

**DECELERATION PARAMETER Q(Z) IN FIVE DIMENSIONAL
GEOMETRIES, AND TO WHAT DEGREE A PARTIAL RE
APPEARANCE OF QUINCESSENCE $\phi(t)$ MAY PLAY A ROLE IN AN
INCREASE IN COSMOLOGICAL ACCELERATION AT $Z \sim .423$**

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The case for a four dimensional graviton mass (non zero) influencing reacceleration of the universe in five dimensions is stated, with particular emphasis upon if five dimensional geometries as given below give us new physical insight as to cosmological evolution. One noticeable datum, that a calculated inflaton $\phi(t)$ may partly re-emerge after fading out in the aftermath of inflation. The inflaton may be a contributing factor to, with non zero graviton mass, in re acceleration of the universe a billion years ago. Many theorists assume that the inflaton is the source of entropy. The inflaton also may be the source of re acceleration of the universe, especially if the effects of a re emergent inflaton are in tandem with the appearance of macro effects of a small graviton mass, leading to a speed up of the rate of expansion of the universe one billion years ago, at red shit value of $Z \sim .423$

1 Introduction

1.1 What can be said about gravitational wave density value detection?

We will start with a first-principle introduction to detection of gravitational wave density using the definition given by Maggiore ¹

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}} \right] \cdot \left(\frac{f}{1kHz} \right)^4 \quad (1)$$

where n_f is the frequency-based numerical count of gravitons per unit phase space. The author suggests that n_f may also depend upon the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund *et al* ². Having said that, the question is, what sort of mechanism is appropriate for considering macro affects of gravitons, and the author thinks that he has one, i.e. reacceleration of the universe, as far as a function of graviton mass, i.e. what Beckwith³ did was to make the following presentation. Assume Snyder geometry and look at use of the following inequality for a change in the HUP, ⁴

$$\Delta x \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p \right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \quad (2)$$

and that the mass of the graviton is partly due to the stretching alluded to by Fuller and Kishimoto,⁵ a supposition the author³ is investigating for a modification of a joint KK tower of gravitons, as given by Maartens⁶ for DM. Assume the stretching of early relic neutrinos that would lead to the KK tower of gravitons--for when $\alpha < 0$, is⁴,

$$m_n(\text{Graviton}) = \frac{n}{L} + 10^{-65} \text{ grams} \quad (3)$$

Note that Rubakov⁷ writes KK graviton representation as, after using the following normalization $\int \frac{dz}{a(z)} \cdot [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$ where J_1, J_2, N_1, N_2 are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \quad (4)$$

This Eq. (4) is for KK gravitons having a TeV magnitude mass $M_z \sim k$ (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$ as an initial starting value for the KK graviton mass, before the KK graviton, as a 'massive' graviton moves with velocity H^{-1} along the RS dS brane. If so, and if $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass lower in value, i.e. of $m_{\text{graviton}}(4 - \text{Dim GR}) \sim 10^{-48} eV$, as opposed to $M_X \sim M_{\text{KK-Graviton}} \sim .5 \times 10^9 eV$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky et.al.⁸ who argue for graviton mass using CMBR measurements, of $M_{\text{KK-Graviton}} \sim 10^{-20} eV$ Dubosky et. al.⁸ results can be conflated with Alves et. al.⁹ arguing that non zero graviton mass may lead to an acceleration of our present universe, in a manner usually conflated with DE, i.e. their graviton mass would be about $m_{\text{graviton}}(4 - \text{Dim GR}) \sim 10^{-48} \times 10^{-5} eV \sim 10^{65}$ grams. Also assume that to calculate the deceleration, the following modification of the HUP is used: $[2] \Delta x \geq [(1/\Delta p) + l_s^2 \cdot \Delta p] \equiv (1/\Delta p) - \alpha \cdot \Delta p$, where the LQG condition is $\alpha > 0$, and brane worlds have, instead, $\alpha < 0$ ⁴. Also Eq. (5) will be the starting point used for a KK tower version of Eq. (6) below. So from Maarten's¹⁰ 2005 paper,

