gravitationally bound, meaning it would require a mass or portion

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[1] It has been shown in Nyambuya (2010b) that the radiation field should not be an impediment to the nascent star while it is in the stellar womb.

thereof to attain the escape velocity of the entire cloud system in-order to escape the gravitational binding imposed on it, thus, we shall take a parcel of matter that has reached and passed its Jeans-limit to constitute an individual system. To evince on the aforesaid: a parcel of matter of mass say $\mathcal{M}$, that has fragmented *via* the Jeans Criterion – from a given cloud system which initially acted as a rigid body with the fragmented unit before the fragmentation took place; will detached from the main body and now act itself too, as a unit conglomerate body – a single unit of mass $\mathcal{M}$. This is what is meant by "a parcel of matter that has reached its Jeans criterion constitutes an individual and independent gravitating system".

3 AZIMUTHALLY SYMMETRIC GRAVITATIONAL JEANS COLLAPSE

Now we are going to investigate the collapse of a rotating cloud. The gravitational field inside this cloud is not spherically symmetric but azimuthally symmetric and from Nyambuya (2010a,b), the gravitational potential of this azimuthally symmetric gravitational field is to first order approximation given by:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}(r)}{r}\left(1 + \frac{\lambda_1 G\mathcal{M}(r)\cos\theta}{rc^2}\right). \tag{14}$$

Given the azimuthally symmetric gravitational potential (14), this means the gravitational energy of a particle inside the cloud is: $\mathcal{U}(r,\theta) = \mu m_H \Phi(r,\theta)$. Now, the kinetic energy ($\mathcal{K}$) of the this particle is given by its thermal energy which is $k_B T$, this means: $\mathcal{K} = k_B \mathcal{T}_{av}$. For Jeans collapse to occur when: $\mathcal{K} < -\mathcal{U}(r,\theta) = |\mathcal{U}(r,\theta)|$.

Now for collapse to occur on a global scale, then, at the surface of the cloud, *i.e.* at $r = \mathcal{R}_{cl}$, the kinetic energy of the particles on this surface must be less than the least (in-terms of the magnitude *i.e.*, $|\mathcal{U}|$) possible gravitational potential energy of the particle on the surface of the cloud, that is: $\mathcal{K} < |\mathcal{U}(\mathcal{R}_{cl},\theta)|_{min}$. The minimum value of $|\mathcal{U}(\mathcal{R}_{cl},\theta)|_{min}$ will occur when $\theta = 180°$, therefore the condition for global collapse is: $\mathcal{K} < |\mathcal{U}(\mathcal{R}_{cl},180°)|_{min}$. We know that $\mathcal{M}(\mathcal{R}_{cl}) = \mathcal{M}_{cl}$ thus the condition: $\mathcal{K} < |\mathcal{U}(\mathcal{R}_{cl},180°)|_{min}$, leads us to:

$$k_B\mathcal{T}_{av} < \left(\frac{\mu G\mathcal{M}_{cl}m_H}{\mathcal{R}_{cl}}\right)\left(1 - \frac{\lambda_1 G\mathcal{M}_{cl}}{\mathcal{R}_{cl}c^2}\right), \tag{15}$$

this can be re-written as:

$$\mathcal{R}_{cl}^2 - \mathcal{R}_J\mathcal{R}_{cl} + \lambda_1\mathcal{R}_J\mathcal{R}_{cl}^S < 0, \tag{16}$$

where $\mathcal{R}_J$ has the same meaning as in the spherically symmet-ric gravitational Jeans collapse and $\mathcal{R}_{cl}^S = G\mathcal{M}_{cl}/c^2$ is the Schwarzschild radius of the MC. To solve this inequality, we have to find the solution to the quadratic equation $\mathcal{R}_{cl}^2 - \mathcal{R}_J\mathcal{R}_{cl} + \lambda_1\mathcal{R}_J\mathcal{R}_{cl}^S = 0$. The two solutions to this equation are:

$$\mathcal{R}_{cl} = \frac{\mathcal{R}_J \pm \sqrt{\mathcal{R}_J^2 - 4\lambda_1\mathcal{R}_J\mathcal{R}_{cl}^S}}{2} \simeq \frac{\mathcal{R}_J \pm (\mathcal{R}_J - 2\lambda_1\mathcal{R}_{cl}^S)}{2}. \tag{17}$$

We have taken these solutions to their first order approximation since we would generally expect $\mathcal{R}_J$ to be much greater than $\lambda_1\mathcal{R}_{cl}^S$, so instead of burdening ourself with the complicated expression, we can work with a less complicated first order approximation. Written more lucidly as $\mathcal{R}_1$ and $\mathcal{R}_2$, these two solutions are given:

$$\mathcal{R}_1 = \mathcal{R}_J - \lambda_1\mathcal{R}_{cl}^S \quad \text{and} \quad \mathcal{R}_2 = \lambda_1\mathcal{R}_{cl}^S. \tag{18}$$

With these solutions, the inequality (16) becomes $(\mathcal{R}_{cl} - \mathcal{R}_1)(\mathcal{R}_{cl} - \mathcal{R}_2) < 0$ and there are two solutions:

(i) **Condition One:** If $[\mathcal{R}_{cl} - \mathcal{R}_1 < 0]$ then $[\mathcal{R}_{cl} - \mathcal{R}_2 > 0]$, and all this leads us to:

$$\mathcal{R}_{cl} < \mathcal{R}_1 \quad \text{and} \quad \mathcal{R}_{cl} > \mathcal{R}_2. \tag{19}$$

(ii) **Condition Two:** If $[\mathcal{R}_{cl} - \mathcal{R}_1 > 0]$ then $[\mathcal{R}_{cl} - \mathcal{R}_2 < 0]$, and all this leads us to:

$$\mathcal{R}_{cl} > \mathcal{R}_1 \quad \text{and} \quad \mathcal{R}_{cl} < \mathcal{R}_2. \tag{20}$$

The second condition is an impossible condition since under all con-ditions of existence $(\mathcal{R}_1 > \mathcal{R}_2)$, hence (19) is our desired solution. Clearly, the condition $\mathcal{R}_{cl} < \mathcal{R}_1$ leads to collapse on a global scale and we see from it that the cloud will have to be smaller (by an amount $\lambda_1\mathcal{R}_{cl}^S$) than the Jeans radius applicable in the case of spher-ically symmetric gravitation. What about the other scale? What does the condition $\mathcal{R}_{cl} > \mathcal{R}_2$ mean?

