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Fermat's Last Theorem has been proved(2)

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In this paper we prove that it is sufficient to prove $S_1^3+S_2^3=1$ for Fermat's last theorem using the complex hyperbolic functions in the hypercomplex variable theory. More than 200 years ago Euler gave a proof of $S_1^3+S_2^3=1$. Fermat's last theorem has been proved.

First we discuss the hyperbolic functions

$$S_1=\text{cht},\qquad S_2=\text{sht} \tag{1}$$

From (1) we get its inverse transformation

$$e^t=S_1+S_2,\quad e^{-t}=S_1-S_2 \tag{2}$$

From (2) we get

$$e^t\cdot e^{-t}=S_1^2-S_2^2=1 \tag{3}$$

If e^t =rational, then (3) has infinitely many rational solutions.

Using above method we prove Fermat's last theorem. From Refs 1 and 2 we introduce the complex hyperbolic functions of order p:

$$S_i=\frac{1}{P}[e^A+2\sum_{j=1}^{\frac{P-1}{2}}(-1)^{(i-1)j}e^{B_j}\cos(\theta_j+(-1)^j\frac{(i-1)j\pi}{P})],$$

where $i=1,2,\cdots,P,\quad P=odd,P>1,$ (4)

$$A=\sum_{e=1}^{\frac{P-1}{2}}(t_e+t_{p-e}).\quad B=\sum_{e=1}^{\frac{P-1}{2}}(t_e+t_{p-e})(-1)^{ej}\cos\frac{ej\pi}{P},t_e,t_{p-e}\in R. \tag{5}$$

$$\theta_j=\sum_{e=1}^{\frac{P-1}{2}}(t_{p-e}+t_e)\sin\frac{ej\pi}{P},\ j=even;$$

$$\theta_j=\sum_{e=1}^{\frac{P-1}{2}}(t_{p-e}+t_e)(-1)^{e+1}\sin\frac{ej\pi}{P},\ j=odd \tag{6}$$

From (4) we get its inverse transformation[3,4]

$$e^A=\sum_{i=1}^{P}S_i \tag{7}$$

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{P-1}(-1)^iS_{i+1}\cos\frac{ij\pi}{P},\ e^{B_j}\sin\theta_j=\sum_{i=1}^{P-1}(-1)^iS_{i+1}\sin\frac{ij\pi}{P},\ j=\text{odd}, \tag{8}$$

$$e^{B_j}\cos\theta_j = S_1+\sum_{i=1}^{P-1} S_{i+1}\cos\frac{ij\pi}{P},\ e^{B_j}\sin\theta_j = -\sum_{i=1}^{P-1} S_{i+1}\sin\frac{ij\pi}{P},\ j = even, \quad (9)$$

Assume $S_1 \neq 0$, $S_2 \neq 0$, $S_3 = \cdots = S_p = 0$. From (7)-(9) we get

$$e^A = S_1 + S_2, \quad (10)$$

$$e^{B_j}\cos\theta_j = S_1 - S_2\cos\frac{j\pi}{P},\ e^{B_j}\sin\theta_j = -S_2\sin\frac{j\pi}{P},\ j = odd, \quad (11)$$

$$e^{B_j}\cos\theta_j = S_1 + S_2\cos\frac{j\pi}{P},\ e^{B_j}\sin\theta_j = -S_2\sin\frac{j\pi}{P},\ j = even, \quad (12)$$

From (11) and (12) we get

$$e^{2B_j} = S_1^2 + S_2^2 - 2S_1S_2\cos\frac{j\pi}{P},\ \ j = odd, \quad (13)$$

$$e^{2B_j} = S_1^2 + S_2^2 + 2S_1S_2\cos\frac{j\pi}{P},\ \ j = even, \quad (14)$$

From (5) we get

$$A + 2\sum_{j=1}^{\frac{P-1}{2}} B_j = 0. \quad (15)$$

From (10), (13), (14) and (15) we may get Fermat's equations

$$P = 3, \exp(A+2B_1) = (S_1+S_2)(S_1^2 - S_1S_2 + S_2^2) = S_1^3 + S_2^3 = 1, \quad (16)$$

$$P = 5, \exp(A+2B_1+2B_2) = (S_1+S_2)(S_1^2+S_2^2-2S_1S_2\cos\frac{\pi}{5})(S_1^2+S_2^2+2S_1S_2\cos\frac{2\pi}{5})$$

$$= S_1^5 + S_2^5 = 1 \quad (17)$$

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$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}} B_j) = (S_1+S_2)\prod_{j=1}^{\frac{P-1}{2}}[S_1^2+S_2^2+(-1)^j 2S_1S_2\cos\frac{j\pi}{P}]$$

$$= S_1^P + S_2^P = 1. \quad (18)$$

Theorem. (18) has no rational solutions for $P > 3$, except $S_1 S_2 = 0$.