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \quad (5)$$

Maartens¹⁰ also gives a 2nd Friedman equation, as

$$\dot{H}^2 = \left[- \left(\frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2 \frac{m}{a^4} + \frac{K}{a^2} \right] \quad (6)$$

Also, if we are in the regime for which $\rho \cong -P$, for red shift values z between zero to 1.0-1.5 with exact equality, $\rho = -P$, for z between zero to .5. The net effect will be to obtain, due to Eq. (6), and use $a \equiv [a_0 = 1]/(1+z)$. As given by Beckwith³

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2 [\rho/m] \cdot (1+z)^4 \cdot (1 + \rho/2\lambda)} \approx -1 + \frac{2}{2 + \delta(z)} \quad (7)$$

Eq. (6) assumes $\Lambda = 0 = K$, and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass done with $\Lambda \neq 0$, even if curvature $\mathbf{K} = 0$

2 Consequences of small graviton mass for reacceleration of the universe

In a revision of Alves *et. al.*,⁹ Beckwith³ used a higher-dimensional model of the brane world and Marsden⁶ KK graviton towers. The density ρ of the brane world in the Friedman equation as used by Alves *et. al.*⁹ is use by Beckwith³ for a non-zero graviton

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G(\hbar=1)^2} \right] \cdot \left(\frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \quad (8)$$

I.e. Eq. (6) above is making a joint DM and DE model, with all of Eq. (6) being for KK gravitons and DM, and 10^{-65} grams being a 4 dimensional DE. Eq. (5) is part of a KK graviton presentation of DM/ DE dynamics. Beckwith¹¹ found at $z \sim .4$, a billion years ago, that acceleration of the universe increased, as shown in Fig. 1.

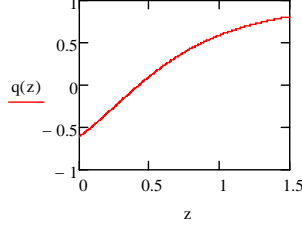


Fig. 1: Reacceleration of the universe based on Beckwith³ (note that $q < 0$ if $z < .423$)

3. Suggesting a non standard way to accommodate small graviton mass in 4 D

If one is adding the small mass of $m_n(\text{Graviton}) = \frac{n}{L} + 10^{-65}$ grams³, with $m_0(\text{Graviton}) \approx 10^{-65}$ grams, then the problem being worked with is a source term problem of the form given by Peskins¹¹ as of the type

$$\psi_n(x) \equiv \int d^3 p \cdot \frac{1}{(2\pi)^3} \cdot \frac{1}{\sqrt{2E_p}} \cdot \left\{ \left(a_p + \frac{i}{\sqrt{2E_p}} \cdot FT(m_0(\text{graviton})) \right) \exp(-ipx) + H.C. \right\} \quad (8)$$

This is, using the language V.A. Rubakov⁷ put up equivalent to^{3, 9},

$$\psi_m(x) \approx h_m(x) + \int d^3 p \cdot \frac{1}{(2\pi)^3} \cdot \left(\frac{1}{\sqrt{2E_p}} \right)^2 \cdot \{ (i \cdot FT(m_0(\text{graviton}))) \exp(-ipx) + H.C. \} \quad (9)$$

If $m_0(\text{graviton})$ is a constant, then the expression (9) has delta functions. This is the field theoretic identification. Another way is to consider an instanton-anti instanton treatment of individual gravitons, and to first start with the supposed stretch out of gravitons to enormous lengths. Assuming $m_0(\text{Graviton}) \approx 10^{-65}$ grams for gravitons in 4 dimensions, the supposition by Bashinsky¹² and Beckwith³ is that density fluctuations are influenced by a modification of cosmological density ρ in the Friedmann equations by the proportionality factor given by Bashinsky,¹² $\left[1 - 5 \cdot (\rho_{\text{neutrino}} / \rho) + \mathcal{G} \left((\rho_{\text{neutrino}} / \rho)^2 \right) \right]$ This proportionality factor for ρ as showing up in the Friedmann equations should be taken as an extension of results from Marklund *et al.*², due to graviton-neutrino interactions as proposed by Marklund *et al.*², where neutrinos interact with plasmons and plasmons interact with gravitons. Thereby

implying neutrino- graviton interactions Also, graviton wavelengths have the same order of magnitude of neutrinos. Note, from Valev,¹³

$$\begin{aligned} m_{graviton} \Big|_{RELATIVISTIC} &< 4.4 \times 10^{-22} h^{-1} eV / c^2 \\ \Leftrightarrow \lambda_{graviton} &\equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} \text{ meters} \end{aligned} \quad (10)$$

