Hubble Redshift due to the Global Non-Holonomity of Space

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Abstract: In General Relativity, the change in energy of a freely moving photon is given by the scalar equation of the isotropic geodesic equations, which manifests the work produced on a photon being moved along a path. I solved the equation in terms of physical observables (Zelmanov A. L., *Soviet Physics Doklady*, 1956, vol. 1, 227–230) and in the large scale approximation, i.e. with gravitation and deformation neglected, while supposing the isotropic space to be globally non-holonomic (the time lines are non-orthogonal to the spatial section, a condition manifested by the rotation of the space). The solution is $E = E_0 \exp(-\Omega^2 a\,t/c)$, where Ω is the angular velocity of the space (it meets the Hubble constant $H_0 = c/a = 2.3\times10^{-18}$ sec^{-1}), a is the radius of the Universe, $t = r/c$ is the time of the photon's travel. Thus, a photon loses energy with distance due to the work against the field of the space non-holonomity. According to the solution, the redshift should be $z = \exp(H_0\,r/c) - 1 \approx H_0\,r/c$. This solution explains both the redshift $z = H_0\,r/c$ observed at small distances and the non-linearity of the empirical Hubble law due to the exponent (at large r). The ultimate redshift in a non-expanding universe, according to the theory, should be $z = \exp(\pi) - 1 = 22.14$.

Contents:

§1. Hubble redshift in a static universe. In this short presentation, I show how the Hubble law, including its non-linearity with distance, can be deduced directly from the equations of the General Theory of Relativity. The Hubble law I have deduced is present in a non-expanding universe. It is also present, in a slightly different form, in an expanding universe and a compressing universe.

In General Relativity, the change of energy of a freely moving photon should be the solution to the scalar equation of isotropic geodesics, which is also known as the equation of energy and manifests the work produced on the photon being moved along the path. In terms of physically observable quantities — chronometric invariants (Zelmanov, 1944), which are the respective projections of four-dimensional quantities onto the time line and spatial section of a given observer — the isotropic geodesic equations are presented with two projections onto the time line and spatial section, respectively [1–3]

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2} F_i c^i + \frac{\omega}{c^2} D_{ik} c^i c^k = 0\,, \tag{1.1}$$

$$\frac{d(\omega c^i)}{d\tau} - \omega F^i + 2\omega \left(D^i_k + A^{\cdot i}_{k\cdot}\right) c^k + \omega \triangle^i_{nk} c^n c^k = 0\,, \tag{1.2}$$

where ω is the proper frequency of the photon, $d\tau$ is the interval of physically observable time, c^i is the chr.inv.-vector of the observable velocity of light ($c_k c^k = c^2$). The physically observable properties of space are presented with the chr.inv.-vector F_i of the gravitational inertial force, the chr.inv.-tensor A_{ik} of the angular velocity of the rotation of space due to its non-holonomity (the non-orthogonality of the time lines to the spatial section, which is expressed as $g_{0i} \neq 0$, and is manifested as the three-dimensional rotation of space), the chr.inv.-tensor D_{ik} of the deformation of space (shows how space deforms with time), and the chr.inv.-Christoffel symbols $\triangle^i_{nk}$ (indicate the non-uniformity of space). All these three-dimensional quantities bear the property of chronometric invariance (i.e. they are invariant in the spatial section of the observer) and are dependent on the gravitational potential $\mathrm{w} = c^2 (1 - \sqrt{g_{00}})$, on the linear velocity $v_i = -\frac{c g_{0i}}{\sqrt{g_{00}}}$ of the rotation of space due to its non-holonomity, and also on the chr.inv.-metric tensor $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$, which characterize the time line and spatial section of the observer.

Integration of the scalar equation of isotropic geodesics (the equation of energy) should give a function $E = E(t)$, where $E = \hbar\omega$ is the proper energy of the photon. However, integration of time in a Riemannian space is not a trivial task. This is because the observable interval of time $d\tau = \sqrt{g_{00}}\, dt - \frac{1}{c^2} v_i dx^i$ depends on the gravitational potential w along the path, on the linear velocity v_i of the rotation of space (due to the non-holonomity of it), and on the displacement dx^i of the observer with respect to his coordinate net during the measurement. The result of integration depends on the integration path, so time is not integrable in a general case. We therefore consider the "large scale approximation", where distances are close to the curvature radius of the Universe; so gravitation and deformation are neglected in the space ($g_{00} = 1$ and $D_{ik} = 0$, respectively), and the observer is resting with respect to his coordinate net ($dx^i = 0$). In such a case, integration of time is allowed, and is simply $d\tau = dt$. We also suppose the isotropic space, the "home space" of isotropic (light-like) trajectories and massless light-like particles (e.g. photons), to be globally non-holonomic ($v_i \neq 0$). With these assumptions, the formula for the gravitational inertial force F_i [1–3], losing the gravitational potential which becomes $\mathrm{w} = c^2 (1 - \sqrt{g_{00}}) = 0$, consists of only the second term

$$F_i = \frac{1}{\sqrt{g_{00}}} \left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t} \right) \simeq -\frac{\partial v_i}{\partial t} \,, \tag{1.3}$$

which is due to the space non-holonomity. This negative (centrifugal) acceleration, experienced by such a photon in the isotropic space, is the solely factor which is still acting on the energy of the photon in the scalar equation of isotropic geodesics in the framework of the "large scale approximation" in a globally non-holonomic isotropic space. It acts on a photon due to the motion (global rotation) of the isotropic space itself.