3.1 Cloud Fragmentation

We shall go back to the afore-described spherically symmetric gravi-tational Jeans collapse, and we go through description we shall mod-ify this description to that is takes into account the two scales of collapse discovered above. As afore-described: in the beginning – a serene MC of radius $\mathcal{R}_{cl}$, with an average temperature $\mathcal{T}_{av}$; and ini-tially under the action of two opposing forces; *i.e.* the inward grav-itational force (whose end and quest is to muzzle all the material to a single point) and the outward resisting thermal force; is such that $\mathcal{R}_{cl} > \mathcal{R}_J$. In the case of azimuthally symmetric gravitational collapse, the global collapse condition now is $\mathcal{R}_{cl} > \mathcal{R}_1$.

As already said, it is acceptable that the Universe began in a state of higher temperature than it is today, that it has cooled to its present temperature of $\sim 2.73\,\text{K}$; a corollary to this, is that MCs have also cooled to their present thermodynamic state. When a gas cools, the mean (average) distance between the particles of this gas will de-crease too, the meaning of which is that the cloud size must decrease as-well.

Now, if the cloud size where to decrease, it is foreseeable that at somepoint in time during its evolution, the critical condition of Jeans-collapse will be reached, that is: $\mathcal{R}_{cl} = \mathcal{R}_1$. Any further cooling of the cloud must lead to Jeans-collapse of the MC because $\mathcal{R}_{cl} < R_1$. From this, it is pristine-clear that thermodynamic cool-ing of the cloud will naturally lead to the Jeans-collapse of the cloud on a global scale. So from a fundamental physics view-point, it is perfectly reasonable to assume that star formation may well be kick started by the thermodynamic cooling of MCs. This assumption does not in any way alter the fragmentation model that we wish to set

(**a**)
(Generation of p-wave)

(**b**)
(Generation of s-wave)
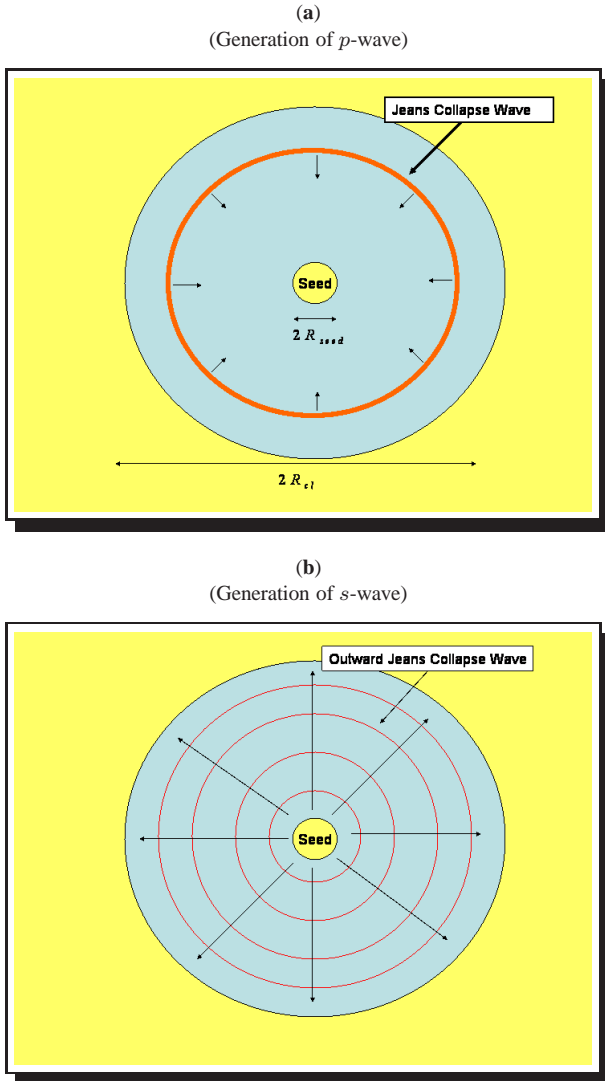
Figure (1). In (a) the initial Jeans-collapse wave generated by the collapse of the cloud travels to the center of the cloud and in (b) this wave arrives at the boundary demarcated by the seed region. Upon arrival at the boundary of the seed region, this seeds a shock that cause this region to collapse and in-turn, this collapse generates a secondary Jeans-collapse ripple wave that moves outward. This secondary wave (s-wave) causes the cloud to be Jeans unstable on the scale of the wavelength of this s-wave and in the process, Jeans-spheres of the size of the wavelength of the s-wave collapse causing the cloud to fragment into Jeans-cores that go on to form stars.

forth. We are simply trying to remove exogenous agents and mechanisms from the star formation podium such as supernova triggered collapse *etc*.

Now we come to the interesting question of the second collapse scale $\mathcal{R}_2$. Just as we argued and accepted that the condition $\mathcal{R}_{cl} < \mathcal{R}_1$, once satisfied, implies that the cloud must collapse on scale $\mathcal{R}_1$ with this scale being measured from the cloud centre, the condition $\mathcal{R}_{cl} > \mathcal{R}_2$ also means that once it is satisfied and as-well the condition $\mathcal{R}_{cl} < \mathcal{R}_1$, then, on this scale $\mathcal{R}_2$, – with it [scale] being measured from the cloud centre, the cloud must collapse! A cloud whose radius is less than $\mathcal{R}_2$ is one that is in a highly relativistic

state since this scale is comparable to the Schwarzschild radius of the MC, so since for ordinary MC, their spatial sizes are far beyond their Schwarzschild radius, then, MCs must collapse on these two scales $\mathcal{R}_1$ and $\mathcal{R}_2$. Thus while the entire cloud collapses, it will collapse internally as-well and in its interior it will collapse on a scale $\mathcal{R}_2$ beginning from the cloud centre. Let us as shown in figure (1) (a) call the internal region where $r \leqslant \mathcal{R}_2$ the seed region.

