Euclid-Euler-Jiang Prime Theorem

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Abstract

Santilli's prime chains: $P_{j+1} = aP_j \pm b$, $j = 1, \cdots, k-1$, $(a,b)=1$, $2 \mid ab$. If $a-1 = P_1^{\lambda_1} \cdots P_n^{\lambda_n}$, $P_1 \cdots P_n \mid b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$. There exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k. It is the Book proof. This is a generalization of Euclid-Euler proof for the existence of infinitely many primes. Therefore Euclid-Euler-Jiang theorem in the distribution of primes is advanced. It is the Book theorem.

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1. Introduction

A new branch of number theory: Santilli's additive isoprime theory is introduced. By using the arithmetic function $J_n(\omega)$ the following prime theorems have been proved. It is the Book proof. [1-10]

1. There exist infinitely many twin primes.
2. The Goldbach's theorem. Every even number greater than 4 is the sum of two odd

primes.

3. There exist finitely many Mersenne primes, that is, primes of the form $2^P - 1$ where P is prime.
4. There exist finitely many Fermat primes, that is, primes of the form $2^{2^n} + 1$.
5. There exist finitely many repunit primes whose digits (in base 10) are all ones.
6. There exist infinitely many primes of the forms: $x^2 + 1$, $x^4 + 1$, $x^8 + 1$, $x^{16} + 1$, $x^{32} + 1$, $x^{64} + 1$.
7. There exist infinitely many primes of the forms: $x^2 + b$, $x^3 + 2$, $x^5 + 2$, $x^7 + 2$
8. There exist infinitely many prime *m*-chains, $P_{j+1} = mP_j \pm (m-1)$, $m = 2,3,\cdots$, including Cunningham chains.
9. There exist infinitely many triplets of consecutive integers, each being the product of k distinct primes, (Here is an example: 1727913=3×11×52361, 1727914=2×17×50821, 1727915=5×7×49369.)
10. There exist infinitely many k-tuples of consecutive integers, each being the product of m primes, where $k > 3, m > 2.$
11. Every integer m may be written in infinitely many ways in the form

$$m = \frac{P_2 + 1}{P_1^k - 1}$$

where $k = 1, 2, 3, \cdots$, P_1 and P_2 are primes.

12. There exist infinitely many Carmichael numbers, which are the product of three primes, four primes, and five primes.
13. There exist infinitely many prime chains in the arithmetic progressions.
14. In a table of prime numbers there exist infinitely many *k*-tuples of primes, where

$k = 2, 3, 4, \cdots, 10^5$.

15. Proof of Schinzel's hypothesis.
16. Every large even number is representable in the form $P_1 + P_2 \cdots P_n$. It is the n primes theorem which has no almost-primes.
17. Diophantine equation

$$P_{n+1}^{\lambda_{n+1}} = \frac{P_{n+2} + \cdots + P_{2n+1} + b}{P_1^{\lambda_1} + \cdots + P_n^{\lambda_n} + b}$$

has infinitely many prime solutions.

18. There are infinitely many primes of the forms: $x^2 + y^n$, $n \ge 2$ and $mP_1^3 + nP_2^3$, $(m,n) = 1$, $2 | mn$, $n \ne \pm b^3$.
19. There are infinitely many prime 5-tuples represented by

$$P^6 - 42^6 = (P-42)(P+42)(P^2 + 42P + 1764)(P^2 - 42P + 1764)$$

20. There are infinitely many prime k-tuples represented by $P^m \pm A^m$.

In this paper by using the arithmetic function $J_2(\omega)$ santilli's prime chains: $P_{j+1} = aP_j \pm b$ are studied. It is a generalization of santillis isoprime m-chains: $P_{j+1} = mP_j \pm (m-1)$ [6].

2. Euclid-Euler-Jiang Prime Theorem: $P_{j+1} = aP_j \pm b$

Theorem 1. An increasing sequence of primes $P_1, P_2, \cdots, P_k$ is called a Santilli's prime chain of the first kind of length k if

$$P_{j+1} = aP_j + b$$

for $j = 1, \cdots, k-1$, $(a,b) = 1$, $2 | ab$.

We have the arithmetic function[6]

$$J_2(\omega) = \prod_{3 \le P \le P_i} (P - \chi(P)),$$

where $\omega = \prod_{2 \le P \le P_i} P$ is called the primorials, P_i the last prime of the primorials.

We now calculate $\chi(P)$. The smallest positive integer such that

$$a^s \equiv 1(\bmod P),\ (a,b)=1.$$

$\chi(P)=k$ if $k<s$; $\chi(P)=s$ if $k\ge s$; $\chi(P)=1$ if $P\mid ab$.