It should be noted that, despite the apparent similarity to the centrifugal force of inertia, this factor is not related to the fictitious forces of inertia. The forces of inertia are observed in a rotating coordinate frame, and are due to the transformation of the coordinates and time which include the angular velocity of the coordinate frame (these were considered in 1909 by Max Born, and are known as the Born coordinates). As a result, the space-time metric being written in the Born coordinates gets additional terms in g_{00} and g_{0i}. The additional terms vanish, in common with the forces of inertia produced due to the terms, by the transformation of the coordinates back to another, non-rotating frame. In contrast, the factor we are considering is due to the basic non-holonomity of space, which is only $g_{0i} \neq 0$, and is invariant in the spatial section of the observer (i.e. this is a chronometrically invariant effect), and cannot therefore be removed by the transformation from one coordinate frame to another one in the spatial section.

Of course, one can derive the inertial force effects in General Relativity, when moving to a rotating coordinate frame (the Born coordinates). These will, however, be only the removable (fictitious) effects, observed on the background of the gravitational potential, the non-holonomity, the deformation, the inhomogeneity, and the curvature of space, whose effects cannot be removed by our choice of the coordinate frame in the spatial section of the observer due to the invariance of the effects in the spatial section.

We consider a single photon travelling in the x-direction. In this case, $c^1 = c$, $c^2 = 0$, $c^3 = 0$. With the "large scale approximation" in a globally non-holonomic isotropic space, and assuming the linear velocity of the space rotation to be $v_1 = v_2 = v_3 = v$ and stationary, i.e. $\frac{\partial v}{\partial t} = B = const$, the scalar equation of isotropic geodesics for such a photon takes the form

$$\frac{dE}{dt} = -\frac{B}{c}\,E\,. \tag{1.4}$$

This is a simple uniform differential equation of the 1st order, like $\dot{y} = -\,ky$, so that we have $\frac{dy}{y} = -k\,dt$ or $d(\ln y) = -k\,dt$. It solves as $\ln y = -kt + \ln C$, where C is the integration constant which can be evaluated when the initial conditions of integration ($y = y_0$, $t_0 = 0$) are substituted. Finally, we obtain $y = y_0\,e^{-kt}$. As a result, the scalar equation of isotropic geodesics (the equation of energy), in the "large scale approximation" in the globally non-holonomic space, gives the solution for the photon's energy (frequency) and the redshift $z = \frac{\omega_0 - \omega}{\omega}$ as depending on the distance $r = ct$ travelled from the observer

$$E = E_0\,e^{-kt}, \qquad z = e^{kt} - 1\,, \tag{1.5}$$

such that at small distances of the photon's travel, i.e. with the exponent $e^x = 1 + x + \frac{1}{2}\,x^2 + \ldots \simeq 1 + x$, it takes the form

$$E \simeq E_0\,(1 - kt)\,, \qquad z \simeq kt\,, \tag{1.6}$$

where $k = \frac{1}{c}\,B = \frac{1}{c}\,\frac{\partial v}{\partial t} = const$. Thus, according to our calculation, which is based on the equations of the General Theory of Relativity, a photon being moved in a non-holonomic space loses its proper energy and frequency due to the work produced by it against the field of the space non-holonomity (or, in other words, the negative work produced by the field on the photon).

We suppose the space (space-time) of our Metagalaxy to be a spherical geometry space, which has a constant curvature and is globally non-holonomic. A constant curvature spherical space, whose metric is sign-definite, is a hypersphere of constant radius (the curvature radius of the space). However, we are considering a four-dimensional spherical space with a sign-alternating metric $(+---)$ or $(-+++)$, which indicates the presence of the special coordinate axis known as time among the four coordinate axes of the space. The sign-alternating metric indicates, in particular, that such a space consists of two subspaces, which are known as the non-isotropic space (the home of non-isotropic trajectories which are the trajectories of mass-bearing particles) and the isotropic space (the home of isotropic trajectories which are the trajectories of massless light-like particles, e.g. photons). We know that, given a point, only one geodesic line can be paved through it in a given direction, and such a unique geodesic line can be either non-isotropic or isotropic (see [4, §6] or [5, §101]). In other words, non-isotropic and isotropic geodesics have no common points. Therefore, the spherical space with the sign-alternating metric we are considering is presented with two concentric hyperspheres — the home of non-isotropic trajectories and that for isotropic ones — which have the same radius of curvature, but are not coinciding with each other.

The constant radius of such a hypersphere manifests that the curvature radius of the space of our Metagalaxy remains unchanged, so the Metagalaxy as a whole does not expand or compress in the framework of this model. Meanwhile, a local volume (a local element of the hypersphere's surface) may experience any stages of the evolution, which could be conceivable in the framework of Zelmanov's theory of a locally inhomogeneous anisotropic universe [2, 3], including the special states of infinite density and infinite rarefraction, if it doesn't change the stationary state of the space (the hypersphere's surface) as a whole.

