Anisotropic Cosmological Models with a Generalized Chaplygin Gas

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Abstract. Anisotropic cosmological models with a generalized Chaplygin gas in a Finsler space-time geometry are considered and a class of exact solutions as well as cosmological parameters behaviors are studied. Moreover, the viability and stability criteria for a general solution are also discussed.

Keywords: Cosmology, Chaplygin gas, Finsler geometry.
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INTRODUCTION

A large number of recent observational data strongly suggest that we live in a flat, accelerating universe which consists of approximatively one third of baryonic and dark matter (baryonic and dark) and two third of an exotic component with a large negative pressure called dark energy. The basic set of confirmation experiments includes: observations from SNeIa [1] , CMB anisotropies[2], large scale structure[3], X-ray data from galaxy clusters[4] etc…Ignoring the exact nature of this new kind of energy, there are many proposed forms namely the cosmological constant[5] (equivalent to the vacuum energy), scalar fields such as quintessence[6] or moduli which are dynamical quantities whose energy density can vary in time and space, Modified theories of gravity (f(R) models etc…)[7], non commutative geometry[8],[9], topological defects[10], Chaplygin gas (or its generalization) [11]etc…The main feature of most of these models is that the dark energy must have a negative pressure. Moreover, during the last few years, considerable studies concerning observable anisotropies of the universe have been investigated. These are connected to the very early state of the universe and related to the estimations of WMAP of CMB[12]-[16] , the anisotropic pressure or the incorporation of a primordial vector field (e. g. magnetic field) to the metrical spatial structure of the universe. The main motivations are based on the fact that the observed anisotropy of the microwave cosmic radiation is of a dipole type[17]- [22]. It is known that although this anisotropy can be explained using the Robertson-walker metric and taking into account the motion of our galaxy with respect to distant galaxies of the universe, still a small contribution is expected, due to the anisotropic distribution of galaxies in our space. Moreover, in the framework of Finsler geometry and as in modified theories of gravity (MOND etc…), the flat rotation curves of spiral galaxies can be deduced naturally without involving dark matter. This has led to a theoretical interest a specially in the so called a Randers-Finsler space of approximate Bewald type where a modified Friedmann model is proposed [23]-[26]. It is shown that the accelerated expanding universe is guaranteed by a constrained Randers-Finsler structure without invoking dark energy and the additional term in the geodesic equation acts as a repulsive force against the gravity. The goal of this paper is to study the effect of the geometry by considering some generalized FRW anisotropic cosmological models with a generalized Chaplygin gas in Finsler geometry and determine some cosmological parameters. Moreover, the viability and the stability criteria of the general solutions are also discussed.

MATHEMATICAL FORMALISM

A Finsler geometry may be considered as a generalization of the Riemanian geometry. It is a physical geometry on which the matter dynamic takes place while Riemannian geometry is the gravitational geometry connecting the metric structure of the space-time to a physical vector field. The latter is of a cosmological origin (which dependents on the position and direction (velocity)), emerging out by a physical source of the universe and becoming incorporated into the geometry causing an anisotropic structure. The latter is expressed in terms of the Cartan torsion tensor of the Finslerian manifold affecting the solutions of Friedmann equations, CMB temperature estimation etc. . . Mathematically, a Finsler space is a metric space where the metric function is defined by a norm $F(x,Y)$ (on a TM tangent bundle of a manifold M) which is a real function of a space-time point and a tangent vector Y belonging to the tangent space $T_x M$ at $x \in M$, playing the role of an internal variable and characterizing the Finslerian field (combined with the concept of anisotropy causes a deviation from the Riemannian geometry). Moreover, a Finsler structure of M is a function: $F : TM \to [0,\infty[$ which has the following properties:

1) Regularity: F is an infinite differentiable function of C^∞
2) Positive homogeneity: $\forall \lambda \rangle 0 \Rightarrow F(x,\lambda Y) = \lambda F(x,Y)$
3) Strong convexity: the Hersian $H_{\mu\nu} = \partial_\mu \partial_\nu (\frac{1}{2} F^2)$ ($\partial_\mu \equiv \partial / \partial Y^\mu$) is positive definite

