Gaps Among Products of m Primes

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function $J_2(\omega)$ we prove gaps among products of m prime:

$d(x)=d(x+1)=d(x+5-3)=d(x+7-3)=\cdots=d(x+P_n-3)=m>1$ infinitely-often,

where P_n denotes the $n-$ th prime.

Theorem 1. Let $P_3=5$, gaps among products of two primes

$$d(x)=d(x+1)=d(x+2)=2 \text{ infinitely-often.} \tag{1}$$

where $d(x)$ represents the number of distinct prime factors of x, $d(x)=\sum_{P|x}1$, $d(3)=1$, $d(15)=2$, $d(105)=3$.

Proof (see[1] p.146 theorem 3.1.154). Prime equations are

$$\beta_1=10\alpha+1,\quad \beta_2=15\alpha+2,\quad \beta_3=6\alpha+1 \tag{2}$$

We have Jiang function

$$J_2(\omega)=3\prod_{7<P}(P-4)\neq 0, \tag{3}$$

where $\omega=\prod_{2\le P}P$

We prove that $J_2(\omega)\neq 0$ there exist infinitely many odd integers α such that β_1, β_2 and β_3 are primes.
We have asymptotic formula

$$\left|\{\alpha\le N:10\alpha+1,15\alpha+2,6\alpha+1\}\right|>\frac{J_2(\omega)\omega}{\varphi^4(\omega)}\frac{N}{\log^4 N}, \tag{4}$$

where $\phi(\omega)=\prod_{2\le P}(P-1)$.

From (2) we have

$$\begin{aligned}&3\beta_1=30\alpha+3,\\&3\beta_1+1=30\alpha+4=2(15\alpha+2)=2\beta_2,\\&3\beta_1+2=30\alpha+5=5(6\alpha+1)=5\beta_3\end{aligned} \tag{5}$$

From (5) we prove

$$d(3\beta_1)=d(3\beta_1+1)=d(3\beta_1+2)=2 \text{ infinitely-often.} \tag{6}$$

We prove that there exist infinitely many triples of consecutive integers, each being the products of two distinct primes.

Theorem 2. Let $P_3=5$, gaps among products of m primes.

$$d(x)=d(x+1)=d(x+2)=m>1 \text{ infinitely-often} \tag{7}$$

Proof (see [1] p.148, theorem 3.1.158). Suppose that $u, u+1$ and $u+2$ are three consecutive integers, each being the products of $m-1$ distinct primes. Let $M=u(u+1)(u+2)$. We define the three prime equations

$$\beta_1=\frac{2M}{u}\alpha+1, \quad \beta_2=\frac{2M}{u+1}\alpha+1, \quad \beta_3=\frac{2M}{u+2}\alpha+1 \tag{8}$$

Using Jiang function $J_2(\omega)$ we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

From (8) we have

$$u\beta_1=2M\alpha+u$$

$$u\beta_1+1=2M\alpha+u+1=(u+1)\left(\frac{2M}{u+1}\alpha+1\right)=(u+1)\beta_2$$

$$u\beta_1+2=2M\alpha+u+2=(u+2)\left(\frac{2M}{u+2}\alpha+1\right)=(u+2)\beta_3 \tag{9}$$

We prove

$$d(x)=d(x+1)=d(x+2)=m>1 \text{ infinitely-often.} \tag{10}$$

Theorem 3. Let $P_4=7$, gaps among products of two primes.

$$d(x)=d(x+2)=d(x+4)=2 \text{ infinitely-often.} \tag{11}$$

Proof [1,2,3]. Prime equations are

$$\beta_1=70\alpha+1, \quad \beta_2=42\alpha+1, \quad \beta_3=30\alpha+1 \tag{12}$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

Frome (12) we have

$$3\beta_1=210\alpha+3,$$

$$3\beta_1+2=210\alpha+5=5(42\alpha+1)=5\beta_2$$

$$3\beta_1+4=210\alpha+7=7(30\alpha+1)=7\beta_3 \tag{13}$$

We prove

$$d(3\beta_1)=d(3\beta_2+2)=d(3\beta_1+4)=2 \text{ infinitely-often.} \quad (14)$$

Theorem 4. Let $P_4=7$, gaps among products of m primes.

$$d(x)=d(x+2)=d(x+4)=m>1 \text{ infinitely-often.} \quad (15)$$

Proof [1, 2, 3]. Suppose that $u, u+2$ and $u+4$ are three odd integers, each being the products of $m-1$ distinct primes. Let $M=u(u+2)(u+4)$

We define three prime equations

$$\beta_1=\frac{2M}{u}\alpha+1, \quad \beta_2=\frac{2M}{u+2}\alpha+1, \quad \beta_3=\frac{2M}{u+4}\alpha+1 \quad (16)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