The non-holonomity of the four-dimensional (non-isotropic or isotropic) space is the basic non-orthogonality of the time lines to the spatial axes on the (non-isotropic or isotropic) hypersphere's surface, and is manifested by its three-dimensional rotation.*

According to the concepts of topology [6, vol. 1], the surface of an $(n+1)$-dimensional sphere is equivalent to the volume of an n-dimensional torus. Thus, the globally non-holonomic spherical space we are considering is representable also with a torus, the home of non-isotropic trajectories and mass-bearing particles, which is coaxial to another torus, the home of isotropic trajectories and massless light-like particles, but is not coinciding with the first.

It is obvious that since the non-holonomity of such a space must be stationary, we can express the acceleration experienced by a photon in the isotropic space due to its non-holonomity, through the angular velocity Ω of the rotation of the isotropic hypersphere and its curvature radius, $a = \frac{c}{H_0}$, which is the same that the curvature radius of our Metagalaxy (H_0 is the Hubble constant). We obtain $\frac{\partial v}{\partial t} = \Omega^2 a = const$. In such a space, the coefficient $k = \frac{1}{c}\frac{\partial v}{\partial t}$ in the solution (1.5) we have

*The non-orthogonality of the time lines to the spatial section is impossible to be in a sign-definite metric space due to the absence of the special coordinate axis known as time. Therefore, all that has been said about holonomic and non-holonomic spaces is valid only for sign-alternating metric spaces such as pseudo-Riemannian spaces (for instance, the four-dimensional pseudo-Riemannian space with the signature $(+---)$ or $(-+++)$, which is the basic space-time of the General Theory of Relativity).

obtained to the scalar equation of isotropic geodesics is

$$k = \frac{1}{c}\,\Omega^2 a = const. \tag{1.7}$$

Then, according to the redshift formula $z \simeq kt$ obtained in the framework of our theory, for the galaxies located at a "small" distance of $r \simeq 630$ Mpc* (the redshift observed on them is $z \simeq 0.16$) we obtain

$$\Omega = \sqrt{\frac{z\,c}{a\,t}} = \sqrt{\frac{z\,c^2}{a\,r}} \simeq 2.4\times10^{-18}\ \text{sec}^{-1}, \tag{1.8}$$

that meets the Hubble constant, which is $H_0 = 72\pm8\times10^5$ cm/sec$\times$Mpc$=$ $= 2.3\pm0.3\times10^{-18}$ sec^{-1} (this is according to the Hubble Space Telescope data, 2001 [7]).

With these we arrive at the following law

$$E = E_0\, e^{-\frac{H_0 r}{c}}, \qquad z = e^{\frac{H_0 r}{c}} - 1\,, \tag{1.9}$$

as a purely theoretical result obtained from our solution to the scalar equation of isotropic geodesics. At small distances of the photon's travel, this law becomes

$$E \simeq E_0\left(1 - \frac{H_0\, r}{c}\right), \qquad z \simeq \frac{H_0\, r}{c}\,. \tag{1.10}$$

As seen, this result provides a complete theoretical ground to the linear Hubble law, empirically obtained by Edwin Hubble for small distances, and also to the non-linearity of the Hubble law observed at large distances close to the size of the Metagalaxy (the non-linearity is explained due to the exponent in our exact solution (1.9), which is becoming a sufficient factor at large r).

Then, proceeding from our solution, we are able to calculate the ultimate redshift, which is allowed in our Universe. It is, according to the exponential law (1.9),

$$z_{\max} = e^{\pi} - 1 = 22.14\,. \tag{1.11}$$

Proceeding from the theoretical considerations presented here, we calculate the linear velocity of the rotation of the isotropic space, which is due to the global non-holonomity of it. It is $\breve{v} = \Omega\, a = H_0\, a = c$, i.e. is equal to the velocity of light. I should note, to avoid misunderstanding, that this linear velocity of rotation is attributed to the isotropic

*1 parsec = 3.0857×10^{18} cm $\simeq 3.1\times10^{18}$ cm.

space, which is the home of isotropic (light-like) trajectories specific to massless light-like particles (e.g. photons). It isn't related to the non-isotropic space of sub-light-speed trajectories, which is the home of mass-bearing particles (e.g. galaxies, stars, planets). In other words, our result doesn't mean that the visible three-dimensional space of cosmic bodies rotates at the velocity of light. The space of galaxies, stars, and planets may be non-holonomic or not, depending on the physical conditions in it.

It is possible to show, by the mathematical methods of orthometric invariants [8] which allow calculation for physically observable quantities in any reference frame of the four-dimensional pseudo-Riemannian space (the basic space-time of the General Theory of Relativity), that the basic non-holonomity of the isotropic space is such that it rotates as a whole with the linear velocity equal to the velocity of light. So, our result concerning the linear velocity of the rotation of the isotropic space meets the basics of geometry of pseudo-Riemannian spaces.

In addition, it should be noted that, according to the theory of chronometric invariants, given the isotropic space rotating at the velocity of light, the observable three-dimensional metric h_{ik} of the space is non-degenerate ($h = \det \| h_{ik} \| \neq 0$). Thus, the four-dimensional metric $g_{\alpha\beta}$ is non-degenerate as well ($g = -h\, g_{00} \neq 0$, where $g = \det \| g_{\alpha\beta} \| \neq 0$). This means that the rotation of the isotropic space at the velocity of light does not lead to a singulary break in it.