To describe the dynamics and produce the geodesics of the 4-dimensional space-time, one has to introduce a Lagrangian $L = F(x,Y)$ of Randers-type where the function $F(x,Y)$ can be written as:

$$F(x,Y) = \sigma + U_\alpha Y^\alpha \tag{1}$$

with

$$\sigma = (g_{\mu\nu} Y_\mu Y^\mu)^{1/2}, \; U_\alpha(x) = \hat{K}_\alpha \Phi(x), \; Y^\mu = \frac{dx^\mu}{ds} \tag{2}$$

and

$$g_{\mu\nu} = diag(1, \frac{-a^2}{(1-Kr^2)}, -a^2 r^2, -a^2 r^2 \sin^2\theta) \tag{3}$$

Here $K = 0, \pm 1$ and s , $a = a(t)$, $\Phi(x)$, $\hat{K}_\alpha$ and $g_{\mu\nu}$ are the proper time, the scale factor, a scalar function, an anisotropy vector and FRW metric respectively. It is to be noted that all the information about the anisotropy is encoded into the U_0 component. It is worth to mention that under a weak field assumption, we can approximate a Finslerian metric as a perturbation of the FRW metric. This metric is referred as an osculating Riemannian metric $\hat{g}_{\mu\nu}(x,Y)$ given by the following expression:

$$\hat{g}_{\mu\nu} = \hat{g}_{\mu\nu}(x,Y) = \frac{1}{2}\frac{\partial^2 F^2}{\partial Y^\mu \partial Y^\nu} = \frac{F}{\sigma} g_{\mu\nu} + \frac{1}{4\sigma}(U_\mu Y_\nu + U_\nu Y_\mu) - \frac{\beta(x,Y)}{\sigma^3} Y_\mu Y_\nu + U_\mu U_\nu \tag{4}$$

where $\beta(x,Y) = U_\alpha(x) Y^\alpha$. It is to be noted also that a Finsler geometry is characterized by a Cartan torsion tensor $C_{\mu\nu\lambda}(x, Y(x))$ such that:

$$C_{\mu\nu\lambda}(x,Y(x)) = \frac{1}{2}\left[\frac{1}{\sigma} S_{(\mu\nu\lambda)}(g_{\mu\nu} U_\lambda) - \frac{1}{\sigma^3} S_{(\mu\nu\lambda)}(Y_\mu Y_\nu U_\lambda) - \frac{\beta}{\sigma^3} S_{(\mu\nu\lambda)}(g_{\mu\nu} Y_\lambda)\right] \tag{5}$$

Here $S_{(\mu\nu\lambda)}$ stands for the symmetrization with respect to the indices μ , ν , λ . In the commoving coordinates where U^ν and Y^ν take the form $U^\nu = (\mu t, 0,0,0)$ and $Y^\nu = (1,0,0,0)$, one can show that the parameter μ is related to the Cartan torsion tensor $C_{\mu\nu\lambda}$ through the relation $\mu = 2\partial_0 C_{000}$ and its lower limit is $-H$ (H is the Hubble constant)[26]. This result is in agreement with the fixing of a similar parameter for a self accelerated brane-world cosmology [27]- [29]. Since the parameter μ is related to the variation of anisotropy, we expect it to have a negative sign (self accelerating universe) as it may control a transition of the universe from a state of anisotropy to a smoother isotropic phase.

QUALITATIVE STUDY WITH A CHAPLYGIN GAS IN FINSLER GEOMETRY

In Finsler geometry and considering a weak linearized anisotropy, the vacuum generalized FRW field equations are given by [26]:

$$\frac{\ddot{a}}{a}+2\left(\frac{\dot{a}}{a}\right)^2+\frac{2K}{a^2}+\frac{11}{4}\frac{\mu\dot{a}}{a}-\frac{(\rho-P)}{2}=0 \tag{6}$$

and

$$\frac{\ddot{a}}{a}+\frac{3}{4}\frac{\mu\dot{a}}{a}+\frac{(\rho+3P)}{6}=0 \tag{7}$$