Proof 1. Since S_i are the functions of $(t_e + t_{p-e}) : S_i = f_i(t_e + t_{p-e})$, we study $(t_e + t_{p-e})$ in (5). Put $P = 3P_1$, where $P_1 =$ odd prime. From (5), (10), (13), (14) and (15) we get

$$\exp(A+2\sum_{j=1}^{\frac{3P_1-1}{2}} B_j) = S_1^{3P_1} + S_2^{3P_1} = 1, \quad (19)$$

$$\exp(A+2B_{P_1}) = S_1^3 + S_2^3 = [\exp(\sum_{\alpha=1}^{\frac{P_1-1}{2}} (t_{3\alpha} + t_{3P_1-3\alpha}))]^3, \quad (20)$$

$$\exp(2\sum B_j) = \sum_{\alpha=1}^{P_1} S_1^{3P_1-3\alpha} S_2^{3(\alpha-1)}(-1)^{\alpha+1} = [\exp(-\sum_{\alpha=1}^{\frac{P_1-1}{2}} (t_{3\alpha} + t_{3P_1-3\alpha}))]^3,$$

$$j \not\equiv 0 (\mathrm{mod}\, P_1), \quad (21)$$

$$\exp(A+2\sum_{\alpha=1}^{\frac{P_1-1}{2}} B_{3\alpha}) = S_1^{P_1} + S_2^{P_1} = [\exp(t_{P_1} + t_{2P_1})]^{P_1}, \tag{22}$$

$$\exp(2\sum B_j) = S_1^{2P_1} - S_1^{P_1} S_2^{P_1} + S_2^{2P_1} = [\exp(-t_{P_1} - t_{2P_1})]^{P_1}, \; j \not\equiv 0(\mathrm{mod}\, 3); \tag{23}$$

(19), (20) and (22) are the Fermat's equations. Since Euler gave a proof of (20), (19). Therefore and (22) have no rational solutions for any odd prime $P_1 > 3$.

From (22) and (23) we get

$$S_1 = \exp(t_{P_1} + t_{2P_1}) \left[\frac{1}{2} \left(1 \pm \sqrt{\frac{4\exp[-3P_1(t_{P_1} + t_{2P_1})] - 1}{3}} \right) \right]^{\frac{1}{P_1}},$$

$$S_2 = \exp(t_{P_1} + t_{2P_1}) \left[\frac{1}{2} \left(1 \mp \sqrt{\frac{4\exp[-3P_1(t_{P_1} + t_{2P_1})] - 1}{3}} \right) \right]^{\frac{1}{P_1}}. \tag{24}$$

If $S_1 =$ rational, then $S_2 =$ irrational and vice versa. Therefore (19), (20) and (22) have no rational solutions for any odd prime P_1.

From (22) and (24) we get

$$a^{P_1} + b^{P_1} = 1, \tag{25}$$

where

$$a = S_1 \exp(-t_{P_1} - t_{2P_1}) = \left[\frac{1}{2} \left(1 \pm \sqrt{\frac{4\exp[-3P_1(t_{P_1} + t_{2P_1})] - 1}{3}} \right) \right]^{\frac{1}{P_1}},$$

$$b = S_2 \exp(-t_{P_1} - t_{2P_1}) \left[\frac{1}{2} \left(1 \mp \sqrt{\frac{4\exp[-3P_1(t_{P_1} + t_{2P_1})] - 1}{3}} \right) \right]^{\frac{1}{P_1}} \tag{26}$$

If $t_{P_1} + t_{2P_2} = 0$, they are two trivial solutions. If $a =$ rational, then $b =$ irrational and vice versa. Therefore (25) has no rational solutions for any odd prime $P_1 > 3$.

Proof 2. Put $P = \prod_{r=1}^{n} P_r$, where P_r =odd prime. From (5), (10), (13), (14) and (15) we get

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}} B_j) = S_1^P + S_2^P = 1, \tag{27}$$

$$\exp(A+2B_{\frac{P}{3}}) = S_1^3 + S_2^3 = [\exp(\sum_{\alpha=1}^{\frac{p-3}{6}} (t_{3\alpha} + t_{P-3\alpha}))]^3, \tag{28}$$

$$\exp(A+2B_{\frac{P}{5}} + 2B_{\frac{2P}{5}}) = S_1^5 + S_2^5 = [\exp(\sum_{\alpha=1}^{\frac{P-5}{10}} (t_{5\alpha} + t_{P-5\alpha}))]^5, \tag{29}$$

$$\exp(A+2B_{\frac{P}{7}}+2B_{\frac{2P}{7}}+2B_{\frac{3P}{7}})=S_1^7+S_2^7=[\exp(\sum_{\alpha=1}^{\frac{P-7}{14}}(t_{7\alpha}+t_{P-7\alpha}))]^7, \quad (30)$$

$$\exp(A+2\sum_{\alpha=1}^{\frac{P_n-1}{2}}B_{\frac{P}{P_n}\alpha})=S_1^{P_n}+S_2^{P_n}=[\exp(\sum_{\alpha=1}^{\frac{P-P_n}{2P_n}}(t_{P_n\alpha}+t_{P-P_n\alpha}))]^{P_n}. \quad (31)$$

