

The Approximate Solutions of Blasius Equation

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Abstract

We find Blasius function to satisfy the boundary condition $f'(\infty)=1$ and obtain the approximate solutions of Blasius equation.

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1. Introduction. In 1967 using Jiang functions we defined Blasius function. Using it we obtain the approximate solutions for Blasius equation [1]. Here we rewrite this paper.

Blasius equation [2] is

$$f'''(x) + f(x)f''(x) = 0, \quad (1)$$

with boundary conditions

$$f(0) = f'(0) = 0, \quad f''(0) = 1, \quad f'(\infty) = 1. \quad (2)$$

Blasius gave a solution in power series

$$f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{A_k}{(3k+2)!} x^{3k+2} \quad (3)$$

where

$$A_0 = A_1 = 1, \quad A_k = \sum_{i=0}^{k-1} \binom{3k-1}{3i} A_i A_{k-i-1}, \quad k \geq 2. \quad (4)$$

From (3) and (4) we have

$$f(x) = \frac{1}{2!}x^2 - \frac{1}{5!}x^5 + \frac{11}{8!}x^8 - \frac{375}{11!}x^{11} + \frac{27897}{14!}x^{14} - \dots \quad (5)$$

To our knowledge, up to now, no one has given a approximate solution of Blasius equation.

2. We find Blasius function to satisfy the boundary condition $f'(\infty)=1$. We define Jiang functions [3]

$$J_1(x) = \frac{1}{3} \left[e^x + 2e^{-x/2} \cos\left(\frac{\sqrt{3}}{2}x\right) \right] = \sum_{i=0}^{\infty} \frac{x^{3i}}{(3i)!}, \quad (6)$$

$$J_2(x) = \frac{1}{3} \left[e^x - 2e^{-x/2} \cos\left(\frac{\sqrt{3}}{2}x - \frac{\pi}{3}\right) \right] = \sum_{i=0}^{\infty} \frac{x^{3i+2}}{(3i+2)!}, \quad (7)$$

$$J_3(x) = \frac{1}{3} \left[e^x + 2e^{-x/2} \cos\left(\frac{\sqrt{3}}{2}x - \frac{2\pi}{3}\right) \right] = \sum_{i=0}^{\infty} \frac{x^{3i+1}}{(3i+1)!}. \quad (8)$$

From (6) and (8) we define Blasius function

$$F'(x) = \frac{J_3(x)}{J_1(x)} = \sum_{k=0}^{\infty} \frac{B_k}{(3k+1)!} x^{3k+1}, \quad (9)$$

where $B_0 = 1, B_1 = -3$,

$$B_k = -3k + \sum_{i=1}^{k-1} (-1)^{i+1} \binom{3k+1}{3(k-i)} |B_i|, \quad k \geq 2 \quad (10)$$

From (9) we have

$$F(x) = \frac{1}{2!}x^2 - \frac{3}{5!}x^5 + \frac{99}{8!}x^8 - \frac{11259}{11!}x^{11} + \dots \quad (11)$$

Blasius function $F(x)$ is the approximate solution of the following Blasius equation

$$F'''(x) + F(x)F''(x) = 0, \quad (12)$$

with boundary conditions

$$F(0) = F'(0) = 0, \quad F''(0) = 1, \quad F'(\infty) = 1. \quad (13)$$

3. We assume $x = \frac{\eta}{\sqrt[3]{3}}$. Substituting it into (11) we have

$$F\left(\frac{\eta}{\sqrt[3]{3}}\right) = \frac{1}{\sqrt[3]{9}} \left[\frac{1}{2!}\eta^2 - \frac{1}{5!}\eta^5 + \frac{11}{8!}\eta^8 - \frac{417}{11!}\eta^{11} + \dots \right] \quad (14)$$

From (14) we define Blasius function

$$\phi(\eta) = \frac{1}{2!}\eta^2 - \frac{1}{5!}\eta^5 + \frac{11}{8!}\eta^8 - \frac{417}{11!}\eta^{11} + \dots \quad (15)$$

Blasius function $\phi(\eta)$ is the approximate solution of the following Blasius equation

$$\phi''' + \phi\phi'' = 0, \quad (16)$$

with boundary conditions

$$\phi(0) = \phi'(0) = 0, \quad \phi''(0) = 1, \quad \phi'(\infty) = 1. \quad (17)$$

4. We define Blasius function

$$\Psi'_{(x)} = \frac{J_3(x) + D(x)}{J_1(x)} \quad (18)$$

From (1) we assume

$$\Psi'(x) = f'(x) \quad (19)$$

Substituting (19) into (18) we have

$$D(x) = f'(x)J_1(x) - J_3(x) \quad (20)$$

From (20) we have

$$D(x) = \sum_{k=1}^{\infty} \frac{C_k}{(3k+1)!} x^{3k+1} \quad (21)$$

where

$$C_k = \left[\sum_{i=0}^k (-1)^{i+1} A_i \binom{3k+1}{3(k-i)} \right] - 1, k \geq 1. \quad (22)$$

From (22) we have

$$C_1 = 2, C_2 = -18, C_3 = 744, C_4 = -61180, C_5 = 11628918, \dots \quad (23)$$

We may prove that

$$\lim_{x \rightarrow \infty} \frac{D(x)}{J_1(x)} = 0. \quad (24)$$

From (18) we have

$$\psi(x) = \int_0^x \frac{J_3(\eta) + D(\eta)}{J_1(\eta)} d\eta \quad (25)$$

Blasius function $\psi(x)$ is the approximate solution of the following Blasius equation

$$\psi''' + \psi\psi'' = 0 \quad (26)$$

with boundary conditions

$$\psi(0) = \psi'(0) = 0, \psi''(0) = 1, \psi'(\infty) = 1. \quad (27)$$

5. We assume that

$$H(x) = \frac{2}{4!}x^4 - \frac{18}{7!}x^7 + \frac{744}{10!}x^{10} - \frac{61180}{13!}x^{13} \quad (28)$$

Using (28) we define Blasius function

$$\phi'(x) = \frac{J_3(x) + H(x)}{J_1(x)} = x - \frac{1}{4!}x^4 + \frac{11}{7!}x^7 - \frac{375}{10!}x^{10} + \frac{27897}{13!}x^{13} - \frac{3817137}{16!}x^{16} + \dots \quad (29)$$

Blasius function $\phi(x)$ is the approximate solution of the following Blasius equation

$$\phi''' + \phi\phi'' = 0, \quad (30)$$

with boundary conditions

$$\phi(0) = \phi'(0) = 0, \phi''(0) = 1, \phi'(\infty) = 1.$$

References

- [1] Chun-Xuan, Jiang, Analytic Solution of Blasius Equation, Unpublished Notebook, (1967).
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- [3] Chun-Xuan, Jiang, Foundations of Santilli's Isonumber Theory, with Applications to New Cryptograms, Fermat's Theorem and Goldbach's Conjecture Inter. Acad, Proess, America-Europe- Asia(2002), 263-268, <http://www.i-b-r.org/docs/jiang.pdf>.