Where ρ and P are the energy density and the pressure respectively. In what follows we set $8\pi G=1$ (G is the Newton constant) and consider a pure generalized Chaplygin gas where its equation of state has the form:

$$P_{chap}=\frac{-A}{\rho_{chap}^{\alpha}} \qquad (A\geq 0, O\leq\alpha\leq 1) \tag{8}$$

The energy-momentum conservation equation reads:

$$\dot{\rho}_{chap}=-(3H+\mu)\rho_{chap}+3A(H+\frac{1}{2}\mu)\rho_{chap}^{-\alpha} \tag{9}$$

One can show easily that the solution of eq. (9) is:

$$\rho_{chap}=\rho_{chap}(\tilde{a},t)=\left[A+\frac{1}{2}\mu(1+\alpha)A\tilde{a}^{-3(\alpha+1)}\int^{\tilde{a}}d\tilde{a}'\frac{\tilde{a}'^{3\alpha+2}}{\tilde{H}(\tilde{a}')}+C\tilde{a}^{-3(\alpha+1)}\right]^{1/(1+\alpha)} \tag{10}$$

where

$$\tilde{a}=ae^{\mu t/3} \tag{11}$$

and C is an integration function. Therefore, in a flat space, the Hubble constant expression as a function of the red shift like parameter $\tilde{z}$ reads:

$$\hat{H}(\tilde{z})\approx\left\{\Omega_{\mu}+\Omega_{chap}\left[\hat{A}+\xi^{3(1+\alpha)}\left(\left(1-\hat{A}\right)+\frac{1}{2}\hat{\mu}(1+\alpha)\Omega_{chap}^{-1/2}\hat{A}.I\right)\right]^{1/(1+\alpha)}\right\}^{1/2} \tag{12}$$

where

$$I=\sigma(\tilde{z})(\phi_1+\phi_2+\phi_3), \qquad \sigma(\tilde{z})=\frac{1}{6}\frac{(1+\tilde{z})^{2\delta}}{\delta(\chi+2)} \tag{13}$$

$$\phi_1={}_2F_1(\chi+3,-\delta;\chi+2,-\nu), \phi_2=\frac{-2\nu\, {}_2F_1(,-\delta+1;\chi+3,-\nu)}{(\chi+2)}, \phi_3=\frac{-\delta(\delta-1)\nu^2\, {}_2F_1(\chi+4,-\delta+2;\chi+4,-\nu)}{[(\chi+2)(\chi+3)]} \tag{14}$$

$$\delta=\frac{1}{2}\frac{1}{(\alpha+1)}, \quad \chi=-\frac{(2+6\alpha+3\alpha^2)}{(1+\alpha)}, \quad \nu=\frac{\hat{A}(1+\tilde{z})^{6\delta}}{(1-\hat{A})} \tag{15}$$

and

$$\hat{A} = \frac{A}{\rho_0^{1+\alpha}}, \quad \hat{\mu} = \frac{\mu}{\tilde{H}_0}, \quad \hat{H} = \frac{\tilde{H}}{\tilde{H}_0}, \quad \tilde{H} = \frac{\dot{\tilde{a}}}{\tilde{a}} = H + \frac{\mu}{3} \tag{16}$$

Here ${}_2F_1(b, d; f, w)$ is the hypergeometric function, $\Omega_\mu = -\mu / 2\tilde{H}_0$, $\Omega_{chap} = \rho_0 / 3\tilde{H}_0$ and $\tilde{a}_0 / \tilde{a} = 1 + \tilde{z}$ (ρ_0 and $\tilde{H}_0$ are the present time energy density and Hubble constant respectively).

TABLE. Validity regions of the W. C. and S. C. conditions in Finsler geometry with a generalized Chaplygin gas.