§2. The rôle of deformation. The exponential redshift law (1.9) and its linear approximation (1.10) were deduced for a static universe, which does not experience expansion or compression, so its space remains non-deforming. Now, we study how the redshift law does change its formulation in a universe which expands or compresses.

The redshift law (1.9) was obtained as a result of integrating the scalar geodesic equation (1.1). According to the equation, the deformation of space is the second factor which, in addition to the gravitational inertial force, changes energy of a freely moving photon. No other factors are manifested. Space deforms while the universe expands or compresses. Thus, integrating the scalar geodesic equation in a non-static universe, we should take the factor of deformation into account.

The chr.inv.-tensor D_{ik} of the deformation of space is formulated [1–3] as the derivative of the chr.inv.-metric tensor h_{ik} by time

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \qquad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,, \tag{2.1}$$

where the tensor's trace (its physical meaning is the volume deformation of space) is

$$D = h^{ik} D_{ik} = \frac{{}^*\partial \ln\sqrt{h}}{\partial t} = \frac{1}{V}\frac{{}^*\partial V}{\partial t}\,, \tag{2.2}$$

where $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$, $h = \det\|h_{ik}\|$, $dV = \sqrt{h}\;dx^1 dx^2 dx^3$ is a differential increment of the volume V of space, while the components of the chr.inv.-metric tensor by definition [1–3] are

$$h_{ik} = -\,g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \quad h^{ik} = -\,g^{ik}, \quad h^i_k = -\,g^i_k = \delta^i_k\,. \tag{2.3}$$

We will consider the redshift law in universes of two kinds, according to two simplest types of deformation*.

First, we will consider the redshift law in a *constant deformation* universe. This means that the volume of space undergoes equal relative changes with time†, so the deformation of space remains constant‡

$$D = \frac{1}{V}\frac{{}^*\partial V}{\partial t} = const \quad\Longrightarrow\quad D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t} = const. \tag{2.4}$$

Deformation of this kind means increase of the linear velocity of the expansion of space in an expanding universe, and decrease of the linear velocity of the compression in a compressing universe. This can be illustrated by calculation of a volume. In the three-dimensional Euclidean space, the volume of a parallelepiped built on the vectors $r^i_{(1)}$, $r^i_{(2)}$, $r^i_{(3)}$ is calculated as $V = \pm\det\|r^i_{(n)}\| = \pm|r^i_{(n)}|$. We obtain the invariant $V^2 = |r^i_{(n)}||r_{(m)i}| = |r^i_{(n)}||h_{ik}\,r^k_{(m)}| = |h_{ik}\,r^i_{(n)}r^k_{(m)}|$. (It should be noted that

*The chr.inv.-quantity D_{ik} takes all changes of the space volume into account. For instance, in a static non-holonomic universe ($v_i \neq 0$), space deforms by its rotation. This is manifested by the derivative from the second term of the chr.inv.-metric tensor h_{ik} (2.3). Meanwhile the coordinate three-dimensional metric g_{ik} changes due to the rotation so that the resulting deformation of space is zero

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t} = 0 \quad\Longleftrightarrow\quad c^2\frac{\partial g_{ik}}{\partial t} = v_i\frac{\partial v_k}{\partial t} + v_k\frac{\partial v_i}{\partial t}\,.$$

In a static holonomic universe ($v_i = 0$), the condition $D_{ik} = 0$ is realized by the conditions $g_{ik} = const$ and $v_i = 0$.

†I refer to this kind of universes as *homotachydioncotic* (ομοταχυδιογκωτικό). This terms originates in *homotachydioncosis* — ομοταχυδιόγκωσης — volume expansion with a constant speed, from όμο which is the first part of όμοιος (omeos) — the same, ταχύτητα — speed, διόγκωση — volume expansion, while compression can be considered as negative expansion.

‡The stationarity of an invariant metric, such as $g_{\alpha\beta}$ or h_{ik}, leads to the stationarity of its determinant, and vice versa. For instance, in the case under consideration, $h_{ik} = const \Longleftrightarrow h = \det\|\,h_{ik}\| = const$.

$h_{ik} \equiv -g_{ik}$ in an Euclidean space.) Concerning a differentially small volume, the invariant is $(dV)^2 = |h_{ik}\, dx^i_{(n)} dx^k_{(m)}| = |h_{ik}||dx^i_{(n)}||dx^k_{(m)}| = h\,|dx^i_{(n)}||dx^k_{(m)}|$. Thus, $dV = \sqrt{h}\,|dx^i_{(n)}|$. Expanding this method onto an n-dimensional pseudo-Riemannian space, we obtain $dV = \sqrt{-g}\,|dx^\alpha_{(\nu)}|$. In particular, a three-dimensional differentially small volume in the four-dimensional space-time of General Relativity is $dV = \sqrt{h}\,|dx^i_{(n)}|$, or, if the basic vectors of the parallelepiped meet the spatial coordinate axes, $dV = \sqrt{h}\, dx^1 dx^2 dx^3$.