From (16) we have

$$u\beta_1=2M\alpha+u,$$

$$u\beta_1+2=2M\alpha+u+2=(u+2)\left(\frac{2M}{u+2}\alpha+1\right)=(u+2)\beta_2,$$

$$u\beta_1+4=2M\alpha+u+4=(u+4)\left(\frac{2M}{u+4}\alpha+1)\right)=(u+4)\beta_3. \quad (17)$$

We prove

$$d(x)=d(x+2)=d(x+4)=m>1 \text{ infinitely-often.} \quad (18)$$

Theorem 5. Let $P_4=7$**,** gaps among products of m primes.

$$d(x)=d(x+1)=d(x+2)=d(x+4)=m>1 \text{ infinitely-often.} \quad (19)$$

Proof. From (12) we have prime equations

$$\beta_1=70\alpha+1, \ \beta_2=105\alpha+2, \ \beta_3=42\alpha+1, \ \beta_4=30\alpha+1 \quad (20)$$

Using Jiang function $J_2(\omega)$ [1] we can prove there exist infinitely many odd integers α such that β_1, β_2, β_3 and β_4 are primes

From (20) we have

$$3\beta_1=210\alpha+3$$

$$3\beta_1+1=210\alpha+4=2(105\alpha+2)=2\beta_2$$

$$3\beta_1+2=210\alpha+5=5(42\alpha+1)=5\beta_3$$

$$3\beta_1+4=210\alpha+7=7(30\alpha+1)=7\beta_4. \quad (21)$$

We prove

$$d(3\beta_1)=d(3\beta_1+1)=d(3\beta_1+2)=d(3\beta_1+4)=2 \text{ infinitely-often.} \quad (22)$$

Using Jiang function we can prove that

$$d(x)=d(x+1)=d(x+2)=d(x+4)=m>1 \text{ infinitely-often.} \quad (23)$$

Theorem 6. Gaps among products of m primes.

$$d(x)=d(x+1)=d(x+5-3)=d(x+7-3)=\cdots=d(x+P_n-3)=m>1 \text{ infintely-often.}$$

（24）

where P_n denotes the $n-$th prime.

Proof. Let $\omega_n = \prod_{2\le P\le P_n} P$. We define the prime equations

$$\beta_1=\frac{\omega_n}{3}\alpha+1,\ \beta_2=\frac{\omega_n}{2}\alpha+2,\ \beta_3=\frac{\omega_n}{5}\alpha+1,\ \beta_4=\frac{\omega_n}{7}\alpha+1,\cdots,\ \beta_n=\frac{\omega_n}{P_n}\alpha+1. \quad (25)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many odd integers α such that β_1, $\beta_2,\cdots$, β_n are primes.

From (25) we have

$$3\beta_1=\omega_n\alpha+3,$$

$$3\beta_1+1=\omega_n\alpha+4=2(\frac{\omega_n}{2}\alpha+2)=2\beta_2,$$

$$3\beta_1+2=\omega_n\alpha+5=5(\frac{\omega_n}{5}\alpha+1)=5\beta_3,$$

$$3\beta_1+4=\omega_n\alpha+7=7(\frac{\omega_n}{7}\alpha+1)=7\beta_4,$$

……

$$3\beta_1+P_n-3=\omega_n\alpha+P_n=P_n(\frac{\omega_n}{P_n}\alpha+1)=P_n\beta_n. \quad (26)$$

From（26）we have

$$d(3\beta_1)=d(3\beta_1+1)=d(3\beta_1+2)=d(3\beta_1+4)=\cdots=d(3\beta_1+P_n-3)=2 \quad \text{infinitely-often.} \quad (27)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that

$$d(x)=d(x+1)=d(x+5-3)=d(x+7-3)=\cdots=d(x+P_n-3)=m>1 \quad \text{infinitely-often.} \quad (28)$$

Theorem 7. Gaps between products of two primes.

$$d(x)=d(x+3)=2 \quad \text{infinitely-often} \quad (29)$$

Proof. We define prime equations

$$\beta_1=30\alpha+1, \qquad \beta_2=12\alpha+1. \quad (30)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1, and β_2 are primes.

From （30）we have

$$2\beta_1=60\alpha+2,$$
$$2\beta_1+3=60\alpha+5=5(12\alpha+1)=5\beta_2 \quad (31)$$

From （31）we prove

$$d(2\beta_1)=d(2\beta_1+3)=2 \quad \text{infinitely-often.} \quad (32)$$

Theorem 8. Gaps between products of m primes.

$$d(x)=d(x+3)=m>1 \text{ infinitely-often} \quad (33)$$

Proof. Suppose that u and $u+3$ are two integers, each being the products of $m-1$ distinct primes.

Let $M=6u(u+3)$. We define prime equations

$$\beta_1=\frac{M}{u}\alpha+1, \quad \beta_2=\frac{M}{u+3}\alpha+1 \quad (34)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1, and β_2 are primes.