Now, at the moment of collapse, an outgoing collapse wave is generated in which process it causes the region engulfed by this wave to contract. In the case of the collapse wave generated at the surface of MC, this wave can not move outward since it is at the boundary. Since it can not move outward, it must be reflected and made to travel down to the nimbus of the MC. Let us call this wave generated at the surface the primary wave and for short let us call it the p-wave. On the other hand, the seed region will collapse in which process it will generate outgoing waves, let these waves be called secondary waves and for short let us call them s-waves.

At this point, let us depart from the deep space of MCs and come back home to mother Earth so that we can learn something from her and then take this lesson into the deep space of MCs. When one casts a pebble into still waters in a pond, a wave with a specific wavelength spreads out (into the outer reaches) from the point of impact of the pebble to the outer reaches of the pond. These waves are seen as spreading concentric rings. Redolently, the seed region will send out s-waves. The seed region acts much the same way as a pebble when it is thrown into the pond. The collapse of the seed region, will generate waves whose wavelength is the same size as the seed itself. This wave will spread outward, just as happens when one casts a pebble into still waters causing the outward spread of waves from the point of impact of the pebble on the surface of the still waters.

Clearly, the generation of the s-wave will dissect the cloud into annular-like regions like an onion and inside these regions, Jeans unstable spheres will be created as the s-wave sweeps through the cloud because as this s-wave sweeps through the cloud, it causes these annular onion shells to undergo compression and given that this regions are already in their critical Jeans condition, they will undergo an immediate Jeans collapse – clearly the cloud must fragment. This is the key idea and forms the bedrock and foundation to our model. If the reader concurs with us, then what follows therefrom, is a given.

From this simple picture and idea of cloud fragmentation, as will be shown in §(4), one can derive the Initial Mass Function (IMF) of stars. In a nutshell, the IMF is posited as originating from the thermodynamic cooling process taking place prior to the initial Jeans-collapse and subsequent-fragmentation originating from the collapse of the seed region. It should be mentioned that, in our derivation of the IMF, we **shall** deliberately neglect the role of turbulence (see *e.g.* Padoan & Nordlund 2003; Padoan *et al.* 2007) and radiation transport – these processes come-in, only after the fragmentation has taken place hence these processes are not relevant at this stage. Before we go on to derive the IMF, we need to compute the time scales on which the s-and-p-waves will last.

3.1.1 Time Scales

As shown in figure (1) (a) the initial Jeans-collapse wave generated by the collapse of the cloud travels to the center of the cloud. We shall make a calculation here of the time it would take to reach the cloud centre. We are doing this so as to get an estimate of the time scale of how long this might last. Because the p-wave must vanish at the cloud surface and center, that is: $\delta\rho = 0$ at $r = 0$ and also at $r = \mathcal{R}_{cl}$; and given that only one cycle of this wave must fit inside the cloud from the center to the edge, it follows that the wavelength (λ_p) of this cloud must equal the size of the cloud, *i.e.*: $\lambda_p = \mathcal{R}_{cl}$.

Given that these s and p-waves are sound waves, from equation (6) and also from the fact the speed of a wave equals λ/τ where τ is time it take for the wave to travel one wavelength, it follows that the p-wave will last for a time:

$$\tau_p = \left(\frac{k_B \mathcal{T}_{av}}{\mu m_H} \right)^{-\frac{1}{2}} \mathcal{R}_{cl}, \tag{21}$$

Taking a typical cloud size of $1.0\,\mathrm{pc}$ and an average typical cloud temperature $\mathcal{T}_{av} = 10\,\mathrm{K}$ one obtains $\tau_p \sim 100\,000\,\mathrm{yrs}$.

The most important time scale is the time is will take the s-wave to reach the surface because then, the fragmentation of the cloud will be complete. What cause the cluster is these fragments this once they are all in place can we say the stage for star formation is set has all the stellar wombs would have formed. Now for the s-wave, this must vanish at the cloud centre and as-well at the cloud edge, that is: $\delta\rho = 0$ at $r = 0$ and $r = \mathcal{R}_{cl}$. The former (*i.e.* $\delta\rho = 0$ at $r = 0$) yields no information and the latter ($\delta\rho = 0$ at $r = \mathcal{R}_{cl}$) yields $2\pi\mathcal{R}_{cl}/\lambda_s = \pi n$. Just as the p-wave, the wavelength of the s-wave must equal the size of the seed, *i.e.* $\lambda_s = \mathcal{R}_{seed}$. It follows that the number of ripple waves that will be generated is:

$$n_{max} = \frac{\mathcal{R}_{cl}}{\mathcal{R}_{seed}} = \frac{\mathcal{R}_1}{\mathcal{R}_2}. \tag{22}$$

We know that the p and s-waves both travel the same distance and that they do so at the same speed, it follows that the s-wave will take the sametime that the p-wave takes to travel down to the centre for the same typical conditions ($\mathcal{R}_{cl} = 1.0\,\mathrm{pc}$ and $\mathcal{T}_{av} = 10\,\mathrm{K}$), hence $\tau_s \sim 100\,000\,\mathrm{yrs}$. So, it means the initial collapse of the cloud leading to the cloud fragmentation will take place on a scale of the order of $0.10\,\mathrm{Myrs}$. Given that stars form on a scale of $10 - 100\,\mathrm{Myrs}$, the time scale on which the collapse and fragmentation occurs is small compared to the time that is takes for the stars to form. This estimate is just a hand-waved estimate, obviously, a most robust calculation will be needed. This has been conducted so that we obtain a rough estimate.

