

SPECIAL ALGEBRAIC STRUCTURES

by Florentin Smarandache
University of New Mexico
200 College Raod
Gallup, NM 87301, USA

Abstract.

New notions are introduced in algebra in order to better study the congruences in number theory. For example, the <special semigroups> make an important such contribution.

Keywords: algebraic structures, semigroup, group, ring, field, lattice, k-module, vectorial space

1991 MSC: 06A99

Introduction.

By <proper subset> of a set A we consider a set P included in A, and different from A, different from the empty set, and from the unit element in A - if any.

We rank the algebraic structures using an order relationship: we say that the algebraic structures $S1 \ll S2$ if:

- both are defined on the same set;
- all $S1$ laws are also $S2$ laws;
- all axioms of an $S1$ law are accomplished by the corresponding $S2$ law;
- $S2$ laws accomplish strictly more axioms than $S1$ laws, or $S2$ has more laws than $S1$.

For example: semigroup \ll monoid \ll group \ll ring \ll field,
or semigroup \ll commutative semigroup, ring \ll unitary ring, etc.

We define a **GENERAL SPECIAL STRUCTURE** to be a structure SM on a set A, different from a structure SN , such that a proper subset of A is an SN structure, where $SM \ll SN$.

1) **The SPECIAL SEMIGROUP** is defined to be a semigroup A, different from a group, such that a proper subset of A is a group (with respect to the same induced operation).

For example, if we consider the commutative multiplicative group
 $SG = \{18^2, 18^3, 18^4, 18^5\} \pmod{60}$
we get the table:

```

x | 24 12 36 48
---|-----
24 | 36 48 24 12
12 | 48 24 12 36
36 | 24 12 36 48
48 | 12 36 48 24

```

Unitary element is 36.

Using the algorithm [Smarandache 1972] we get that 18^2 is congruent to $18^6 \pmod{60}$.

Now we consider the commutative multiplicative semigroup $SS = \{18^1, 18^2, 18^3, 18^4, 18^5\} \pmod{60}$ and we get the table:

```

x | 18 | 24 12 36 48
---|---|-----
18 | 24 | 12 36 48 24
---|---|-----
24 | 12 | 36 48 24 12
12 | 36 | 48 24 12 36
36 | 48 | 24 12 36 48
48 | 24 | 12 36 48 24

```

Because SS contains a proper subset SG , which is a group, then SS is a Special Semigroup. This is generated by the element 18. The powers of 18 form a cyclic sequence: 18, 24,12,36,48, 24,12,36,48,

Similarly are defined:

2) **The SPECIAL MONOID** is defined to be a monoid A , different from a group, such that a proper subset of A is a group (with respect with the same induced operation).

3) **The SPECIAL RING** is defined to be a ring A , different from a field, such that a proper subset of A is a field (with respect with the same induced operations).

We consider the commutative additive group $M = \{0, 18^2, 18^3, 18^4, 18^5\} \pmod{60}$ [using the module 60 residuals of the previous powers of 18], $M = \{0, 12, 24, 36, 48\}$, unitary additive unit is 0. $(M, +, x)$ is a field.

While $(SR, +, x) = \{0, 6, 12, 18, 24, 30, 36, 42, 48, 54\} \pmod{60}$ is a ring whose

proper subset $\{0,12,24,36,48\} \pmod{60}$ is a field.
Therefore $(SR,+,x) \pmod{60}$ is a Special Ring.
This feels very nice.

4) **The SPECIAL SUBRING** is defined to be a Special Ring B which is a proper subset of a Special Ring A (with respect with the same induced operations).

5) **The SPECIAL IDEAL** is defined to be an ideal A, different from a field, such that a proper subset of A is a field (with respect with the same induced operations).

6) **The SPECIAL SEMILATTICE** is defined to be a lattice A, different from a lattice, such that a proper subset of A is a lattice (with respect with the same induced operations).

7) **The SPECIAL FIELD** is defined to be a field $(A,+,x)$, different from a K-algebra, such that a proper subset of A is a K-algebra (with respect with the same induced operations, and an external operation).

8) **The SPECIAL R-MODULE** is defined to be an R-MODULE $(A,+,x)$, different from an S-algebra, such that a proper subset of A is an S-algebra (with respect with the same induced operations, and another "x" operation internal on A), where R is a commutative unitary ring and S is its proper subset field.

9) **The SPECIAL K-VECTORIAL SPACE** is defined to be a K-vectorial space $(A,+,.)$, different from a K-algebra, such that a proper subset of A is a K-algebra (with respect with the same induced operations, and another "x" operation internal on A), where K is a commutative field.

Craiova, 1973

References:

[1] Smarandache, Florentin, "A generalization of Euler's Totient Function", manuscript, 1972.

In the mean time the following papers, inspired by this subject, have been published or presented to international conferences:

[2] Castillo, J., "The Smarandache Semigroup", <International Conference on Combinatorial Methods in Mathematics>, II Meeting of the project 'Algebra, Geometria e Combinatoria', Faculdade de Ciencias da Universidade do Porto, Portugal, 9-11 July 1998.

[3] Padilla, Raul, "Smarandache Algebraic Structures", <Bulletin of Pure

and Applied Sciences>, Delhi, India, Vol. 17E, No. 1, 119-121, 1998.

- [4] Padilla, Raul, "Smarandache Algebraic Structures", <Smarandache Notions Journal>, USA, Vol. 9, No. 1-2, 36-38, Summer 1998;
<http://www.gallup.unm.edu/~smarandache/alg-s-tx.txt>;
Presented to the <International Conference On Semigroups>, Universidade do Minho, Braga, Portugal, 18-23 June, 1999.