From (34) we have

$$u\beta_1=M\alpha+u,$$
$$u\beta_1+3=M\alpha+u+3=(u+3)\left(\frac{M}{u+3}\alpha+1\right)=(u+3)\beta_2 \quad (35)$$

From (35) we prove

$$d(x)=d(x+3)=m>1 \text{ infinitely-often.} \quad (36)$$

Theorem 9. Gaps between products of two primes.

$$d(x)=d(x+6)=2 \text{ infinitely-often} \quad (37)$$

Proof. We define prime equations

$$\beta_1=66\alpha+1, \qquad \beta_2=30\alpha+1. \quad (38)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1, and β_2 are primes.

From (38) we have

$$5\beta_1=330\alpha+5,$$
$$5\beta_1+6=330\alpha+11=11(30\alpha+1)=11\beta_2 \quad (39)$$

From (39) we prove

$$d(5\beta_1)=d(5\beta_1+6)=2 \text{ infinitely-often.} \quad (40)$$

Theorem 10. Gaps between products of m primes.

$$d(x)=d(x+6)=m>1 \text{ infinitely-often} \quad (41)$$

Proof. Suppose that u and $u+6$ are two integers, each being the products of $m-1$ distinct primes. Let $M=30u(u+6)$. We define prime equations

$$\beta_1=\frac{M}{u}\alpha+1, \quad \beta_2=\frac{M}{u+6}\alpha+1 \quad (42)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1, and β_2 are primes.

From (42) we have

$$u\beta_1=M\alpha+u,$$
$$u\beta_1+6=M\alpha+u+6=(u+6)\left(\frac{M}{u+6}\alpha+1\right)=(u+6)\beta_2 \quad (43)$$

From（43）we prove

$$d(x)=d(x+6)=m>1 \text{ infinitely-often.} \quad (44)$$

Theorem 11. Gaps between products of two primes.

$$d(x)=d(x+5040)=2 \text{ infinitely-often} \quad (45)$$

Proof. Suppose $M=2\times3\times11\times5051$. We define prime equations

$$\beta_1=\frac{M}{11}\alpha+1, \qquad \beta_2=\frac{M}{5051}\alpha+1. \quad (46)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1 and β_2 are primes.

From （46）we have

$$\begin{aligned}&11\beta_1=M\alpha+11,\\&11\beta_1+5040=M\alpha+5051=5051\left(\frac{M}{5051}\alpha+1\right)=5051\beta_2\end{aligned} \quad (47)$$

From （47）we prove that

$$d(x)=d(x+5040)=2 \text{ infinitely-often.} \quad (48)$$

Using Jiang function we can prove that

$$d(x)=d(x+5040)=m>1 \text{ infinitely-often} \quad (49)$$

Theorem 12. Gaps between products of two primes.

We study general solutions of

$$d(x)=d(x\pm5040)=2 \text{ infinitely-often} \quad (50)$$

Proof. We define a prime equation

$$P_2=P_1\pm5040. \quad (51)$$

Using Jiang function $J_2(\omega)$ we can prove that there exist infinitely many prime P_1 such that P_2 is a prime.

From （51）suppose $M=2\times3\times P_1\times P_2$. We define prime equations

$$\beta_1=\frac{M}{P_1}\alpha+1, \qquad \beta_2=\frac{M}{P_2}\alpha+1 \quad (52)$$

Using Jiang function $J_2(\omega)$ we can prove that there exist infinitely many integers α such that β_1 and β_2 are primes.

From （52）we have

$$\begin{aligned}&P_1\beta_1=M\alpha+P_1,\\&P_1\beta_1\pm5040=M\alpha+P_1\pm5040=M\alpha+P_2=P_2\left(\frac{M}{P_2}\alpha+1\right)=P_2\beta_2.\end{aligned} \quad (53)$$

We prove that

$$d(x)=d(x\pm5040)=2 \text{ infinitely-often.} \quad (54)$$

Theorem 13. Gaps between products of two primes.

$$d(x)=d(x+5)=2 \text{ infinitely-often} \quad (55)$$

Using $2\times 43+5=7\times 13$ we define prime equations,

$$\beta_1=39\alpha+4,\quad \beta_2=6\alpha+1 \tag{56}$$

From (56) we have

$$\begin{aligned}&2\beta_1=78\alpha+8,\\&2\beta_1+5=78\alpha+13=13(6\alpha+1)=13\beta_2.\end{aligned} \tag{57}$$

We redefine prime equations

$$\beta_1=42\alpha+1,\quad \beta_2=12\alpha+1 \tag{58}$$

From (58) we have

$$\begin{aligned}&2\beta_1=84\alpha+2,\\&2\beta_1+5=84\alpha+7=7(12\alpha+1)=7\beta_2.\end{aligned} \tag{59}$$

