There are infinitely many prime triplets

$P, 3P-2, 3P+2$

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Abstract

Using Jiang's function we prove that there are infinitely many primes P such that $3P-2$ and $3P+2$ are primes.

In studying Williams numbers Echi conjectures that there are infintely many prime triplets $P, 3P-2, 3P+2$ [1]. In this paper using Jiang's function we prove this conjecture and find the best asymptotic formula for the number of primes P.

Theorem 1. The prime equations are

$$P_1 = 3P-2 \text{ and } P_2 = 3P+2 \qquad (1)$$

There are infinitely many primes P such that P_1 and P_2 are primes.

Proof. Jiang's function is

$$J_2(\omega) = \prod_P (P-1-x(P)) \qquad (2)$$

where $\omega = \prod_P P$, $x(P)$ is the number of solutions of the following congruence

$$(3q-2)(3q+2) = 0 (\bmod P) \qquad (3)$$

where $q = 1, 2, \cdots P-1$

From (3) we obtain

Table 1

q	$3q-2$	$3q+2$	
1	1	5	$x(2)=0, J_2(2)=1$
2	4	8	$x(3)=0, J_2(3)=2$
3	7	11	
4	2×5	2×7	$x(5)=2, J_2(5)=2$
5	13	17	
6	16	20	$x(7)=2, J_2(7)=4$
7	19	23	
8	2×11	2×13	
9	25	29	
10	28	32	$x(11)=2, J_2(11)=8$
…	…	…	…
$P-1$	$3P-5$	$3P-1$	$x(P)=2, J_2(P)=P-3$

In order to understand the Jiang's function we explain the table 1.

Let $\omega = 2$, Euler function $\phi(2) = 1$. There is the prime equation

$$2n+1 \qquad (4)$$

where $n = 1, 2, \cdots$.

$J_2(2) = 1$, there is the prime equation

$$P = 2n + 1 \qquad (5)$$

Substituting (5) into (1) we obtain

$$P_1 = 6n + 1 \text{ and } P_2 = 6n + 5 \qquad (6)$$

There are infinitely many integers *n* such that P, P_1 and P_2 are primes.

Let $\omega = 6$, $\phi(6) = 2$. There are the prime equations

$$6n + h \qquad (7)$$

where $n = 0, 1, 2, \cdots h = 1, 5$.

$J_2(6) = 2$, there is the prime equation

$$P = 6n + h \qquad (8)$$

Substituting (8) into (1) we obtain

$$P_1 = 18n + 3h - 2 \text{ and } P_2 = 18n + 3h + 2. \qquad (9)$$

There are infintely many integers n such that P, P_1 and P_2 are primes.

Let $\omega = 30$, $\phi(30) = 8$. There are the prime equations

$$30n + h \qquad (10)$$

where $h = 1, 3, 11, 13, 17, 19, 23, 29$, $n = 0, 1, ...$

$J_2(30) = 4$, there is the prime equation

$$P = 30n + u, \qquad (11)$$

where $u = 7, 13, 17, 23$.

Substituting (11) into (1)

$$P_1 = 90n + 3u - 2 \text{ and } P_2 = 90n + 3u + 2 \qquad (12)$$

There are infinitely many integers n such that P, P_1 and P_2 are primes.

Let $\omega = 210$, $\phi(210) = 48$. There are the prime equations

$$210n + u \qquad (13)$$

where $u =$ 1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 121, 127, 131, 137, 139, 143, 149, 151, 157, 163, 167, 169, 173, 179, 181, 187, 191, 193, 197, 199, 209, $n = 0, 1, 2...$

$J_2(210) = 16$, there is the prime equations

$$P = 210n + u \qquad (14)$$

where u=13, 23, 37, 43, 47, 83, 97, 103, 107, 113, 127, 163, 167, 173, 187, 197.

Substituting (14) into (1) we obtain

$$P_1 = 630n + 3u - 2 \quad \text{and} \quad P_2 = 630n + 3u + 2 \qquad (15)$$

There are infinitely many integers n such that P, P_1 and P_2 are primes.

Let $\omega = 2310$, $\phi(2310) = 480$. There are the prime equations

$$2310n + h, \qquad (16)$$

where $n = 0,1,\cdots, h = 13,17,...,...,2309$.

$J_2(2310) = 128$, there is the prime equation

$$P = 2310n + u, \qquad (17)$$

where u=13, 17,… 2287, 2297.

Substituting (17) into (1)

$$P_1 = 6930n + 3u - 2 \quad \text{and} \quad P_2 = 6930n + 3u + 2. \qquad (18)$$

There are infinitely many integers n such that P, P_1 and P_3 are primes.

$J_2(\omega) \to \infty$ as $\omega \to \infty$, there infinitely many prime equations such that P, P_1 and P_2 are primes.

We prove the theorem 1

From (2) we obtain

$$J_2(\omega) = 2\prod_{5 \le P}(P-3) \qquad (19)$$

We obtain the best asymptotic formula for the number of primes P [2]

$$\pi_3(N,2) = \left|\{P \le N, 3P-2 = prime, 3P+2 = prime\}\right| \sim \frac{J_2(\omega)\omega^2}{\phi^3(\omega)} \frac{N}{\log^3 N}$$

$$= 9\prod_{5 \le P}\left(1 - \frac{3P-1}{(P-1)^3}\right)\frac{N}{\log^3 N} \sim 5.77\frac{N}{\log^3 N} \qquad (20)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (20) we obtain

Table 2

N	10^2	10^3	10^4	10^5	10^6	10^7	10^8
$\pi_3(N,2)$ (20)	6	18	74	378	2188	13780	92312
$\pi_3(N,2)$ (exact)	7	21	89	445	2420	14828	98220

From table 2 we consider that prime distribution is order rather than random [2].

References

[1] Othman Echi, Williams numbers, C. R. Math. Rep. Acad. Sci. Canada Vol. 29(2) 2007, PP. 41-47

[2] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. Inter Acad. Press. America-Europe-Asia, 2002.