$\rho \in$	**W.C.**	**W.C.**	**S.C.**	**S.C.**
$\left[0, A^{1/(1+\alpha)}\right[$	yes if $H \prec 0$	no if $H \succ 0$	yes if $H \succ \Delta \succ 0$	no if $H \prec \Delta \prec 0$
$\left[A^{1/(1+\alpha)}, (\frac{3}{2}A)^{1/(1+\alpha)}\right[$	yes if $H \prec 0$	no if $H \succ 0$	no if $H \succ \Delta \succ 0$	yes if $H \prec \Delta \prec 0$
$\left[(\frac{3}{2}A)^{1/(1+\alpha)}, (3A)^{1/(1+\alpha)}\right[$	yes if $H \prec 0$	no if $H \succ 0$	no if $\Delta \prec H \prec 0$ or $H \succ 0$	yes if $0 \prec H \prec \Delta$ or $H \prec 0$
$\left[(3A)^{1/(1+\alpha)}, \infty\right[$	yes if $H \succ 0$	no if $H \prec 0$	no if $\Delta \prec H \prec 0$ or $H \prec 0$	yes if $0 \prec H \prec \Delta$ or $H \prec 0$

Moreover, we have displayed in the table (where $\Delta = -\mu(3P + 2\rho)/6(P + \rho)$) the various validity regions of the strong and weak conditions in Finsler geometry with a generalized Chaplygin gas denoted by W. C and S. C respectively. Furthermore, the regions where we have open or closed as well as accelerated or decelerated universe are presented in figs. 1(a),(b).

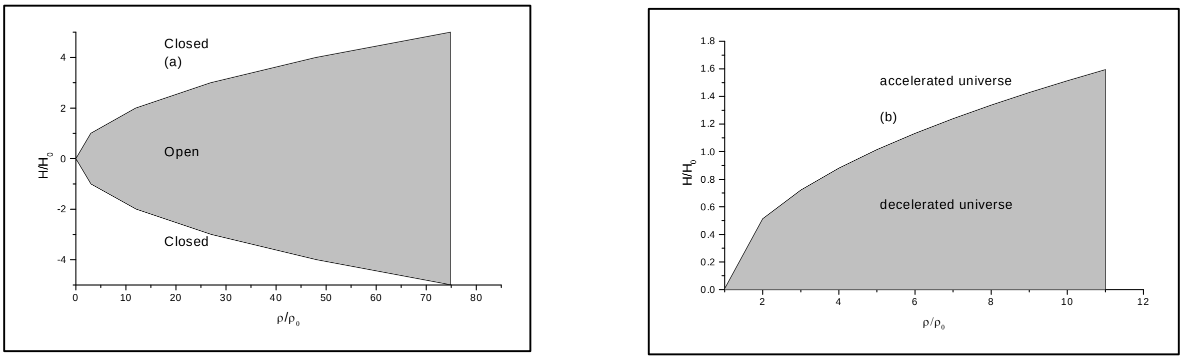

FIGURE 1. (a) regions of open and closed universes, (b) regions of accelerated and decelerated universes

In the qualitative theory of dynamical systems, the evolution of a system is represented by trajectories in the (ρ, H) space and it is uniquely determined by the initial conditions. Instead of finding and analyzing an individual solution of a model, a space of all possible solutions is investigated. Thus, the phase plane analysis is an important tool in studying the qualitative behavior of nonlinear two dimensional systems where there is often no analytical solution. Furthermore, knowing the stabilities and instabilities of the system provides additional informations about the solutions and improve our understanding of the model. Now, if we take (ρ, H) as phase variables and apply the dynamical system approach to study the phase portrait and critical points, one has to consider for the congruence of the world lines of matter in Finsler geometry, the Rychaudhuri eq. (7) written as:

$$\dot{H} = -H^2 - \frac{3}{4}H\mu - \frac{1}{6}(\rho + 3P) \tag{17}$$

and the energy-momentum tensor conservation condition:

$$\dot{\rho} = -3H(\rho + P) - \frac{1}{2}\mu(2\rho + 3P) \tag{18}$$

Then, one can show easily that the equilibrium points (ρ_c, H_c) verify the following nonlinear equation:

$$H_c^{1+\alpha}(4H_c + 3\mu)^{\alpha}(2H_c + \mu)^{\alpha}(H_c + \mu)(8H_c + 3\mu) = \frac{2^{\alpha-1}}{3^{\alpha}} A(4H_c + \mu)^{1+\alpha} \tag{19}$$