Theorem 14. Gaps between products of two primes.

$$d(x)=d(x+7)=2 \text{ infinitely-often} \tag{60}$$

Using $2\times 29+7=5\times 13$ we define prime equations,

$$\beta_1=30\alpha-1,\quad \beta_2=12\alpha+1 \tag{61}$$

From (61) we have

$$\begin{aligned}&2\beta_1=60\alpha-2,\\&2\beta_1+7=60\alpha+5=5(12\alpha+1)=5\beta_2.\end{aligned} \tag{62}$$

We redefine prime equations

$$\beta_1=26\alpha+3,\quad \beta_2=4\alpha+1 \tag{63}$$

From (63) we have

$$\begin{aligned}&2\beta_1=52\alpha+6,\\&2\beta_1+7=52\alpha+13=13(4\alpha+1)=13\beta_2.\end{aligned} \tag{64}$$

Theorem 15. Gaps between products of two primes.

$$d(x)=d(x+9)=2 \text{ infinitely-often} \tag{65}$$

Using $2\times 13+9=5\times 7$ we define prime equations,

$$\beta_1=15\alpha-2,\quad \beta_2=6\alpha+1 \tag{66}$$

From (66) we have

$$\begin{aligned}&2\beta_1=30\alpha-4,\\&2\beta_1+9=30\alpha+5=5(6\alpha+1)=5\beta_2.\end{aligned} \tag{67}$$

We redefine prime equations

$$\beta_1=42\alpha-1,\quad \beta_2=12\alpha+1 \tag{68}$$

From (68) we have

$$\begin{aligned}&2\beta_1=84\alpha-2,\\&2\beta_1+9=84\alpha+7=7(12\alpha+1)=7\beta_2.\end{aligned} \tag{69}$$

Theorem 16. Gaps between products of two primes.

$$d(x-11)=d(x)=2 \text{ infinitely-often} \quad (70)$$

Using $2\times 23-11=5\times 7$ we define prime equations,

$$\beta_1=20\alpha+3, \quad \beta_2=8\alpha-1 \quad (71)$$

From （71）we have

$$2\beta_1=40\alpha+6,$$
$$2\beta_1-11=40\alpha-5=5(8\alpha-1)=5\beta_2. \quad (72)$$

We redefine prime equations

$$\beta_1=21\alpha+2, \quad \beta_2=6\alpha-1 \quad (73)$$

From（73）we have

$$2\beta_1=42\alpha+4,$$
$$2\beta_1-11=42\alpha-7=7(6\alpha-1)=7\beta_2. \quad (74)$$

Goldston *et. al* proved only

$$d(x)=d(x+n\le 6)=2 \text{ infinitely-often [4-5].} \quad (75)$$

References

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美国匈牙利土耳其数学家正在研究 x 和 $x+n\le 6$ 两个数。有无限多个 x 使得 x 和 $x\le 6$ 两个数每个都是两个素数相乘[4]。例如 $x=14=2\times 7$，$x+1=15=3\times 5,\cdots$，$x=86=2\times 43$，$x+6=91=7\times 13$,这就是当代国际数学最高水平，受到当代数学界的关注。他们并没有证明这个问题，它比哥德巴赫猜想难一万倍。蒋春暄看到[4] 以后，国外就这么点水平吹到天上去，决定写本文，在国内外散发。蒋春暄 2002 年结果[1]。从定理一得出 $x=93=3\times 31$，$x+1=94=2\times 47$，$x+2=95=5\times 19$，有无限多个 x 使得 x，$x+1$，$x+2$ 三个数每个数都是两个素数相乘。从定理二得出，$x=1727913=3\times 11\times 52361$，$x+1=1727914=2\times 17\times 50821$，

$x+2=1727915=5\times 7\times 49369$，有无限多个 x 使得每个数都是 m 个素数相乘。定理六，x，$x+1$，$x+2$，$x+4,\cdots$，$x+P_n-3$ 有 n 个数,有无限多个 x 使得每个数都是 m 个素数相乘，这样成果在过去没有数学家想象过，这是素数分布一个重要规律。将来一定会有广泛的应用。这是数学美！这是人类数学中最伟大成就, 在中国被评为最大伪科学, 中国不承认蒋春暄成就。母校北航不承认蒋春暄是北航的学生, 献给北航母校被拒绝！2009 年 12 月中科院数学院与北航联合创办" 华罗庚数学班" ，中国全面封杀蒋春暄成果。这样事件只能在中国才存在, 全世界任何国家都不会发生"蒋春暄现象" 事件！
It was therefore a great surprise for Erdos (and pobably for other number theorists as well) when C.Spiro proved in 1981 that d(x)=d(x + 5040) infinitely-often. It is theorem 11.There cannot modern prime theory without Jiang function.