If $J_2(\omega)=0$, there exist finitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k. If $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k. It is the Book proof. This is a generalization of the Euclid-Euler proof for the existence of infinitely many primes.

We have the best asymptotic formula of the number of primes $P_1 \le N$

$$\pi_k(N,2)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}(1+O(1)),$$

where $\phi(\omega)=\prod_{2\le P\le P_i}(P-1)$ is called the Euler function of the primorials.

The $P_{j+1}=aP_j-b$ is called a Santilli's prime chain of the second kind of length k.

Both $P_{j+1}=aP_j\pm b$ have the same arithmetic function $J_2(\omega)$. If $a=m$ and $b=m-1$, it is Santilli's isoprime m-chains[6].

Theorem 2. $P_{j+1}=2P_j\pm b$, $j=1,\cdots,k-1$, b is an odd number.

We have the arithmetic function [6]

$$J_2(\omega)=\prod_{3\le P\le P_i}\left(P-\chi(P)\right)\ne 0$$

We now calculate $\chi(P)$. The smallest positive integer s such that

$$2^s \equiv 1(\bmod P),$$

$\chi(P)=k$ if $k<s$; $\chi(P)=s$ if $k\ge s$; $\chi(P)=1$ if $P\mid b$.

Since $J_3(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k. This is the Book proof.

We have the best asymptotic formula of the number of primes $P_1 \le N$.

$$\pi_k(N,2) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}(1+O(1))$$

The $P_{j+1} = 2P_j \pm 1$ are Cunningham prime chains [6].

Example 1. $P_{j+1} = 2P_j + 7$, $j = 1, 2, 3, 4, 5$.

We have the arithmetic function

$$J_2(\omega) = 6 \prod_{11 \le P \le P_i} \left(P - 6 - \chi(P)\right) \ne 0,$$

where $\chi(31) = -1$, $\chi(p) = 0$ otherwise.

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_6$ are primes.

We have the best asymptotic formula of the number of primes $P_1 \le N$,

$$\pi_6(N,2) = \frac{1}{8}\left(\frac{35}{8}\right)^5 \prod_{11 \le P \le P_i} \frac{P^5(P-6-\chi(P))}{(P-1)^6} \frac{N}{\log^6 N}\left(1+O(1)\right).$$

Theorem 3. $P_{j+1} = 3P_j \pm b$, $j = 1, \cdots, k-1$, $(3,b) = 1$, $2|b$.

We have the arithmetic function

$$J_2(\omega) = \prod_{3 \le P \le P_i} \left(P - \chi(P)\right) \ne 0.$$

We now calculate $\chi(P)$. The smallest positive integer s such that

$$3^s \equiv 1 (\bmod P),$$

$\chi(P) = k$ if $k < s$; $\chi(P) = s$ if $k \ge s$; $\chi(3) = 1$; $\chi(P) = 1$ if $P|b$.

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k .

We have the best asymptotic formula of the number of primes $P_1 \le N$,

$$\pi_k(N,2) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}(1+O(1)).$$

Example 2. $P_{j+1} = 3P_j + 4$, $j = 1, 2, 3, 4, 5$.

We have the arithmetic function

$$J_2(\omega) = 96 \prod_{17 \le P \le P_i} (P-6) \ne 0.$$

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_6$ are primes.

We have the best asymptotic formula of the number of primes $P_1 \le N$,

$$\pi_6(N,2) = \frac{1}{60}\left(\frac{1001}{192}\right)^5 \prod_{17 \le P \le P_i} \frac{P^5(P-6)}{(P-1)^6} \frac{N}{\log^6 N}\left(1+O(1)\right).$$

Theorem 4. $P_{j+1} = 4P_j \pm b$, $j = 1, \cdots, k-1$, b is an odd number.

(1) $3|b$, we have the arithmetic function

$$J_2(\omega) = \prod_{3 \le P \le P_i} \left(P - \chi(P)\right) \ne 0.$$

We now calculate $\chi(P)$. The smallest positive integer s such that

$$4^s \equiv 1 (\bmod P).$$

$\chi(P) = k$ if $k < s$; $\chi(P) = s$ if $k \ge s$; $\chi(P) = 1$ if $P|b$.

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

We have the best asymptotic formula of the number of primes $P_1 \le N$,

$$\pi_k(N,2) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}(1+O(1)).$$

(2) $3 \mid b$, $k = 3$, we have $J_2(3) = 0$.

(3) $3 \mid b$, $k = 2$, we have $P_2 = 4P_1 \pm b$. Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that P_2 is a prime.

Theorem 5. $P_{j+1} = 5P_j \pm b$, $j = 1, \cdots, k-1$, $(5, b) = 1$, $2 | b$.

