Experiment to test the quantum effect of a waveguide (I)

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Abstract: The waveguide can be regarded as a potential barrier to microwaves and we apply quantum mechanics to study the coefficient of reflection R and transmission T. An initial experimental result is also presented in this paper that the transverse momentum of the electromagnetic field in a waveguide is zero which is no longer in proportion to the transverse wave vector. We're preparing to detect under other conditions and will report as soon as possible.

Key words: quantum theory; waveguide; microwave; tunneling effect

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The coefficient of reflection R and transmission T of energy flow depends on the momentum as

$$R = \frac{(p_{z1} - p_{z2})^2}{(p_{z1} + p_{z2})^2} \quad (1)$$

$$T = \frac{4 p_{z1}\, p_{z2}}{(p_{z1} + p_{z2})^2} \quad (2)$$

$$R + T = 1 \quad (3)$$

For example, p_{z1} is $\sqrt{2m_0 E}$ to a free and non-relativistic particle moving along the z axis and then $p_{z2} = \sqrt{2m_0(E-\varphi)}$ in a step barrier where the potential energy is φ, [1]

$$R = \frac{(\sqrt{E} - \sqrt{E-\varphi})^2}{(\sqrt{E} + \sqrt{E-\varphi})^2} \quad (1')$$

$$T = \frac{4\sqrt{E(E-\varphi)}}{(\sqrt{E} + \sqrt{E-\varphi})^2} \quad (2')$$

The form is still tenable to relativistic particles. In optics, the momentum of a photon is in direct proportion to the refractive index n of the medium (Minkowski formulation) . Therefore [2],

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} \quad (1'')$$

$$T = \frac{4 n_1\, n_2}{(n_1 + n_2)^2} \quad (2'')$$

We use a rectangular waveguide that the cross section is $a = 16mm$, $b = 8mm$ and the length is $d = 153mm$ whose cut-off frequency is $f_c = \frac{c}{2\times 16mm} = 9.37\ GHz$. The terminal is connected to a microwave generator and power meter respectively through two commutators. The input momentum of a quantum in the coaxial cable is $p_{z1} = \hbar\omega\sqrt{\varepsilon\mu} = \frac{\hbar\omega}{c}$ ($\varepsilon_r \approx 1, \mu_r \approx 1$). As to the rectangular waveguide, $p_{z2} = \hbar \mathrm{k}_z = \frac{\hbar\omega}{c}\sqrt{1-\omega_c^2/\omega^2}$ when the mentioned state (i) is true [3] and hence

$$R_i = \frac{(1 - \sqrt{1-\omega_c^2/\omega^2})^2}{(1 + \sqrt{1-\omega_c^2/\omega^2})^2} \quad (1''')$$

$$T_i = \frac{4\sqrt{1-\omega_c^2/\omega^2}}{(1 + \sqrt{1-\omega_c^2/\omega^2})^2} \quad (2''')$$

By contrast, if the state (ii) is correct that is $p_{z2} = \hbar k_z \sqrt{1-\omega_c^2/\omega^2} = \frac{\hbar\omega}{c}(1-\omega_c^2/\omega^2)$,

$$R_{\text{ii}} = \frac{\omega_c^4/\omega^4}{(2-\omega_c^2/\omega^2)^2} \tag{1''''}$$

$$T_{\text{ii}} = \frac{4(1-\omega_c^2/\omega^2)}{(2-\omega_c^2/\omega^2)^2} \tag{2''''}$$

The initial data are as bellows,

f (GHz)	P_{output} (μW)	P_{input} (μW)	P_{output}/P_{input}	T_{i}($theory$)
<9	<10	522	<2%	—
9.7	24.9	522	4.8%	—
9.8	47.6	522	9.1%	—
9.9	191	522	36.6%	—
10	195	522	37.4%	—
10.1	256	522	49.0%	—
10.2	390	522	74.7%	80%
10.4	409	522	78.4%	84%
11	472	522	90.4%	90%
12	491	522	94.1%	94%
13	493	522	94.4%	96%