We have used the Pplane Java applet to plot the trajectories of the two dimensional system of autonomous differential equations in eqs. (17)-(18). If we set $x = \rho / \rho_0$ and $y = H / H_0$ and take as a normalization $\rho_0 / H_0^2 = A / \rho_0^{\alpha+1} = 1$, we have found for $\alpha \approx 0.4$, $\mu / H_0 \approx -0.01$ and the range of our interest the following equilibrium (critical) points:
i) A nodal sink point $A(0.99, 0.57)$ where the linear stability matrix M_A is given by:

$$M_A = \begin{pmatrix} -2.43 & 0.008 \\ -0.37 & -1.15 \end{pmatrix} \tag{20}$$

with real eigenvalues $\lambda_{A_1} \approx -1.16$ and $\lambda_{A_2} \approx -2.43$ of the same negative sign. The corresponding eingenvectors are $\vec{V}_{\lambda_{A_1}}(0.007, 0.999)$ and $\vec{V}_{\lambda_{A_2}}(0.96, 0.28)$ respectively.
ii) A nodal source $B(1.002, -0.57)$ with a linear stability matrix M_B is given by:

$$M_B = \begin{pmatrix} 2.42 & -0.008 \\ -0.37 & 1.15 \end{pmatrix} \tag{21}$$

The eingenvalues $\lambda_{B_1} \approx 2.42$ and $\lambda_{B_2} \approx 1.15$ are real and have the same positive sign. The eigenvectors are $\vec{V}_{\lambda_{B_1}}(0.96, -0.28)$ and $\vec{V}_{\lambda_{B2}}(0.007, 0.999)$.
iii) A saddle point $C(2.19, 0.002)$ where the linear stability matrix M_C is given by:

$$M_C = \begin{pmatrix} 0.0035 & -4.38 \\ -0.233 & 0.0025 \end{pmatrix} \tag{22}$$

The corresponding eingenvalues and eigenvectors are $\lambda_{C_1} \approx 1.014$, $\lambda_{C_2} \approx -1.008$ and $\vec{V}_{\lambda_{C_1}}(0.97, -0.22)$, $\vec{V}_{\lambda_{C2}}(0.97, 0.22)$ respectively.

It is worth to mention that in the phase space region under consideration we do not have a static critical point. By analyzing the phase plane trajectories we have identified the following regions:
1- Trajectories starting from the line $\rho = 0$ with a negative infinite pressure P correspond to an open decelerated expanding universe. After that $|P|$ decreases and ρ increases to reach ρ_c (global attractor stable nodal sink) where near this critical point, the deceleration parameter q and the geometry type index K decrease and change sign to end up with a closed accelerated expanding universe.
2- Trajectories beginning first from a singularity at $\rho \to \infty$ and a vanishing pressure P than ρ decreases to the equilibrium value ρ_c (global attractor stable nodal sink). The deceleration parameter q increases without changing sign. These models correspond to an open accelerated expanding universe.
3- Trajectories starting from the critical point ρ_c (global repulsor unstable nodal source) and evolve to $\rho = 0$ with a negative infinite pressure P where the deceleration parameter q and geometry type index K do not change sign to

obtain at the end an accelerated collapsing universe.

4- Trajectories which begin from ρ_c (global repulsor unstable nodal source) and evolve to a singularity at $\rho \to \infty$ where the deceleration parameter q decreases without changing sign to end up with an accelerated collapsing universe.

5- Trajectories starting from ρ_c (global repulsor unstable nodal source) and reach ρ_c (global attractor stable nodal sink) after a bounce (a decrease than an increase in the value of ρ) where K changes sign. They correspond to an accelerated collapsing universe.

6- Trajectories which start from ρ_c (global repulsor unstable nodal source) corresponding to an accelerated collapsing universe reach ρ_c (global attractor stable nodal sink) after a bounce (an increase than a decrease in the value of ρ) where K and the Hubble constant H change sign. They correspond to an accelerated universe.

7- Trajectories which begin from ρ_c (global repulsor unstable nodal source) and evolve to ρ_c (unstable saddle point) where q increases and changes sign correspond to a collapsing universe. After that $\rho \to \infty$ (Chaplygin pressure vanishes) and again the deceleration parameter q changes sign getting at the end a closed accelerated collapsing universe.