The volume of a finite space comes with the integration of dV, wherein the differential lengths dx^i, and also the scale of x^i we integrate, do not depend on time (integration with respect to the spatial coordinates is instant). Thus, we obtain

$$D = \frac{{}^*\partial \ln\sqrt{h}}{\partial t} = \frac{1}{\sqrt{h}}\frac{{}^*\partial \sqrt{h}}{\partial t} = \frac{1}{V}\frac{{}^*\partial V}{\partial t} = \gamma\,\frac{1}{a}\frac{{}^*\partial a}{\partial t} = \gamma\,\frac{\mathrm{v}}{a}\,, \tag{2.5}$$

where $V \sim a^3$ as for any three-dimensional volume, a is the radius of the universe (equal to the curvature radius in a constant curvature space), $\mathrm{v} = \pm|\mathrm{v}|$ is the linear velocity of the expansion or compression of space (positive in an expanding universe and negative in a compressing universe), and γ-factor is a constant numerical coefficient which is specific to the shape of space ($\gamma = 3$ in the homogeneous isotropic models [2,3]). As seen from this formula under $D = const$, in a constant deformation expanding universe, the linear velocity of its expansion increases with the growing radius of space (this means accelerated expansion of the universe). In contrast, in a constant deformation compressing universe, the linear velocity of its compression decreases with the shrinking radius of space (decelerated compression).

Second, we will consider the redshift law in a *constant speed deforming* universe*, i.e. in a universe which expands or compresses with a constant linear velocity $\mathrm{v} = \frac{{}^*\partial a}{\partial t} = const$. In a universe of this kind, the radius of space changes linearly with time $a = a_0 \pm \mathrm{v}\,t$ (here the upper sign is attributed to the expansion of space, while the lower sign characterizes the compression), while the deformation of space (2.5) is

$$D = \gamma\,\frac{1}{a_0 \pm \mathrm{v}\,t}\frac{{}^*\partial a}{\partial t} \simeq \gamma\,\frac{1}{a_0}\left(1 \mp \frac{\mathrm{v}\,t}{a_0}\right)\frac{{}^*\partial a}{\partial t} \simeq \gamma\,\frac{\mathrm{v}}{a_0} \mp \gamma\,\frac{\mathrm{v}^2 t}{a_0^2}\,, \tag{2.6}$$

*I refer to this kind of universes as *homotachydiastolic* (ομοταχυδιαστολικός). It's origin is *homotachydiastoli* — ομοταχυδιαστολή — linear expansion with a constant speed, from όμο which is the first part of όμοιος — the same, ταχύτητα — speed, and διαστολή — linear expansion (compression can be considered as negative expansion).

and is a linear function of time: $D = D_0 \mp \mu t$, $\mu = const$. Thus, in a constant speed expanding universe, the deformation decreases with time, while it grows with time in a constant speed compressing universe

$$D \simeq D_0 - \gamma \frac{\mathrm{v}^2 t}{a_0^2} \quad \text{in the case of expansion,} \tag{2.7}$$

$$D \simeq D_0 + \gamma \frac{\mathrm{v}^2 t}{a_0^2} \quad \text{in the case of compression,} \tag{2.8}$$

where $D_0 \simeq \gamma \frac{\mathrm{v}}{a_0}$ is the deformation of space and a_0 is the radius of the universe at the start of measurement.

It should be noted that all that has been said here about the deformation of space is valid to both a finite and an infinite universe. This is because, according to the theory of an inhomogeneous anisotropic universe (Zelmanov, 1944 [2,3]), not only a whole universe can be a subject of evolution, but also any volume element of it, including even differentially small volume elements.

§3. Redshift in a constant deformation universe. In a universe of this kind, $D = const$ and $D_{ik} = const$. We neglect gravitation ($g_{00} = 1$), i.e. the gravitational potential is $\mathrm{w} = c^2 (1 - \sqrt{g_{00}}) = 0$ as in the "large scale approximation". As in our consideration of a static non-holonomic universe, we consider a single photon travelling in the x-direction (in this case, $c^1 = c$, $c^2 = 0$, $c^3 = 0$) and the linear velocity of the space rotation to be $v_1 = v_2 = v_3 = v$. However, v is not stationary in this case. (It is stationary only in a static universe, because it does not change its volume during the rotation.)

We consider the function $v = v(t)$. The relation $\frac{\partial v}{\partial t} = \Omega^2 a$ is obvious in a spherical space. The conservation of angular momentum of the universe therefore means that $\Omega a^2 = const$. These relations lead to $\frac{\partial v}{\partial t} = \frac{\Omega^2 a^4}{a^3} = \frac{\chi}{V}$, where $\chi = \sigma \Omega^2 a^4 = const$. Here σ is a constant structural coefficient specific to the shape of space so that the volume of space is expressed as $V = \sigma a^3$ at any stage of the evolution of the universe (we assume that space does not change its shape being homogeneous expanding or compressing). The constant deformation condition $D = \frac{1}{V} \frac{\partial V}{\partial t} = \frac{\partial \ln V}{\partial t} = A = const$ (here $\frac{{}^*\partial}{\partial t} = \frac{\partial}{\partial t}$ because no gravitation) gives $\ln V = At + \ln C$. Thus, $V = V_0 \, e^{At}$, where $A = \gamma \frac{\mathrm{v}}{a}$ according to (2.5). Thus we obtain

$$\frac{\partial v}{\partial t} = \frac{\chi}{V_0} \, e^{-At}. \tag{3.1}$$

With these, we adopt the scalar equation of isotropic geodesics (1.1) to a photon travelling in a constant deformation universe. We obtain

$$\frac{dE}{dt} = -\left(\frac{\chi}{cV_0}\, e^{-At} + D_{11}\right) E\,, \tag{3.2}$$

or $\dot{y} = -\,k\,y$, where $k = \frac{\chi}{c\,V_0}\, e^{-At} + D_{11}$.