They satisfy (2''') in the region of high frequencies approaching to full transmission. Since the waveguide is in a circuit and affected by other factors, we will measure the open-ended waveguide according to the following diagram,

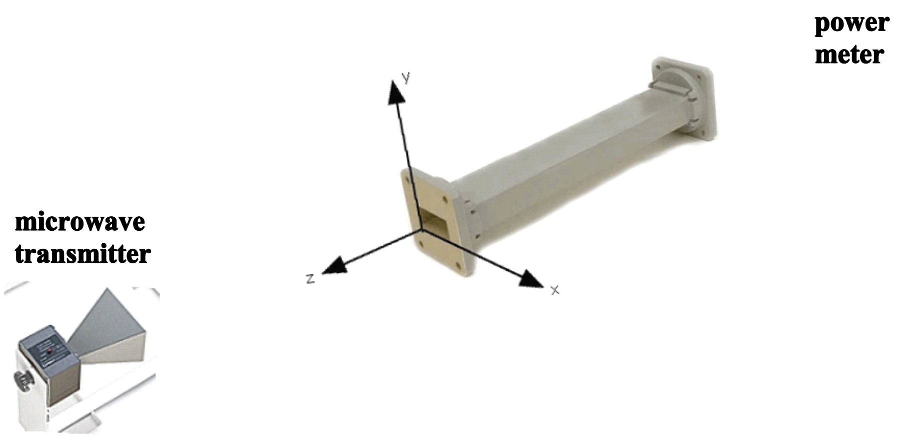

Figure.1

The waveguide can be regarded as an finite square well and the solution is [4]

$$R = \frac{(p_{z1}^2 - p_{z2}^2)^2 \sin^2(p_{z1}d/\hbar)}{(p_{z1}^2 - p_{z1}^2)^2 \sin^2(p_{z2}d/\hbar) + 4p_{z1}^2 p_{z2}^2} \tag{4}$$

$$T=\frac{4p_{z1}^2\,p_{z2}^2}{(p_{z1}^2-p_{z2}^2)^2\sin^2(p_{z2}d/\hbar)+4p_{z1}^2p_{z2}^2} \tag{5}$$

The momentum of an incident photon moving along the z axis from the unbounded space is $p_{z1}=\frac{\hbar\omega}{c}$ and $p_{z2}=\hbar k_z=\frac{\hbar\omega}{c}\sqrt{1-\omega_c^2/\omega^2}$ corresponding to the state (i) ,

$$R=\frac{\frac{\omega_c^4}{\omega^4}\sin^2\left\{\omega\sqrt{1-\omega_c^2/\omega^2}\ d/c\right\}}{\frac{\omega_c^4}{\omega^4}\sin^2\left\{\omega\sqrt{1-\omega_c^2/\omega^2}\ d/c\right\}+4(1-\omega_c^2/\omega^2)} \tag{4'}$$

$$T=\frac{4(1-\omega_c^2/\omega^2)}{\frac{\omega_c^4}{\omega^4}\sin^2\left\{\omega\sqrt{1-\omega_c^2/\omega^2}\ d/c\right\}+4(1-\omega_c^2/\omega^2)} \tag{5'}$$

Especially, the transmission is still non-zero even in the classical forbidden zone $\omega<\omega_c$,

$$R=\frac{\frac{\omega_c^4}{\omega^4}sh^2\left\{\omega\sqrt{\omega_c^2/\omega^2-1}\ d/c\right\}}{\frac{\omega_c^4}{\omega^4}sh^2\left\{\omega\sqrt{\omega_c^2/\omega^2-1}\ d/c\right\}+4(\omega_c^2/\omega^2-1)} \tag{6}$$

$$T=\frac{4(\omega_c^2/\omega^2-1)}{\frac{\omega_c^4}{\omega^4}sh^2\left\{\omega\sqrt{\omega_c^2/\omega^2-1}\ d/c\right\}+4(\omega_c^2/\omega^2-1)} \tag{7}$$

Conclusion

The initial result implies the longitudinal momentum $p_{//}$ of a quantum in the waveguide is $\hbar k_{//}$ which equals the total momentum $p=\hbar k_{//}$ proposed in [3]. Whereby the transverse momentum $p_\perp=\sqrt{p^2-p_{//}^2}=0$ is not in direct proportion to $k_\perp=\frac{2\pi}{32mm}\neq 0$.