Extending M. Marklund *et al.*² and Valev¹³, some gravitons may become larger¹⁴, i.e.

$$\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 10^4 \text{ meters or larger} \quad (11)$$

A way to accommodate this wave length has been suggested by Beckwith,³ as to an instanton-anti instanton packaging of gravitons, was to start with an analogy between Giovannini,¹⁵ from a least action version of the Einstein – Hilbert action for ‘quadratic’ theories of gravity involving Euler- Gauss-Bonnet. Then Giovannini’s¹⁵ equation 6 corresponds to

$$\phi = \tilde{v} + \arctan((bw)^v) \quad (12)$$

Givannini¹⁵ represents of Eq. (12) as a kink, and makes references to an anti-kink solution, in Fig. 1 in Givannini¹⁵. Furthermore the similarity between Eq. (12) and

$$\phi_+(z, \tau) = 4 \cdot \arctan \left(\exp \left\{ \frac{z + \beta \cdot \tau}{\sqrt{1 - \beta^2}} \right\} \right)$$

in Beckwith’s^{3, 16} treatment with regards to density wave physics instantons is obvious. If $\arctan((bw)^v)$ is part of representing a graviton as

a kink-anti-kink combination, arising from a 5 dimensional line element,¹⁵

$$dS^2 = a(w) \cdot [\eta_{uv} dx^u dx^v - dw^2] \quad (13)$$

Then, noting as Beckwith³ mentioned, there is the possibility of using t’Hoofts¹⁷ classical embedding of “deterministic quantum mechanics” as a way to embed a nearly four dimensional graviton as having almost zero mass, in a larger non linear theory.

4. What if an inflaton partly re-emerges in space-time dynamics? At $\mathbf{z} \sim .423$?

Padmanabhan¹⁸ has written up how the 2nd Friedman equation as of Eq. (5), which for $\mathbf{Z} \sim .423$ may be simplified to read as

$$\dot{H}^2 \cong \left[-2 \frac{m}{a^4} \right] \quad (14)$$

would lead to an inflaton value of , when put in, for scale factor behavior as given by $a(t) \propto t^\lambda, \lambda = (1/2) - \varepsilon^+, 0 \leq \varepsilon^+ \ll 1$, of, for the inflaton¹⁸ and inflation of

$$\phi(t) = \int dt \cdot \sqrt{-\frac{\dot{H}}{4\pi G}} \quad (15)$$

Assuming a decline of $a(t) \propto t^\lambda, \lambda = (1/2) - \varepsilon^+, 0 \leq \varepsilon^+ \ll 1$, Eq. (15) yields

$$\phi(t) \sim \sqrt{\frac{2m}{4\pi G}} \cdot [2\varepsilon^+] \cdot t^{2\varepsilon^+} \quad (16)$$

As the scale factor of $a(t) \propto t^\lambda, \lambda = (1/2) - \varepsilon^+, 0 \leq \varepsilon^+ \ll 1$ had time of the value of roughly $a(t) \propto t^\lambda, \lambda = (1/2) - \varepsilon^+, 0 \leq \varepsilon^+ \ll 1$ have a power law relationship drop below $a(t) \propto t^{1/2}$, the inflaton took Eq. (16) 's value which may have been a factor as to the increase in the rate of acceleration, as noted by the q factor , given in Fig. 1. Note that there have been analytical work projects relating the inflaton, and its behavior to entropy via noting that inflation stopped when the inflaton field settled down into a lower lower energy state. The way to relate an energy state to the inflaton is , if $a(t) = a_0 t^\lambda$, then in the early universe, one has a potential energy term of¹⁹

$$V(\phi) = V_0 \cdot \exp \left[-\sqrt{\frac{16\pi G}{\lambda}} \cdot \phi(t) \right] \quad (17)$$