4 JEANS-CRITERION & THE CENTRAL MASSIVE STAR

Our model, which is wholly a model based on the Jeans mechanism will have to explain, how it is possible for massive stars to form in the center given the nature of the Jeans condition [see equation (1) & (2)]. In the center of these MC, the density of material is high; the meaning of which is that there will not be enough mass to form a

massive star since the Jeans mass varies as the inverse of the square-root of the density *i.e.* $\mathcal{M}_J \propto \rho^{-1/2}$. This is a question that has been raised by the authors Bonnell *et al.* (1998); Bonnell & Bate (2002, 2004) for example; they have used this as a very strong argument against the Jeans mechanism to serve as a vehicle leading to the formation of star clusters – because as already said, massive stars are found deep in the centers of these cluster; so how does the Jeans mechanism generate massive stars in the center if this central region is without enough mass to form these massive stars? Obviously, as will be seen shortly, these authors have assumed throughout the MC a uniform density and temperature; this is certainly not the case as we have already know that MCs do exhibit a radial density profile given by:

$$\rho = \rho_0 \left(\frac{r}{R_0} \right)^{-\alpha_\rho}, \tag{23}$$

where ρ_0 and R_0 are normalization constants particular to the cloud in question. Additionally, MCs exhibit a radial temperature profile similar in form to the density profile, *i.e.*:

$$\mathcal{T} = \mathcal{T}_0 \left(\frac{r}{R_0} \right)^{-\alpha_T}, \tag{24}$$

where $\mathcal{T}_0$ is another normalization constant which is particular to the cloud. Now, inserting equation (23) and (24) into equation (1) and (2), we get:

$$\mathcal{M}_J = \mathcal{M}_{*J} \left(\frac{r}{R_1} \right)^{-\left(\frac{3\alpha_T - \alpha_\rho}{2} \right)}, \tag{25}$$

and:

$$R_J = \mathcal{R}_{*J} \left(\frac{r}{R_1} \right)^{-\left(\frac{\alpha_T - \alpha_\rho}{2} \right)}, \tag{26}$$

where $\mathcal{M}_{*J}$ and $\mathcal{R}_{*J}$ are normalization constants. From this we see that if:

$$\alpha_\rho > 3\alpha_T, \tag{27}$$

then, on a scale of R_0 from the center of the cloud, the Jeans mass is much higher in the cloud center than in the outer reaches of the MC, the meaning of which is that the claim (Bonnell *et al.* 1998; Bonnell & Bate 2002, 2004) that Jeans fragmentation **can not** lead to massive stars forming in the center may not necessarily hold as has been demonstrated above. For typical clouds, we have: $1 \leqslant \alpha_\rho < 2$ and $0 < \alpha_T \leqslant 1$, thus the condition ($\alpha_\rho > 3\alpha_T$) can be satisfied. Clearly, the scenario just present runs contrary to the central arguments posited by Bonnell *et al.* (1998); Bonnell & Bate (2002, 2004) in their pioneering work(s). In light of the just demonstrated, one sees that they (Bonnell *et al.* 1998; Bonnell & Bate 2002, 2004), have assumed a constant density and temperature cloud profile ($\alpha_T = \alpha_\rho = 0$), in which case according to (25), the Jeans mass is the same throughout the cloud and if $\alpha_\rho < 3\alpha_T$, then the Jeans mass is smaller in the center and it grows as one zooms out of the cloud thus if ($\alpha_\rho < 3\alpha_T$), then the arguments advanced by these researcher holds.

We have just paved the way and we hope the reader accepts that when ($\alpha_\rho > 3\alpha_T$), the Jeans mass will be higher in the cloud centre than in the outer reaches of the cloud. As a first step, this allows us to entertain the idea that massive stars can indeed form in the centre *via* the Jeans collapse process for as long as the emergent fragments have enough mass needed to form these massive stars.

5   DERIVATION OF THE MASS FUNCTION

With the above presented model of cloud fragmentation, not only is the IMF easily explained, but also the spatial distribution of the stars. What we have is a cloud that has been dissected into annular like regions of equal breath and this breath is the size of the wavelength of the s-wave. From the cloud center, the boundary of the annular regions is given by $r = n\mathcal{R}_{seed}$ where $n = 1, 2, 3, ..., n_{max}$. Inside these annular regions, Jeans unstable spheres of radius $R_{core} = \mathcal{R}_{seed}$ have collapsed and have become the stellar wombs from which the next generation of stars will form.

Our first question is; how many Jeans-cores are generated in the nth shell? To answer this, we know that the volume of the nth shell $V_n = 4\pi r_n^3/3 - 4\pi r_{n-1}^3/3 = V_1(3n^2 - 3n + 1)$ where $V_1 = 4\pi\mathcal{R}_{seed}^3/3$ is the volume of the each of the Jeans-cores. It follows then that the number $N(n)$ of fragments emerging from the Jeans fragmentation process in the nth shell is:

$$N(n) = 3n^2 - 3n + 1. \tag{28}$$

This number is simple obtained from $N(n) = V_n/V_1$. The total amount of mass in each of these spheres in the nth shell can be computed. In this case, this MDF reduces to:

$$\mathcal{M}(r_n) = \mathcal{M}_{csl}\left(\frac{r_n^{3-\alpha_\rho} - \mathcal{R}_{core}^{3-\alpha_\rho}}{\mathcal{R}_{cl}^{3-\alpha_\rho} - \mathcal{R}_{core}^{3-\alpha_\rho}}\right) + \mathcal{M}_{seed}, \tag{29}$$