This is a simple uniform differential equation of the same kind as the equation of isotropic geodesics (1.4) we deduced for a static (non-deforming) non-holonomic universe with the only difference that being $k = \frac{\chi}{c\,V_0}\, e^{-At} + D_{11}$. Expanding the contants A and χ, and taking into account that $\Omega = H_0$, $a = \frac{c}{H_0}$, $t = \frac{r}{c}$ (H_0 is the Hubble constant, r is the distance of the photon's travel), and that $D_{11} = D = A$ in the case under consideration, we obtain

$$k = H_0\left(e^{-\gamma\,\frac{H_0\, r\, \mathrm{v}}{c^2}} + \gamma\,\frac{\mathrm{v}}{c}\right), \tag{3.3}$$

where the linear velocity of the expansion or compression of space is $\mathrm{v} = \pm|\mathrm{v}|$, becoming positive in an expanding universe and negative in a compressing universe.

The equation (3.2) can be solved in the same way as (1.4). The solution will have only k according to the formula (3.3) instead of $k = H_0$ from the solution (1.9) we have obtained in a static (non-deforming) universe.

Thus (3.2) solves as

$$E = E_0\, e^{-\frac{H_0 r}{c}\left(e^{-\gamma\,\frac{H_0 r|\mathrm{v}|}{c^2}} + \gamma\,\frac{|\mathrm{v}|}{c}\right)} \quad \text{in an expanding universe,} \tag{3.4}$$

$$E = E_0\, e^{-\frac{H_0 r}{c}\left(e^{\gamma\,\frac{H_0 r|\mathrm{v}|}{c^2}} - \gamma\,\frac{|\mathrm{v}|}{c}\right)} \quad \text{in a compressing universe.} \tag{3.5}$$

The redshift in a deforming non-holonomic universe (in the "large scale approximation", where gravitation is neglected) arrives with the sum of two terms. First, the redshift due to the non-holonomity of space, which is resulted from the solution to a photon's scalar equation of motion. Second, the relativistic Doppler redshift, which is an effect of the photon's motion with respect to the observer. Thus, with the obtained solutions (3.4) and (3.5), we obtain the redshift law in a constant deformation non-holonomic universe in the cases of the expansion and compression, respectively

$$z = \left(e^{\frac{H_0 r}{c}\left(e^{-\gamma\frac{H_0 r|\mathrm{v}|}{c^2}} + \gamma\frac{|\mathrm{v}|}{c}\right)} - 1\right) + \left(\frac{1+\frac{|\mathrm{v}|}{c}}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} - 1\right), \qquad (3.6)$$

$$z = \left(e^{\frac{H_0 r}{c}\left(e^{\gamma\frac{H_0 r|\mathrm{v}|}{c^2}} - \gamma\frac{|\mathrm{v}|}{c}\right)} - 1\right) + \left(\frac{1-\frac{|\mathrm{v}|}{c}}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} - 1\right), \qquad (3.7)$$

where the main goal at sub-relativistic velocities* is due to the first term (a result of the non-holonomity of space), while the numerical value of the second (Doppler-effect) term is much less and consequently plays an auxiliary rôle in the redshift law.

At small distances of the photon's travel and sub-relativistic velocities of the expansion or compression, the redshift law (3.6) and (3.7) takes the linear approximation form

$$z \simeq \frac{H_0 r}{c}\left[1 - \gamma\frac{|\mathrm{v}|}{c}\left(\frac{H_0 r}{c} - 1\right)\right] + \frac{|\mathrm{v}|}{c} \quad \text{in an expanding universe,} \qquad (3.8)$$

$$z \simeq \frac{H_0 r}{c}\left[1 + \gamma\frac{|\mathrm{v}|}{c}\left(\frac{H_0 r}{c} - 1\right)\right] - \frac{|\mathrm{v}|}{c} \quad \text{in a compressing universe.} \qquad (3.9)$$

What is curious in the obtained law is that it will be blueshifted ($z < 0$) at only small distances $r \ll a$ in a compressing universe. This is because the first (exponential) term will be positive in any case due to the exponent. At large distances, the first (always positive) term in the law (3.7) is much bigger than the second (Doppler-effect) negative term. For instance, let a photon travel at a distance r equal to the curvature radius of space $a = \frac{c}{H_0} \simeq 1.3\times10^{28}$ cm $\approx 4\times10^9$ parsec, while the universe compresses with a linear velocity of 100,000 km/sec. We assume also the shape-factor of space $\gamma = 3$ as for the inhomogeneous isotropic models [2, 3], but this is not principal in the calculation (the numerical value of γ depends weakly from the space of space). In this case, the first term in the redshift law (3.7) is $z_1 = +4.6$, while the second term (the relativistic Doppler blueshift) is $z_2 = -0.29$. If the universe compresses with a velocity of 10,000 km/sec, for a photon at the same distance $r = a$, we