8- Trajectories starting from a singularity at $\rho \to \infty$ correspond to an accelerated expanding universe than ρ decreases to a minimum value $\rho_{\min}$ in an accelerated expansion. After that ρ increases again to reach a singularity at $\rho \to \infty$ corresponding to an accelerated collapsing universe.

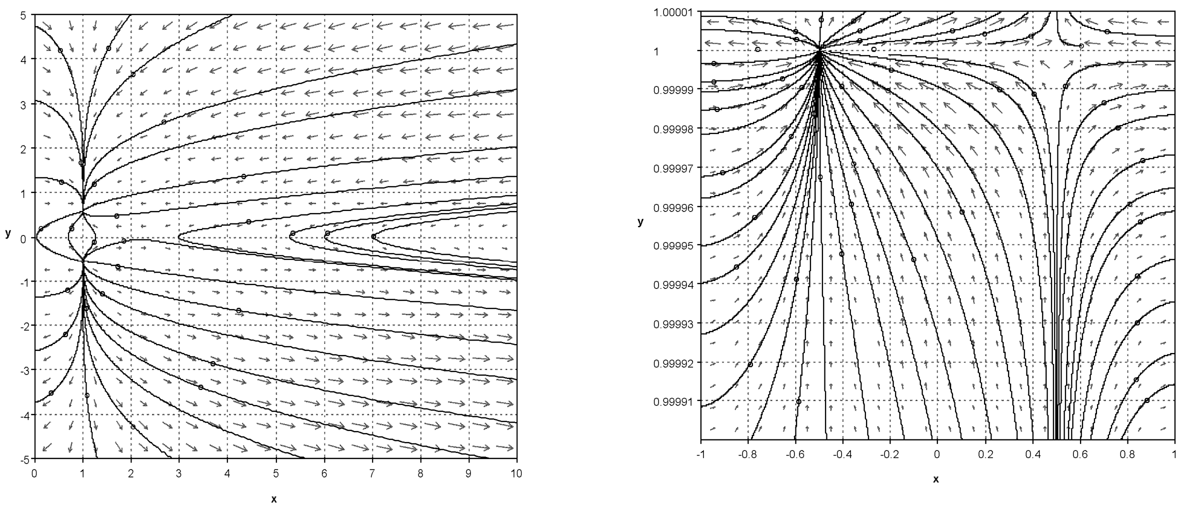

FIGURE 2. Phase portrait with: (left) (ρ, H), (right) $(\tilde{r}, v)$ near infinity as dynamical variables.

The model under consideration is structurally stable in its physical domain because there are no separatrices connecting saddle points, all critical points are hyperbolic, non degenerate and their number is finite . In our case we have obtained the following separatrices of equations $y \approx 0$, $x \approx 1$ and $y \approx 114x^2 - 343x + 297$ in the finite region. We have made similar study near infinity (asymptotic limit) and check the existence of fixed points. Physically such points represent regimes in which one or more of the terms in generalized Friedman equations become dominant. The asymptotic analysis can be performed by compactifying the phase space portrait to $(\tilde{r}, v)$ using Poincaré method. If we set $v = \cos\theta$ and $\tilde{r} = r/(1+r)$ where θ and r are the polar coordinates (v and $\tilde{r}$ are denoted in fig. 2 (right) by x and y respectively), the generalized FWR equations lead to the following non linear differential equations:

$$\dot{\tilde{r}} = v\left\{-3v\Xi\Sigma_1 - \frac{1}{2}v(1-\tilde{r})\Sigma_3\right\} - \Xi\left\{\Xi^2 + \frac{3}{4}(1-\tilde{r})\Xi + \frac{1}{6}v(1-\tilde{r})\Sigma_2\right\} \tag{23}$$

and

$$\dot{v} = \Xi^2 \left\{ \frac{\Xi^2}{(1-\tilde{r})} + \frac{3}{4}\Xi + \frac{1}{6} v\Sigma_2 \right\} - v\Xi \left\{ \frac{3v\Xi\Sigma_1}{(1-\tilde{r})} + v\Sigma_3 \right\} \tag{24}$$

where

$$\Sigma_1 = 1 - \left(\frac{(1-\tilde{r})}{v} \right)^{\alpha+1}, \quad \Sigma_2 = -2 + 3\Sigma_1, \quad \Sigma_3 = 1 + \Sigma_2, \quad \Xi = \sqrt{1-v^2} \tag{25}$$