We have the arithmetic function

$$J_2(\omega) = \prod_{3 \le P \le P_i} \left(P - \chi(P)\right) \ne 0$$

We now calculate $\chi(P)$. The smallest positive integer s such that

$$5^s = 1 (\bmod P).$$

$\chi(P) = k$ if $k < s$; $\chi(P) = s$ if $k \ge s$; $\chi(5) = 1$; $\chi(P) = 1$ if $P | b$.

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

We have the best asymptotic formula of the number of primes $P_1 \le N$,

$$\pi_k(N, 2) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}(1 + O(1))$$

Theorem 6. $P_{j+1} = 6P_j \pm b$, $j = 1, \cdots, k-1$, $(3, b) = 1$, b is an odd number.

(1) $5 | b$, we have the arithmetic function

$$J_2(\omega) = \prod_{3 \le P \le P_i} \left(P - \chi(P)\right) \ne 0.$$

We calculate $\chi(P)$. The smallest positive integer s such that

$$6^s \equiv 1 (\bmod P).$$

$\chi(P) = k$ if $k < s$; $\chi(P) = s$ if $k \ge s$; $\chi(3) = 1$; $\chi(P) = 1$ if $P | b$.

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

We have the best asymptotic formula of the number of primes $P_1 \le N$,

$$\pi_k(N, 2) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}(1 + O(1)).$$

(2) $5 \mid b$, $k = 5$, we have $J_2(5) = 0$.

(3) $5 \nmid b$, $k \le 4$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 7. $P_{j+1} = 7P_j \pm b$, $j = 1, \cdots, k-1$, $(7, b) = 1$, $2 | b$.

(1) $6 | b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $6 \nmid b$, $k = 3$, we have $J_2(3) = 0$.

(3) $6 \nmid b$, $k = 2$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 8. $P_{j+1} = 8P_j \pm b$, $j = 1, \cdots, k-1$, b is an odd number.

(1) $7 | b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $7 \nmid b$, $k = 7$, we have $J_2(7) = 0$.

(3) $7 \nmid b$, $k \le 6$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 9. $P_{j+1} = 9P_j \pm b$, $j = 1, \cdots, k-1$, $(3, b) = 1$, $2 | b$.
We have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

Theorem 10. $P_{j+1} = 10P_j \pm b$, $j = 1, \cdots, k-1$, b is an odd number. $(5, b) = 1$.

(1) $3 | b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $3 \nmid b$, $k = 3$, we have $J_2(3) = 0$.

(3) $3 \nmid b$, $k = 2$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 11. $P_{j+1} = 11P_j \pm b$, $j = 1, \cdots, k-1$, $2 | b$, $(11, b) = 1$.

(1) $5 | b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $5 \nmid b$, $k = 5$, we have $J_2(5) = 0$.

(3) $5 \nmid b$, $k \le 4$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 12. $P_{j+1} = 12P_j \pm b$, $j = 1, \cdots, k-1$, $(3, b) = 1$, b is an odd number.

(1) $11|b$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k.

(2) $11\nmid b$, $k=11$, we have $J_2(11)=0$.

(3) $11\nmid b$, $k\le 10$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$.

Theorem 13. $P_{j+1}=16P_j\pm b$, $j=1,\cdots,k-1$, b is an odd number.

(1) $15|b$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k.

(2) $3\nmid b$, $k=3$, we have $J_2(3)=0$. $k=2$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$.

(3) $3|b$, $k=5$, we have $J_2(5)=0$. $k\le 4$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$.

Theorem 14. $P_{j+1}=17P_j\pm b$, $j=1,\cdots,k-1$, $2|b$, $(17,b)=1$.

We have $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k.

Theorem 15. $P_{j+1}=(2^{\lambda}+1)P_j\pm b$, $j=1,\cdots,k-1$, $2|b$, $((2^{\lambda}+1),b)=1$.

We have $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k.

Theorem 16. $P_{j+1}=(3^{\lambda}+1)P_j\pm b$, $j=1,\cdots,k-1$, $((3^{n}+1),b)=1$, b is an odd number.

(1) $3|b$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k.

(2) $3\nmid b$, $k=3$, we have $J_2(3)=0$. $k=2$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$.

Theorem 17. $P_{j+1}=(2^{\lambda_1}\cdot 3^{\lambda_2}+1)P_j\pm b$, $j=1,\cdots,k-1$, $2|b$,

$((2^{\lambda_1}\cdot 3^{\lambda_2}+1),b)=1$.

(1) $6|b$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many primes P_1

such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $6 \nmid b$, $k=3$, we have $J_2(3)=0$. $k=2$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 18. $P_{j+1} = (3^{\lambda_1} \cdot 5^{\lambda_2} + 1)P_j \pm b$, $j = 1, \cdots, k-1$, b is an odd number.