(see the Appendix section of Nyambuya 2010b) therefore the amount of mass in the nth shell [$\mathcal{M}(n) = \mathcal{M}(r_n) - \mathcal{M}(r_{n-1})$] is given:

$$\mathcal{M}(n) = \left(\frac{\mathcal{M}_{csl}\mathcal{R}_{core}^{3-\alpha_\rho}}{\mathcal{R}_{cl}^{3-\alpha_\rho} - \mathcal{R}_{core}^{3-\alpha_\rho}}\right)\left(n^{3-\alpha_\rho} - (n-1)^{3-\alpha_\rho}\right). \tag{30}$$

and assuming n is large enough, then to first order approximation we will have: $n^{(3-\alpha_\rho)} - (n-1)^{(3-\alpha_\rho)} = n^{(3-\alpha_\rho)}[1 - (1 - 1/n)^{3-\alpha_\rho}] \simeq (3 - \alpha_\rho)n^{(2-\alpha_\rho)}$ and inserting this into the above, we will have:

$$\mathcal{M}(n) = \left(\frac{(3-\alpha_\rho)\mathcal{M}_{csl}\mathcal{R}_{core}^{3-\alpha_\rho}}{\mathcal{R}_{cl}^{3-\alpha_\rho} - \mathcal{R}_{core}^{3-\alpha_\rho}}\right)n^{(2-\alpha_\rho)} = m_* n^{(2-\alpha_\rho)}. \tag{31}$$

where m_* is defined as can be deduced from above. Now, from this mass $\mathcal{M}(n)$, there will emerge $N(n)$ cores and these cores will go on to form stars of mass $\mathcal{M}_{star}(n)$. If $\mathcal{M}_{core}(n)$ is the mass of the core in the nth shell – from this core the star of mass $\mathcal{M}_{star}(n)$ will emerge, then: $N(n)\mathcal{M}_{core}(n) = \mathcal{M}(n)$.

Now we shall make an extrapolation from Larson's 1982 result

namely that the mass of the most massive star $\log\mathcal{M}_{max} \propto \log\mathcal{M}_{cl}$. If we consider that the massive star sees the entire cloud as a core from which it will accrete its mass competitively with the other stars, then, our extrapolation is that Larson law suggests an extension to the effect that the mass of star and the core from which it form may very well have a similar relation namely: $\log\mathcal{M}_{star} \propto \log\mathcal{M}_{core}$ hence $\mathcal{M}_{star} = (\mathcal{M}_{core}/\mathcal{M}_{sfr})^{\alpha_{sfr}}$ where α_{sfr} is the star formation rate index and $\mathcal{M}_{sfr}$ is a characteristic constant. Applying this idea to the fragments, we will have:

$$\mathcal{M}_{core}(n) = \mathcal{M}_{sfr}\left(\frac{\mathcal{M}_{star}(n)}{\mathcal{M}_\odot}\right)^{1/\alpha_{sfr}}. \tag{32}$$

Now, given that: $\mathcal{M}(n) = N(n)\mathcal{M}_{core}(n)$ and taking $N(n) \simeq 3n^2$, this implies: $m_* n^{2-\alpha_\rho} = 3n^2\mathcal{M}_{core}(n)$ and further this implies:

$$n = \left(\frac{3\mathcal{M}_{core}(n)}{m_*}\right)^{-1/(\alpha_\rho)} = \left(\frac{3\mathcal{M}_L}{m_*}\right)\left(\frac{\mathcal{M}_{star}(n)}{\mathcal{M}_\odot}\right)^{-\frac{1}{\alpha_\rho\alpha_{sfr}}}. \tag{33}$$

Now, from (31):

$$\frac{d\mathcal{M}(n)}{\mathcal{M}(n)} = d\log\mathcal{M}(n) = m_*\left(\frac{dn}{n}\right), \tag{34}$$

and from (28) we have: $dN(n) = 3(2n - 1)dn \simeq 6ndn$. Putting all things together, we are lead to:

$$\frac{dN(n)}{d\log\mathcal{M}(n)} = \left(\frac{6}{m_*}\right)n^2 = \left(\frac{54\mathcal{M}_{sfr}^2}{m_*^3}\right)\left(\frac{\mathcal{M}_{star}(n)}{\mathcal{M}_\odot}\right)^{-\frac{2}{\alpha_\rho\alpha_{sfr}}}. \tag{35}$$

Now, clearly, if we are to look on any mass range of stars, $dN(n)/d\log\mathcal{M}(n) = dN/d\log\mathcal{M}$. Combing all this, it follows that:

$$\frac{dN}{d\log\mathcal{M}} = k_{imf}\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-\Gamma}, \tag{36}$$

where:

$$k_{imf} = \left(\frac{54\mathcal{M}_{sfr}^2}{m_*^3}\right), \tag{37}$$

and:

$$\Gamma = \frac{2}{\alpha_\rho\alpha_{sfr}}. \tag{38}$$

From a mathematical stand-point, we have just derived the IMF and it is seen that it is dependent on just two parameters, the mass density index α_ρ and Larson's exponent α_{sfr}. From a physical stand-point, this will only be acceptable if and only if it reproduces what we know. Of great importance is: does this derived IMF give a slope comparable to the measured slope? According to our model, the Salpeter slope of $\Gamma = 1.35$ is reproduced by for density profile whose index lays in the (acceptable) range $(1.00 \leqslant \alpha_\rho < 2.00)$, by a SFR-index which is in the range $(0.75 \leqslant \alpha_{sfr} < 1.00)$.

When collapse of the cloud occurs, the cloud will prior to this moment be in thermal equilibrium. It this state, the gravitational and thermal forces will hang in balance on a knife edge. It is expected that the density profile will most likely be $\rho \sim \rho_0(r/R_0)^{-2}$: *i.e.* $\alpha_\rho \sim 2$. If this is the case, this means $\alpha_{sfr} \sim 0.75$.

6 KING PROFILES

Generally, if the cloud fragmentation occurs before the entire cloud system reaches its Jeans limit, a loose gravitationally unbound cluster of stars will emerge and this cluster has the means to disperse. If the opposite occurs as is in the posited IMF model, *i.e.*, cloud fragmentation occurs concurrent with the collapse of the cloud system, then, a gravitationally bound cluster of stars will emerge from this and this cluster will not disperse easily like the one afore described.

Now, if it is assumed that stars do not move significantly outward in the radial direction but inward (if they ever move significantly – in the radial direction that is), then the present model predicts that the most massive stars will be found preferential in the centers of clusters. This assumption appears natural and reasonable and thus explains the observational fact why it is that massive stars or in general, the most massive member(s) of a cluster are found preferentially found in the centers of these clusters.