A situation where both $\lambda = (1/2) - \varepsilon^+$ grows smaller, and, temporarily, $\phi(t)$ takes on Eq. (16)'s value, even if the time value gets large, and also, if acceleration of the cosmic expansion is taken into account, then there is infusion of energy by an amount dV. The entropy $dS \cong dV/T$, will lead, if there is an increase in V, as given by Eq. (17) a situation where there is an effective increase in entropy. If there is, as will be related to later, in page eight, circumstances, where $S \approx N =$ number of graviton states^{3,18} as will be derived in Eq. (27), then at least in higher dimensions, we have an argument that the emergence of an inflaton, with a corresponding reduction of Eq. (17) in magnitude may be part of gravitons playing a role in the re acceleration of the universe.

5. Other than five dimensions for cosmology? Problems which need resolutions

If a way to obtain a graviton mass in four dimensions is done which fits in with the as given higher 5 dimensions specified by a slight modification of brane theory, or Maarten's cosmological evolution^{3,10} equations, what benefits could this approach accrue for other outstanding problems in cosmology? Beckwith³ claims that a re do of the Friedmann equations would result in deceleration parameter $q(z)$ similar to Fig. 1 above. Snyder geometry for the four dimensional case with would specify Friedmann equations along the lines of $\alpha > 0$ in Eq. (2) above. If one follows $\alpha < 0$, then the Friedmann equations appear as giving details to the following equation^{3,20}

$$\mathfrak{S} = -\frac{1}{2} \int d^4 x h_{uv}^0 \cdot T^{uv} \sim L^2 \approx \delta^+ \geq 0 \quad (18)$$

The construction done from sections 1 to 3 are for $\alpha < 0$. When $\alpha > 0$, the claim is that almost all the complexity is removed $\alpha > 0$, and what is left is a Taveras²⁰ treatment of the Friedmann equations, where he obtains, to first order, if ρ is a scalar field density,

$$\left(\frac{\dot{a}}{a}\right)^2 = [\kappa/3] \cdot \rho \quad (19)$$

and

$$\left(\frac{\ddot{a}}{a}\right) = -[2 \cdot k/3] \cdot \rho \quad (20)$$

The interpretation of ρ as a scalar field density²⁰, and if one does as Alves et al⁹ uses Eq. (7) above. We need to interpret the role of ρ . In the LQG version by²¹, Eq. (20) may be rewritten as follows: If conjugate momentum is in many cases, "almost" or actually a constant, using $\dot{\phi} = -[\hbar/i] \cdot [\partial/\partial p_\phi]$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv [\kappa/6] \cdot [p_\phi^2/a^6] \quad (21)$$

Beckwith¹¹ claims that the deceleration parameter $q(z)$ incorporating Eq. (19), Eq. (20) and Eq. (21) should give much the same behavior as Fig. 1 above. If so, then if one is differentiating between four and five dimensions by what is gained, in cosmology, one needs having it done via other criteria. The following is a real problem. As given by Maggiore¹, the massless equation of the graviton evolution equation takes the form

$$\partial_\mu \partial^\sigma h_{\mu\nu} = \sqrt{32\pi G} \cdot \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\mu_\mu \right) \quad (22)$$

When $m_{graviton} \neq 0$, the above becomes

$$\left(\partial_\mu \partial^\sigma - m_{graviton}\right) \cdot h_{\mu\nu} = \left[\sqrt{32\pi G} + \delta^+\right] \cdot \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T_\mu^\mu + \frac{\partial_\mu \partial_\nu T_\mu^\mu}{3m_{graviton}}\right) \quad (23)$$

The mismatch between these two equations, when $m_{graviton} \rightarrow 0$, is due to $m_{graviton} h_\mu^\mu \neq 0$ as $m_{graviton} \rightarrow 0$, which is due to setting a value of $m_{graviton} \cdot h_\mu^\mu = -\left[\sqrt{32\pi G} + \delta^+\right] \cdot T_\mu^\mu$. The semi classical method by t'Hooft¹⁷, using Eq. (12) is the solution. We generalize to higher dimensions the following diagram as given by Beckwith³. Use an instanton- anti instanton structure, and t'Hooft¹⁷ equivalence classes along the lines of Eq. (24) below with equivalence class structure in the below wave functional to be set by a family of admissible values³ $\phi_0(x)$

$$\Psi_{i,f} [\phi(\mathbf{x})]_{\phi=\phi_{ci,f}} = c_{i,f} \cdot \exp\left\{-\int d\mathbf{x} \alpha \left[\phi_{ci,f}(\mathbf{x}) - \phi_0(\mathbf{x})\right]^2\right\} \quad (24)$$