We have obtained the following separatrices of equations $x \approx 0.5$ and $y \approx 1$. For the critical points we have got unstable saddle point at $x \approx 0.5$ and asymptotically stable node at $x \approx -0.5$ and $y \approx 1$. Figure 2 (left) displays all trajectories or orbits in phase space portraits (where (ρ, H) are taken as dynamical variables) converging or diverging to nodal sink or nodal source points. Similarly, for fig.(2) (right) but with $(\tilde{r}, v)$ as dynamical variables near infinity (asymptotic limit).

SOME SIMPLE MODELS WITH EXACT SOLUTIONS

In order to understand the anisotropic cosmological models in Finsler geometry, we consider two simple cases showing the pure effect of the geometry and leading to dark energy scenarios.

Model 1

Let us consider a model where the relation $\rho_{tot} + 3P_{tot} = 0$ holds. Here $\rho_{tot} = \rho_m + \rho_{chap}$ and $P_{tot} = P_m + P_{chap}$ (ρ_m and P_m are the matter density and pressure respectively). If we use the generalized Friedman equations of eqs. (6) and (7), one can show that the Hubble constant, scale factor and deceleration parameter have the following exact expressions:

$$H = -\frac{3}{4}\mu H_0 e^{\theta-\theta_0} / [H_0(e^{\theta-\theta_0} - 1) - \frac{3}{4}\mu] \tag{26}$$

and

$$q = -e^{-(\theta-\theta_0)}[(H_0(e^{\theta-\theta_0} - 1) - \frac{3}{4}\mu]/H_0 \tag{27}$$

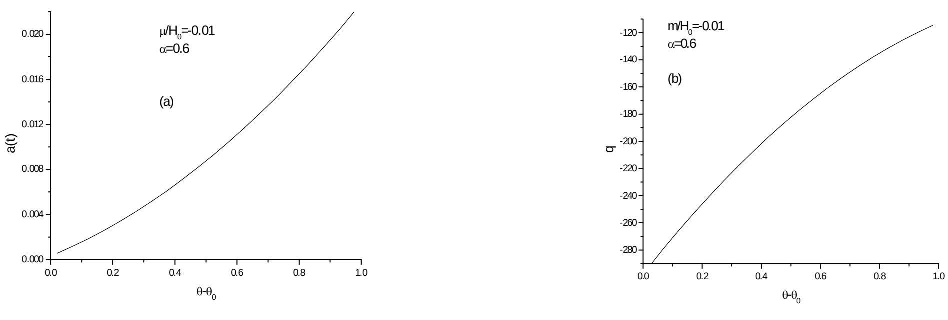

FIGURE 3. Time evolution of: (a) scale parameter (b) deceleration parameter in model 1.

where $\theta = -3\mu t/4$. Notice that q (resp. H) is negative (resp. positive). This will account for an expanding accelerated universe (dark energy). If take the matter pressure $P_m = 0$ (matter dominated universe) the Chaplygin gas pressure P_{chap} gets the form:

$$P_{chap} = \frac{1}{3} g(\theta) \tag{28}$$

where

$$g(\theta) = \mu^2 H_0 e^{\theta-\theta_0} \frac{\left[-\frac{1}{4} H_0 e^{\theta-\theta_0} + H_0 + \frac{3}{4}\mu\right]}{\left[H_0(e^{\theta-\theta_0} - 1) + \frac{3}{4}\mu\right]^2} \tag{29}$$

This is a pure Finsler geometry result. Infact, in the Riemanian geometry and with the same kind of fluid one gets a static universe instead of an accelerating one. Moreover, If we consider a Finsler space-time without a Chaplygin gas, one can get an induced time dependent cosmological constant given by the expression $\Lambda(t) = g(\theta)/3$. Figure 3 (resp. 4) displays for $\mu / H_0 = -0.01$ and Chaplygin gas parameter $\alpha = 0.6$., the scale factor $a(t)$ (fig.3(a)) and the deceleration parameter $q(t)$ (fig.3(b)) (resp. induced cosmological constant) as functions of the time in arbitrary units. Notice the Finsler space cosmological parameters $a(t)$ and $q(t)$ are increasing functions of time, however the induced cosmological constant $\Lambda(t)$ is a decreasing function and vanishes for large values of t .