Emergent from the above thesis is the prediction that a cluster will exhibit a Distance-Mass power law since stars of a certain mass will be found within a radial distance not greater than the radial distance of their birth site. Since $r = n\mathcal{R}_{seed}$ and (33), we have:

$$r(\mathcal{M}_{star} \geqslant \mathcal{M}) = \mathcal{R}_{seed} \left(\frac{3\mathcal{M}_{sfr}}{m_*} \right) \left(\frac{\mathcal{M}}{\mathcal{M}_\odot} \right)^{-1/\alpha_\rho \alpha_{sfr}}, \quad (39)$$

that is, any star of mass $\mathcal{M}_{star} \geqslant \mathcal{M}$ will be found within a radius $r(\mathcal{M}_{star} \geqslant \mathcal{M})$ from the cloud center. For a gravitationally bound cluster, the above curve(s) will form an upper bound for a plot of the mass of stars verse-us their distance from the cluster center. For example, this kind of behavior is clearly exhibited by the Pleiads cluster (see *e.g.* Raboud & Mermilliod 1998) and many other open cluster systems (see *e.g.* Schilbach *et al.* 2006). The question is can this give rise to the well known King profiles (King 1962) and see for example Bonatto *et al.* 2006; Hill & Zaritsky 2006; Slawson *et al.* 2007; Sánchez & Alfaro 2009. To try and answer this question we have to find the total number of stars in a given volume sphere centred about the cloud centre. Let $N_{tot}(n)$ be the total number of stars in the nth concentric sphere. We know this we be given by:

$$N_{tot}(n) = 3\sum_1^n n^2 - 3\sum_1^n n + \sum_1^n 1. \quad (40)$$

We know that $\sum_1^n n^2 = n(n+1)(2n+1)/6$, $\sum_1^n n = n(n+1)/2$ and $\sum_1^n 1 = n$, hence from all this information, we will have:

$$N_{tot}(n) = n^3. \quad (41)$$

The above relationship suggests that there must be a well behaved sort of relationship between the star density – *i.e.*, number of stars per unit projected surface area; and the radial distance from the cloud centre. As (41) stands, it does not take into account projection effects. It should be possible *via* Monte Carlo simulations to deduce the expected number density of stars with distance. This exercise is reserved for a future reading. It is hoped that such a study may unlock the origins of the King Profiles. Stars do exhibit a well behaved spatial distribution as can be seen in the studies of Bonatto *et al.* 2006; Hill & Zaritsky 2006; Slawson *et al.* 2007; Sánchez & Alfaro 2009; for example, and to study them, the King model (King 1962) is normally used to deduce some parameters of the cluster.

7 DISCUSSION & CONCLUSIONS

The posited model of the IMF is based on simultaneous Jeans collapse and fragmentation. The collapse occurs on two scales and the condition for this to occur is that the MC must possess some finite spin angular momentum. One scale is significantly smaller than the other and collapse on the smaller scale causes the cloud to fragment sequentially from the cloud centre on the given scale in concentric and annular regions. This collapse and fragmentation takes place concurrently on a scale of the order of 0.10 Myrs, the corollary of which is that, star formation across the entire cloud system will take place simultaneously, *i.e.*, the most massive stars in he cloud centre will form together with the least massive stars in the outer limbs of the cloud.

If the most massive stars are to form together with the least massive stars, then, "we have a serious problem". It is accepted that massive stars, have much smaller Kelvin-Helmholtz timescales, thus they will burn up and turn supernova well before the LMSs come into being. If this where the case, then massive stars would never be observed co-habiting with LMSs. If at all massive stars are to form together with the LMSs, the accretion rate of the most massive stars while they are in their stellar wombs must be much less than the accretion rate of LMSs while they are in their stellar wombs. This means, the free-fall time scale of the most massive cores must at the very be equal to that of the least massive cores. In this way, LMSs can complete their accretion while massive stars are still accreting. This solution does not at all appear neat nor elegant.

The other solution would the radical approach. Since these massive stars do possess some spin angular momentum, we must perhaps revise the Kelvin-Helmholtz timescales. If our revision is to produce a plausible solution, it must be one such that the Kelvin-Helmholtz timescales of massive can be greater or comparable to that of LMSs.

In accordance with Newtonian gravitation, the Kelvin-Helmholtz timescales is given by $t_{KH}^{NG} = \left| \mathcal{E}_{gpe}^{NG} \right| / \mathcal{L}_{star}$ where $\mathcal{E}_{gpe}^{NG}$ is the total Newtonian gravitational energy and $\mathcal{L}_{star}$ is luminosity of the star. The superscript NG in $\mathcal{E}_{gpe}^{NG}$ stands for "Newtonian gravitation" and the subscript gpe for "gravitational energy". The total Newtonian gravitational potential energy $\mathcal{E}_{gpe}^{NG}$ stored by a non-rotating star of radius $\mathcal{R}_{star}$ is:

$$\mathcal{E}_{gpe}^{NG} = \int_0^{\mathcal{M}_{star}} \int_{\Phi(\mathcal{R}_{star})}^{\Phi(\infty)} d\Phi(r) d\mathcal{M} = -\frac{G\mathcal{M}_{star}^2}{2\mathcal{R}_{star}}, \quad (42)$$

where $\Phi(r) = -G\mathcal{M}_{star}/r$ and for a spinning star possessing an azimuthal gravitational field, the total gravitational potential energy $\mathcal{E}_{gpe}^{AG}$ is given by:

$$\mathcal{E}_{gpe}^{AG} = \int_0^{\mathcal{M}_{star}} \int_{\Phi(\mathcal{R}_{star,0})}^{\Phi(\infty, 2\pi)} d\Phi(r, \theta) d\mathcal{M}, \quad (43)$$

where the superscript AG in $\mathcal{E}_{gpe}^{NG}$ stands for "azimuthal gravitation" and likewise the subscript gpe for "gravitational energy". Evaluating (43), one finds:

$$\mathcal{E}_{gpe}^{AG} = -\frac{G\mathcal{M}_{star}^2}{2\mathcal{R}_{star}} \left(1 + \frac{2\lambda_1 G\mathcal{M}_{star}}{3c^2 \mathcal{R}_{star}} \right). \quad (44)$$