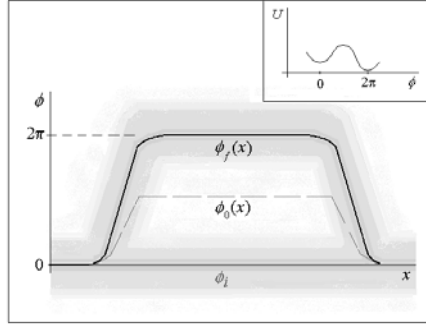


Fig. 2: The pop up effects of an instanton-anti-instanton in Euclidian space^{3,17}

6. Conclusion. Examining information exchange between different universes?

Beckwith³ has concluded that the only way to give an advantage to higher dimensions as far as cosmology would be to look at if a fifth dimension may present a way of actual information exchange to give the following parameter input from a prior to a present universe, i.e. the fine structure constant, as given by³

$$\tilde{\alpha} \equiv e^2/\hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc} \quad (25)$$

The wave length as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be put below, namely:

Argue that when taking the log, that the 1/N term drops out. As used by Ng¹⁷

$$Z_N \sim (1/N!) \cdot (V/\lambda^3)^N \quad (26)$$

This, according to Ng,¹⁸ leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith³

$$S \approx N \cdot (\log[V/\lambda^3] + 5/2) \approx N \quad (27)$$

Eventually, the author hopes to put on a sound foundation what 'tHooft¹⁷ is doing with respect to t'Hooft¹⁷ deterministic quantum mechanics and equivalence classes embedding quantum particle structures.. Doing so will answer the questions Kay²² raised about particle creation, and the limitations of the particle concept in curved and flat space, i.e. the global hyperbolic space time which is flat everywhere expect in a localized "bump" of curvature. Furthermore, making a count of gravitons with $S \approx N \sim 10^{20}$ gravitons^{3,17}, with $I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} \sim 10^{20}$ as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced graviton³. What the author, Beckwith, sees is that since instanton- anti instanton pairs do not have to travel slowly, as has been proved by authors in the 1980s, that gravitons if nucleated in a fashion as indicated by Fig. 2, may be able to answer the following. The stretch-out of a graviton wave, greater than the size of the solar system, gives, an upper limit of a graviton mass due to wave length $\lambda_{graviton} > 300 \cdot h_0 kpc \Leftrightarrow m_{graviton} < 2 \times 10^{-29} h_0^{-1} eV$. I. e. stretched graviton wave, at ultra-low frequency, may lead to a low mass limit. However, more careful limits due to experimental searches, as presented by Buonanno²² have narrowed the upper limit to $10^{-20} h_0^{-1} eV$. An instanton – anti instanton structure to the graviton, if confirmed, plus experimental confirmation of mass, plus perhaps $n \sim 10^{20}$ gravitons $\approx 10^{20}$ entropy counts, Eq. (23) implies up to $\approx 10^{27}$ operations. If so, there is a one-to-one relationship

between an operation and a bit of information, so a graviton has at least one bit of information. And that may be enough to determine the conditions needed to determine if Eq. (21) gives information and structure from a prior universe to our present cosmos. **Finally, the datum referred to in Eq. (14) to Eq. (17) as combined with $S \approx N$ as referenced on pages 5 and 6 as a way to relate the graviton count with entropy may be a way to make inter connection between the inflaton picture of entropy generation and entropy connected/ generated with a numerical count of gravitons. This datum needs experimental confirmation and may be important to astro physics linkage of DE with DM, in the future. Eq. (14) to Eq. (17) if confirmed for $Z \sim .423$ may prove, in part, that higher dimensions are necessary for cosmology.**

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