$((3^{\lambda_1} \cdot 5^{\lambda_2} + 1), b) = 1$

(1) $15|b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $3 \nmid b$, $k=3$, we have $J_2(3)=0$. $k=2$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

(3) $3|b$, $k=5$, we have $J_2(5)=0$. $k \le 4$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 19. $P_{j+1} = (3^{\lambda_1} \cdot 5^{\lambda_2} \cdot 7^{\lambda_3} + 1)P_j \pm b$, $j = 1, \cdots, k-1$, b is an odd number, $((3^{\lambda_1} \cdot 5^{\lambda_2} \cdot 7^{\lambda_3} + 1), b) = 1$

(1) $105|b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $3|b$, $k=5$, we have $J_2(5)=0$. $k \le 4$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

(3) $3 \nmid b$, $k=3$, we have $J_2(3)=0$. $k=2$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 20. $P_{j+1} = (3^{\lambda_1} \cdot 5^{\lambda_2} \cdot 7^{\lambda_3} \cdot 11^{\lambda_4} + 1)P_j \pm b$, $j = 1, \cdots, k-1$, b is an odd number, $((3^{\lambda_1} \cdot 5^{\lambda_2} \cdot 7^{\lambda_3} \cdot 11^{\lambda_4} + 1), b) = 1$

(1) $1155|b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $3|b$, $k=5$, we have $J_2(5)=0$. $k \le 4$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

(3) $3 \nmid b$, $k=3$, we have $J_2(3)=0$. $k=2$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$.

Theorem 21. $P_{j+1} = (7^{\lambda_1} \cdot 19^{\lambda_2} \cdot 31^{\lambda_3} + 1)P_j \pm b$, $j = 1, \cdots, k-1$, b is an odd number, $((7^{\lambda_1} \cdot 19^{\lambda_2} \cdot 31^{\lambda_3} + 1), b) = 1$

(1) $4123|b$, we have $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes for arbitrary length k.

(2) $7|b$, $k=19$, we have $J_2(19)=0$. $k\le 18$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$.

(3) $7\nmid b$, $k=7$, we have $J_2(7)=0$. $k\le 6$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$.

Theorem 22. $P_{j+1}=aP_j\pm b$, $j=1,\cdots,k-1$, $(a,b)=1$, $2|ab$.

If $a-1=P_1^{\lambda_1}\cdots P_n^{\lambda_n}, P_1\cdots P_n|b$, we have $J_2(\omega)\to\infty$ as $\omega\to\infty$. There exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes for arbitrary length k [6].

3. Euclid-Euler-Jiang Prime Theorem

Around 300BC by using the equation

$$(\omega+1,\omega)=1 \quad \text{as} \quad \omega\to\infty,$$

Euclid proved that there are infinitely many primes.

In 1748 by using the equation

$$\frac{\omega}{\phi(\omega)}=\prod_{i=1}^{\infty}\left(1-\frac{1}{P_i}\right)^{-1}=\sum_{n=1}^{\infty}\frac{1}{n}\to\infty \quad \text{as} \quad \omega\to\infty,$$

Euler proved that there are infinitely many primes.

By using the equation [1-10]

$$J_2(\omega)\to\infty \quad \text{as} \quad \omega\to\infty.$$

Jiang has proved that there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes[1-10]. It is a generalization of Euclid-Euler theorem. Therefore Euclid-Euler-Jiang theorem in the distribution of primes is advanced. It is the Book theorem.

From [6] we have

$$\frac{\phi(\omega)}{\omega}=\prod_{2\le P\le N}\left(1-\frac{1}{P}\right)\sim\frac{C_1}{\log N}.$$

Therefore we have the prime number theorem.

$$\pi(N)\sim\frac{N}{\log N}$$

where $\pi(N)$ denotes the number of primes $\leq N$.

From [6] we have

$$\frac{J_2(\omega)}{\omega} \sim B_1 \prod_{k<P\leq N}\left(1-\frac{k}{P}\right) \sim \frac{B_2}{\log^k N}.$$

Therefore we have the prime k-tuples theorem

$$\pi_k(N,2) \sim C_k \frac{N}{\log^k N},$$

where $\pi_k(N,2)$ denotes the number of primes $P_1 \leq N$.

If the arithmetic constant $C_k = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \neq 0$, that is $J_2(\omega) \neq 0$, there exist infinitely many primes P_1 such that $P_2, \cdots, P_k$ are primes. $\pi_k(N,2)$ have the same form $\frac{N}{\log^k N}$, but differ in C_k.

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