Now, in the reading Nyambuya (2010b), we did show that one can write $\lambda_1 = \epsilon_1 \mathcal{R}_{star}/\mathcal{R}_S$ where $\mathcal{R}_S = 2G\mathcal{M}_{star}/c^2$ is the Schwazchild radius of the star and ϵ_1 is a new replacement parameter with the same role as λ_1. With this, (44) becomes:

$$\mathcal{E}_{gpe}^{AG} = -\frac{(1+\epsilon_1/3)G\mathcal{M}_{star}^2}{\mathcal{R}_{star}}. \tag{45}$$

Now, the Kelvin-Helmholtz timescales applicable to azimuthal gravitation is: $t_{KH}^{AG} = \left|\mathcal{E}_{gpe}^{AG}\right|/\mathcal{L}_{star}$. From all this, clearly:

$$t_{KH}^{AG} = \left(1 + \frac{\epsilon_1}{3}\right)t_{KH}^{NG}. \tag{46}$$

Since $\epsilon_1 = \epsilon_1(\omega_{star}\mathcal{R}_{star})$ is an evolutionary parameter, what (46) means is that the Kelvin-Helmholtz timescale of a spinning star is a variable and dynamic parameter. If the star spins faster, ϵ_1 becomes larger and the Kelvin-Helmholtz timescale increases. This means, the faster a star spins, the more gravitational potential energy it has hence the longer it lives! This kind of conclusion is nothing short of a radical departure from our current understanding and for this reason, we ask our reader to carefully go through the readings Nyambuya (2010a,b,c) in order to appreciate this conclusion.

We did argue in Nyambuya (2010b) that during the period of outflow activity, ϵ_1: $1 \leqslant \epsilon_1 \leqslant 3.32 \times 10^6$. Outflows will switch on when $\epsilon_1 = 1$ and these same outflows are expected to switch off when $\epsilon_1 = 3.32 \times 10^6$. The point that we are putting across is the magnitude of ϵ_1. For example, in accordance with Newtonian gravitation, a $100\mathcal{M}_\odot$ should have a life span of about 3500 yrs. This same star, if it has the right amount of spin angular momentum, it can live for $1\,\mathrm{Myrs}$. For this to be so, its spin angular momentum must be such that $\epsilon_1 \sim 900$, which is well within the range $1 \leqslant \epsilon_1 \leqslant 3.32 \times 10^6$.

So, if the cores from which the massive stars form are to amass the right amount of spin at the time of fragmentation, they will be able to live long enough to see their siblings reach their full maturity. This strongly points to the need to study the distribution of spin angular momentum of the cores. Hence thus, in a future study, there is need to consider the distribution of spin angular momentum of the fragments and as well how much of this angular momentum is transfered to the nascent star. We believe thus far we have set a road map on how to tackle the contentious question: "Since massive stars are expected to form quickly and exhaust their nuclear fuel well before lower mass stars form; how, then is it possible for massive stars to form simultaneously with their lower mass counter parts such that they are found co-habiting?"

Now, we shall move on to the question of the piecewise nature of the IMF and the characteristic mass. As we have done above, we shall simply set a road map on how we think these questions can (hopefully) be tackled from the posited model. So far the posited IMF theory has been silent on the unexplained evidence that the IMF breaks at $\mathcal{M}_{ch} \sim 0.5 - 1.0\,\mathcal{M}_\odot$ causing it to flatten in the regime of very low mass stars and stepper slope for higher mass stars appears twofold soluble. The first explanation would be by considering a cloud density structure exhibiting is piecewise power law that is consistent with this kind of flattening. We feel that such a piecewise density law, while not an impossibility, it is very difficult to justify because the universality of the IMF will require that MCs exhibit similar piecewise density power laws. This would require special

configurations of the material in these MCs so that one can explain the universality and the behavior of the IMF. Postulating such a scenario is not at all convincing thus we are of the view that this is not the right path to take.

The second explanation could be a piecewise SFR-index (α_{sfr}) law. On the average, the piecewise IMF has the index:

$$\Gamma = \begin{cases} 0.33 & for & 0.001\mathcal{M}_\odot \leqslant \mathcal{M}_{star} \leqslant 0.5\mathcal{M}_\odot \\ 1.35 & for & 0.5\mathcal{M}_\odot \leqslant \mathcal{M}_{star} \leqslant 10\mathcal{M}_\odot \\ 2.33 & for & \mathcal{M}_{star} \geqslant 10\mathcal{M}_\odot \end{cases}. \tag{47}$$

For stars emerging from a cloud whose density index $\alpha_\rho \sim 2$, the SFR-index that conforms with this kind of IMF index is:

$$\alpha_{sfr} \simeq \begin{cases} 1.00 & for & 0.001\mathcal{M}_\odot \leqslant \mathcal{M}_{star} \leqslant 0.5\mathcal{M}_\odot \\ 0.75 & for & 0.5\mathcal{M}_\odot \leqslant \mathcal{M}_{star} \leqslant 10\mathcal{M}_\odot \\ 0.43 & for & \mathcal{M}_{star} \geqslant 10\mathcal{M}_\odot \end{cases}. \tag{48}$$

How can one justify or derive this kind piecewise SFR-index? Clearly, one physical quantity of the molecular cores cores that can significantly affect the SFR is the gas pressure of these cores. A cloud core with a high pressure will have a high SFR and a cloud with a low pressure will likewise, have a lower SFR. If these molecular cloud core are in a barotrophic state, then is foreseeable how such a piecewise SFR-index can arise. A cloud core in a barotrophic state has its gas pressure described by the barotropic equation of state, $\mathcal{P} = k\rho^\gamma$ where $\mathcal{P}$ is the gas pressure, ρ the gas density and the index γ is such that:

$$\gamma \simeq \begin{cases} 0.75 & for & \rho/\mathrm{gcm}^{-3} \leqslant 5.50 \times 10^{-19} \\ 1.00 & for & 5.50 \times 10^{-19} \leqslant \rho/\mathrm{gcm}^{-3} \leqslant 5.50 \times 10^{-15} \\ 1.40 & for & 5.50 \times 10^{-15} \leqslant \rho/\mathrm{gcm}^{-3} \leqslant 2.30 \times 10^{-13} \\ 1.00 & for & \rho/\mathrm{gcm}^{-3} \geqslant 2.30 \times 10^{-13} \end{cases} \tag{49}$$