*It is unbelievable that a universe expands or compresses with a velocity close to the velocity of light. On the other hard, such "ultimate cases" of ultra-relativistic expansion or compression would be interested from purely theoretical viewpoint.

obtain $z_1 = +1.7$ and $z_2 = -0.033$. In contrast, at small distances $r \ll a$, the first term approaches to zero, while the second (Doppler-effect blueshift) term becomes valuable. For instance, if the universe compresses at 300 km/sec, at a short distance of 10^6 parsec (the Andromeda Galaxy is located at a distance of $\sim$780,000 parsec) we obtain $z_1 = +0.00023$ and $z_2 = -0.001$, so the resulting shift of a photon's frequency at this distance is negative (the photon is definitely blueshifted).

Therefore, I suggest the same name "redshift law" for the obtained law in both expanding universe and compressing universe.

The redshift law (3.6, 3.7) and its linear approximation (3.8, 3.9) were obtained in a constant deformation non-holonomic universe. It is obvious that, in the absence of expansion or compression of space ($\mathrm{v} = 0$), these formulae transform into the redshift law (1.9) and its linear approximation (1.10) as deduced in a static (non-deforming) non-holonomic universe.

§4. Redshift in a constant speed deforming universe. A universe of this kind expands or compresses with a constant linear velocity $\mathrm{v} = const$.

In this case, neglecting gravitation as in the "large scale approximation" ($g_{00} = 1$), and taking the conservation of the angular momentum of the universe ($\Omega\, a^2 = const$) into account, we obtain

$$\frac{\partial v}{\partial t} = \Omega^2 a = \frac{\Omega^2 a^4}{(a_0 + \mathrm{v}t)^3} \simeq \frac{\Omega^2 a^4}{a_0^3}\left(1 - \gamma\,\frac{\mathrm{v}t}{a_0}\right), \tag{4.1}$$

where $\mathrm{v} = \pm|\mathrm{v}|$ (the positive velocity characterizes the expansion of space, while the sign minus characterizes the compression).

With (4.1) and the formula of deformation with $\mathrm{v} = const$ (2.6), we apply the scalar equation of isotropic geodesics (1.1) to a photon travelling in a constant speed deforming non-holonomic universe. As a result we obtain

$$\frac{dE}{dt} = -\left(\frac{\Omega^2 a^4}{c a_0^3} + \gamma\,\frac{\mathrm{v}}{a_0}\right) E + \gamma\,\frac{\mathrm{v}}{a_0}\left(\frac{\Omega^2 a^4}{c a_0^3} + \frac{\mathrm{v}}{a_0}\right) E\,t\,, \tag{4.2}$$

i.e. a separable first order ordinary differential equation $\dot{y} = -ay - byt$, which solves by separation of variables (moving the y terms to one side and the t terms to the other side). Thus, we transform this equation into $\frac{dy}{y} = -(a + bt)\,dt$, then obtain $d\ln y = -\left(at + \frac{1}{2}bt^2\right) + \ln C$. Finally, we have the solution $y = y_0\, e^{-\left(at + \frac{1}{2}bt^2\right)}$. As a result, the scalar equation of isotropic geodesics (4.2) solves as

$$E = E_0\, e^{-\frac{H_0 r}{c}\left\{1+\gamma\frac{|\mathrm{v}|}{c}-\gamma\frac{H_0 r|\mathrm{v}|}{2c^2}\left(1+\frac{|\mathrm{v}|}{c}\right)\right\}} \quad \text{in an expanding universe,} \tag{4.3}$$

$$E = E_0\, e^{-\frac{H_0 r}{c}\left\{1-\gamma\frac{|\mathrm{v}|}{c}+\gamma\frac{H_0 r|\mathrm{v}|}{2c^2}\left(1-\frac{|\mathrm{v}|}{c}\right)\right\}} \quad \text{in a compressing universe.} \tag{4.4}$$

Accordingly, the redshift law in a constant speed deforming non-holonomic universe is the sum of the redshift proceeded from the solutions to a photon's scalar equation of motion and the relativistic Doppler redshift. With these solutions (4.3) and (4.4) we obtain the redshift law in a constant speed deforming non-holonomic universe, the the cases of expansion and compression, respectively

$$z = \left(e^{\frac{H_0 r}{c}\left\{1+\gamma\frac{|\mathrm{v}|}{c}-\gamma\frac{H_0 r|\mathrm{v}|}{2c^2}\left(1+\frac{|\mathrm{v}|}{c}\right)\right\}} - 1\right) + \left(\frac{1+\frac{|\mathrm{v}|}{c}}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} - 1\right), \tag{4.5}$$

$$z = \left(e^{\frac{H_0 r}{c}\left\{1-\gamma\frac{|\mathrm{v}|}{c}+\gamma\frac{H_0 r|\mathrm{v}|}{2c^2}\left(1-\frac{|\mathrm{v}|}{c}\right)\right\}} - 1\right) + \left(\frac{1-\frac{|\mathrm{v}|}{c}}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} - 1\right). \tag{4.6}$$