(27)-(31) are the Fermat's equations. Since Euler gave a proof of (28), therefore (27), (29), (30) and (31) have no rational solutions for any odd prime $P_r > 3$.

Example. $P=15$ From (5) we get

$$A=(t_1+t_{14})+(t_2+t_{13})+(t_3+t_{12})+(t_4+t_{11})+(t_5+t_{10})+(t_6+t_9)+(t_7+t_8)$$

$$2B_1=-2(t_1+t_{14})(1)+2(t_2+t_{13})(2)-2(t_3+t_{12})(3)+2(t_4+t_{11})(4)-2(t_5+t_{10})(5)$$

$$+2(t_6+t_9)(6)-2(t_7+t_8)(7)$$

$$2B_2=-2(t_1+t_{14})(2)+2(t_2+t_{13})(4)+2(t_3+t_{12})(6)+2(t_4+t_{11})(8)+2(t_5+t_{10})(10)$$

$$+2(t_6+t_9)(12)+2(t_7+t_8)(14)$$

$$2B_3=-2(t_1+t_{14})(3)+2(t_2+t_{13})(6)-2(t_3+t_{12})(9)+2(t_4+t_{11})(12)-2(t_5+t_{10})(15)$$

$$+2(t_6+t_9)(12)-2(t_7+t_8)(21)$$

$$2B_4=2(t_1+t_{14})(4)+2(t_2+t_{13})(8)+2(t_3+t_{12})(12)+2(t_4+t_{11})(16)+2(t_5+t_{10})(20)$$

$$+2(t_6+t_9)(24)+2(t_7+t_8)(28)$$

$$2B_5=-2(t_1+t_{14})(5)+2(t_2+t_{13})(10)-2(t_3+t_{12})(15)+2(t_4+t_{11})(20)-2(t_5+t_{10})(25)$$

$$+2(t_6+t_9)(30)-2(t_7+t_8)(35)$$

$$2B_6=2(t_1+t_{14})(6)+2(t_2+t_{13})(12)+2(t_3+t_{12})(18)+2(t_4+t_{11})(24)+2(t_5+t_{10})(30)$$

$$+2(t_6+t_9)(36)+2(t_7+t_8)(42)$$

$$2B_7=-2(t_1+t_{14})(7)+2(t_2+t_{13})(14)-2(t_3+t_{12})(21)+2(t_4+t_{11})(28)-2(t_5+t_{10})(35)$$

$$+2(t_6+t_9)(42)-2(t_7+t_8)(49) \quad (32)$$

where $(i)=\cos\frac{i\pi}{15}$

From (10), (13), (14), (15) and (32) we get

$$\exp(A+2\sum_{j=1}^{7}B_j)=S_1^{15}+S_2^{15}=1, \quad (33)$$

$$\exp(A+2B_5)=S_1^3+S_2^3=[\exp(t_3+t_{12}+t_6+t_9)]^3, \quad (34)$$

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5=[\exp(t_5+t_{10})]^5. \quad (35)$$

(33)-(35) are the Fermat's equations. Since Euler gave a proof of (34), therefore (33) and (35) have no rational solutions.

Remark. In order to prove Fermat's last theorem, it is sufficient to prove $S_1^4+S_2^4=1$ and $S_1^3+S_2^3=1$. The former was due to Fermat and the latter was due to Euler. Fermat's last theorem has been proved. On Oct. 25, 1991 without using any number theory we have proved Fermat last theorem.

References

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中国著名数论专家乐茂华教授看到本文 1992 年 1 月 21 日

来信"至于您这次寄来的文章，不知已在何种刊物发表？望来信告知。"他是第一位看到本文的专家，对本文肯定，以后他多次来信支持蒋春暄的工作，他是广东湛江师范学院教授，本文已在全世界散发 600 份。

十七年后蒋春暄重读这篇论文，他可以说他最大贡献是证明费马大定理（1992），这是天才的证明，伟大的证证明，绝后的证明，费马的证明，最高级的证明。1995 年怀尔斯(Wiles proof of Fermat last theorem)费马大定理证明是可怜的，没有意义的，张冠李戴的，无法理解的，骗人的，最低级的。

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