(see *e.g.* Smith *et al.* 2008). This barotropic equation of state is used for basic heating and cooling to ensure that the Jeans mass at the point of fragmentation matches the characteristic mass (see *e.g.* Smith *et al.* 2008). With such a piecewise pressure law and the fact that gas pressure must have a significant effect on the SFR, clearly, this piecewise pressure law must lead to a piecewise SFR-index. So, in any future model, we believe one must look into the work by Smith *et al.* (2008) or similar works. With this kind of piecewise pressure law, (potentially) one solves the problem of the characteristic mass and as-well the piecewise nature of the IMF. We believe we have set a road map to the resolution of how the proposed IMF can explain the characteristic mass and the piecewise nature of the IMF, now we have to move to other things.

The posited model supports observations from dust continuum measurements which seem to suggest that the IMF is a direct descendant of the CoMF where stars are thought to form (as in the present IMF model) directly from the dense cores of gas observed in MCs (see *e.g.* Johnstone *et al.* 2000; Testi & Sargent 1998; Johnstone *et al.* 2006; Nutter & Ward-Thompson 2007; Enoch *et al.* 2001), thus the proposed IMF has the potency to also contribute to the explanation of these observations. The close similarity in shape between the CoMF and the IMF supports a general concept of a $1-$ to -1 correspondence between the individual dense cores and soon to be formed stars (Motte *et al.* 1998; Alfev *et al.* 2006), this scenario emerges naturally in this model.

The posited model seems to gives rise to a mass-distance function in star cluster systems. This distance-mass functions explains why massive stars will be found in the cloud centre and LMSs in at the edge. Because our observations at affected by projection effects, LMSs will be found smeared out across the cluster will the most massive stars will be found in a small smeared region around the cloud centre. This distance power should have a relationship with the King profile. We have not conducted this exercise to investigate this. We left this for a future reading.

We are of the view that proposed IMF model complements the accretion model which assumes that a star will form wholly from the material resident in its core in that it furnishes the missing part of this model by explaining how the IMF will come into being from the fragmentation process without having to resort to competitive accretion (see *e.g.* Zinnecker 1982; Clarke 1998; Bonnell et al. 2001a,b; Bate & Bonnell 2004) to generate the IMF. With growing evidence of disks around massive stars (see *e.g.* Chini *et al.* 2005; Beltrán *et al.* 2004), it appears more logical to assume that massive stars will form in isolation, in the manner as low mass stars. If isolated accretion *via* the disk (Yorke 2001; Yorke 2002; Yorke & Sonnhalter 2002) and not competitive accretion (Bonnell *et al.* 2001) is to lead to the formation of massive stars, the accretion model (Yorke 2001; Yorke 2002; Yorke & Sonnhalter 2002) then ought to explain how the IMF comes into being. For isolated accretion *via* the disk, the mass of a star can only come from the fragmentation processes and this part is explained here.

One the question of the Universality of the IMF. It is not unreasonable to imagine that this same mechanism that we have proposed may be a universal mechanism by which interstellar material on all scales will collapse and fragment. This tentative and tacit thought brings one to the idea that if galaxies in the past collapsed and fragmented in this way, then galaxies will exhibit a mass function as well. It is natural to go to further and think that when the galaxies themselves collapsed, they should have given rise to a MCMF and in-turn these MC also should collapse giving rise to a ClMF. The suggestion is clear, the present ideas about IMF are easily extendable to much larger systems, that is cores, clumps, MCs & GMCs. If it can be confirmed that a clusters of galaxies do exhibit a mass function, this would be support the universality of the mechanism by which the mass functions come into being.

On other interesting matters, depending on the mass of the cloud, the present ideas suggest that it is possible for there to exist a cluster of massive stars. Lets make a crude calculation. Assuming $\xi_{SFR} = 100\%$ and taking for example $\mathcal{M}_{max} = 20\mathcal{M}_\odot$, the first shell of this cloud will go on to form the $20\mathcal{M}_\odot$ and the next shell will form 7 stars in the mass range $7.0\mathcal{M}_\odot < \mathcal{M} < 20\mathcal{M}_\odot$ which is a cluster of massive stars. Examples of massive star clusters are the stellar cluster near the luminous star LBV $1806 - 20$ which is one of the most unusual clusters in the Galaxy given that it is host to number of group of massive stars. This associated object LBV $1806 - 20$ may be the most luminous single star in the Galaxy, with $\mathcal{L} > 6 \times 10^6 \mathcal{L}_\odot$. This cluster is home to two blue hypergiants and the soft gamma-ray repeater $SGR\ 1806 - 20$, all in a volume of about less than $2\,pc$ in diameter (Eikenberry *et al.* 2001). The other example is IRAS $05358 + 3543$ with a rich system of collimated outflows resembling those typically associated with low-mass star-forming regions. This system of massive stars supports the hypothesis that massive star

formation occurs similarly to low-mass star formation (Beuther *et al.* 2002).

In a nutshell, basic idea behind the present IMF theory is that the IMF may originate from the thermodynamic cooling and Jeans fragmentation process of the MC. The cloud cools until such a time that on the global scale the Jeans criterion is attained causing the MC to collapse on two scales. The large scale leads to global collapse of the cloud and the smaller collapse leads to the fragmentation of the cloud and the mass of the fragments is already set to generate the CoMF and from there the IMF emerges as a descendant.

In closing, allow us to say that the present work, while calling for a paradigm shift in our understanding of gravitation, it is only tentative and requires further work to cement it. It being tentative does not at all mean it is without a basis because the soils on which is is founded (Nyambuya 2010a,b,c) seem very much to have something to do with reality. Actually, the proposed azimuthally symmetric gravitational theory (Nyambuya 2010a,b,c) need to be taken seriously.

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