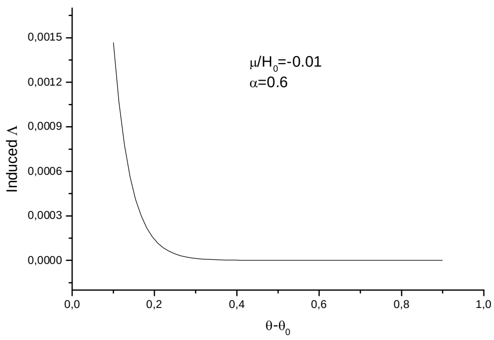

FIGURE. 4. Induced cosmological constant as a function of time in model 1.

Model 2

As a second model, we consider a Finsler space-time with a Chaplygin gas and a non vanishing cosmological constant. Then, one can show that the energy-momentum stress tensor conservation equation compatible with the Bianchi identities, has the form:

$$\dot{\rho} = \frac{2}{3} \rho [\Omega_1 + \frac{\Omega_2}{(\alpha A + \rho^{\alpha+1})}] \tag{30}$$

where

$$\Omega_1 = -3H - \mu\,, \quad \Omega_2 = A(3H(1-\alpha) + \mu\alpha) \tag{31}$$

Now, if we assume that the fluid under consideration verifies the property $\tilde{\rho} + 3\tilde{P} = 0$ where the redefined energy density $\tilde{\rho}$ and pressure $\tilde{P}$ such that $\tilde{\rho} = \rho + \Lambda$ and $\tilde{P} = P - \Lambda$ respectively, then one can show easily that the deceleration parameter q has the following expression:

$$q = -\frac{9\mu^2 e^{\theta}}{(32 \sinh \theta)} \tag{32}$$

where

$$\theta = -\frac{3}{8}\mu t + \frac{1}{2}B \tag{33}$$

and

$$B = \frac{3}{4}\mu t_0 + \ln\left(\frac{H_0}{(H_0 + \frac{3}{4}\mu)}\right) \tag{34}$$

It is clear from eq. (32) that q is negative (acceleration) and proportional to the anisotropy parameter μ. This is again a pure Finsler geometry result where in the Riemannian space-time we obtain a static universe. Figure 5 presents the variation of the cosmological constant Λ as a function of the time. Notice that it is an increasing function accounting for a dark energy.

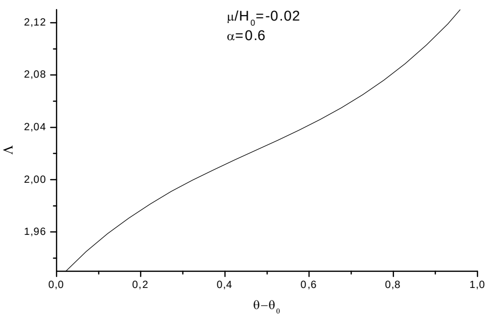

FIGURE. 5. Time evolution of the cosmological constant in model 2

CONCLUSION

From our a qualitative study, we conclude that as a first step towards an understanding of the geometry effect on the cosmology dynamics and testing the possibility to get more realistic and viable models, we have discused general behaviors (stabilities, unstabilities, critical points etc…) of a Chaplygin gas in a Finsler geometry. An exact expression of the Hubble constant is obtained explicitly as a function of the redshift-like parameter $\tilde{z}$ (comparaison with the present data and observational tests are under investigations). Moreover, two simple pure Finsler space-time models with exact solutions accounting for expanding accelerated universe are considered and some of the cosmological parameters expressions are obtained. The most interesting in this theoretical models is that the geometry affects the dynamics of the cosmology. In fact a static or decelerating universe in Riemannian geometry may correspond to an accelerating expanded universe in Finsler space-time. Moreover, an induced time dependent cosmological constant can also be a result of the non Riemannian geometry.

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