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Postscript:

The equation (6) is deduced from the postulate that the momentum of $\omega < \omega_c$ in the waveguide is imaginary on the basis of the relativistic mechanical theory [3]

$\omega > \omega_c$

rest mass $m_0 = \hbar\omega_c / c^2$

velocity $V = c\sqrt{1-\omega_c^2/\omega^2} < c$

energy $E = \dfrac{m_0 c^2}{\sqrt{1-V^2/c^2}} = \hbar\omega$

momentum $p = \dfrac{m_0 V}{\sqrt{1-V^2/c^2}} = \dfrac{\hbar\omega}{c}\sqrt{1-\omega_c^2/\omega^2}$

Actually, a superluminal theory can be applied[4] if the quanta passing through the waveguide in the above tunnel effect are faster than light. In this case,

$\omega < \omega_c$

rest mass $m_0 = \hbar\omega_c / c^2$

velocity $V = c\sqrt{1+\omega_c^2/\omega^2}$

energy $E = \dfrac{m_0 c^2}{\sqrt{V^2/c^2-1}} = \hbar\omega$

momentum $p = \dfrac{m_0 V}{\sqrt{V^2/c^2-1}} = \dfrac{\hbar\omega}{c}\sqrt{1+\omega_c^2/\omega^2}$

where the momentum is still a real quantity in the classical forbidden zone and the coefficient of transmission should be

$$T = \frac{4(1+\omega_c^2/\omega^2)}{\dfrac{\omega_c^4}{\omega^4}\sin^2\left\{\omega\sqrt{1+\omega_c^2/\omega^2}\; d/c\right\} + 4(1+\omega_c^2/\omega^2)} \qquad (8)$$

On the other hand, if the following tachyonic equations are tenable to the quanta $\omega < \omega_c$,

rest mass $m_0 = \hbar\omega_c / c^2$

velocity $V = c\sqrt{\omega_c^2/\omega^2 - 1}$

energy $E = \frac{m_0 c^2}{\sqrt{1+V^2/c^2}} = \hbar\omega$

momentum $p = \frac{m_0 V}{\sqrt{1+V^2/c^2}} = \frac{\hbar\omega}{c}\sqrt{\omega_c^2/\omega^2 - 1}$ (energy-momentum relation $E = \sqrt{-p^2c^2 + m_0{}^2c^4}$)

The coefficient is

$$T = \frac{4(\omega_c^2/\omega^2 - 1)}{(2-\omega_c^2/\omega^2)^2 \sin^2\left\{\omega\sqrt{\omega_c^2/\omega^2 - 1}\ d/c\right\} + 4(\omega_c^2/\omega^2 - 1)} \tag{9}$$

In addition,

rest mass $m_0 = \hbar\omega_c / c^2$

energy $E = \frac{m_0 c^2}{\sqrt{-1-V^2/c^2}} = \hbar\omega$

velocity $V = ic\sqrt{1+\omega_c^2/\omega^2}$

momentum $p = \frac{m_0 V}{\sqrt{-1-V^2/c^2}} = i\frac{\hbar\omega}{c}\sqrt{1+\omega_c^2/\omega^2}$ (energy-momentum relation $E = \sqrt{-p^2c^2 - m_0{}^2c^4}$)

$$T = \frac{4(1+\omega_c^2/\omega^2)}{(2+\omega_c^2/\omega^2) sh^2\left\{\omega\sqrt{1+\omega_c^2/\omega^2}\ d/c\right\} + 4(1+\omega_c^2/\omega^2)} \tag{10}$$

To make a comparison with the experimental result and (7)~(10) is helpful to judge whether speeds of electromagnetic quanta in the tunneling effect are faster than $c = 1/\sqrt{\varepsilon_0\mu_0}$ or not.

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