In the case, where the photon travels at a small distance, while space expands or compresses with a sub-relativistic velocity, the redshift law (4.5, 4.6) takes the linear approximation form

$$z \simeq \frac{H_0 r}{c}\left[1-\gamma\frac{|\mathrm{v}|}{c}\left(\frac{H_0 r}{2c}-1\right)\right] + \frac{|\mathrm{v}|}{c} \quad \text{in an expanding universe,} \tag{4.7}$$

$$z \simeq \frac{H_0 r}{c}\left[1+\gamma\frac{|\mathrm{v}|}{c}\left(\frac{H_0 r}{2c}-1\right)\right] - \frac{|\mathrm{v}|}{c} \quad \text{in a compressing universe.} \tag{4.8}$$

As seen, the formulae (4.7, 4.8) differ from the linear form redshift law in a constant deformation universe (3.8, 3.9) by only the numerical multiplier $\frac{1}{2}$ in the brackets of the second term, which is due to the hon-holonomity of space, while the second term (due to the Doppler-effect) remains the same. This means that the redshift in a universe which expands with a constant linear velocity is less that the redshift in a universe whose space expands so that its deformation remains unchanged.

In a constant speed compressing universe, this difference leads to a blueshift (due to the Doppler-effect, manifested by the second term of the redshift law) which is observed at a distance larger than in a constant deformation compressing universe. Then the first term (due to the non-holonomity of space), which is always positive due to the exponent, increases with the distance, so that it exceeds the second (Doppler-effect blueshift) term and the summary shift in a photon's frequency becomes positive: the photon becomes definitely redshifted in a compressing universe.

This tendency is still valid in the exponential redshift law (4.5, 4.6), which takes an account of the large distances and the ultra-relativistic velocity of the expansion or compression.

If no expansion or compression of space ($\mathrm{v} = 0$), these formulae transform into the redshift law (1.9) and its linear approximation (1.10) we have deduced in a static (non-deforming) non-holonomic universe.

§5. Conclusions. To better view of the results obtained in this paper, they have been collected into a Table shown on Page 16. Actually, this is the redshift law and its linear approximation. These have been theoretically deduced in the framework of a globally non-holonomic universe, where the isotropic space (the "home space" of isotropic trajectories and massless light-like particles, e.g. photons) rotates with the velocity of light and at an angular velocity equal to the Hubble constant. In summary, the following results are emphasized:

1. The empirical Hubble law, including its non-linearity at large distances, is completely explained in a static (non-deforming) universe due to the redshift produced by the global non-holonomity of the isotropic space (a photon being moved in a non-holonomic space loses its proper energy/frequency due to the work produced by it against the field of the space non-holonomity).
2. The non-linearity of the Hubble law, observed at large distances close to the curvature radius of space, is explained due to the exponent in the redshift law deduced for a static universe.
3. The ultimate redshift in a static spherical universe, according to the theory, should be $z = \exp(\pi) - 1 = 22.14$.
4. Deformation (expansion or compression) of space results changes in the redshift law. In a deforming universe, it consists of two terms: the first term is due to the non-holonomity of space, while the second term manifests the relativistic Doppler effect observed on a photon due to the rapid expansion or compression of space.
5. In an expanding universe, according to the redshift law, the near objects must be redshifted (on the average) due to the Doppler-

	The Redshift Law	Its Linear Approximation
Static universe	$z = e^{\frac{H_0 r}{c}} - 1$	$z \simeq \frac{H_0 r}{c}$
Constant deformation expansion	$z = \left(e^{\frac{H_0 r}{c}} \left(e^{-\gamma \frac{H_0 r \lvert \mathrm{v} \rvert}{c^2}} + \gamma \frac{\lvert \mathrm{v} \rvert}{c} \right) - 1 \right) + \left(\frac{1 + \frac{\lvert \mathrm{v} \rvert}{c}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} - 1 \right)$	$z \simeq \frac{H_0 r}{c} \left[1 - \gamma \frac{\lvert \mathrm{v} \rvert}{c} \left(\frac{H_0 r}{c} - 1 \right) \right] + \frac{\lvert \mathrm{v} \rvert}{c}$
Constant deformation compression	$z = \left(e^{\frac{H_0 r}{c}} \left(e^{\gamma \frac{H_0 r \lvert \mathrm{v} \rvert}{c^2}} - \gamma \frac{\lvert \mathrm{v} \rvert}{c} \right) - 1 \right) + \left(\frac{1 - \frac{\lvert \mathrm{v} \rvert}{c}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} - 1 \right)$	$z \simeq \frac{H_0 r}{c} \left[1 + \gamma \frac{\lvert \mathrm{v} \rvert}{c} \left(\frac{H_0 r}{c} - 1 \right) \right] - \frac{\lvert \mathrm{v} \rvert}{c}$
Constant speed expansion	$z = \left(e^{\frac{H_0 r}{c} \left\{ 1 + \gamma \frac{\lvert \mathrm{v} \rvert}{c} - \gamma \frac{H_0 r \lvert \mathrm{v} \rvert}{2c^2} \left(1 + \frac{\lvert \mathrm{v} \rvert}{c} \right) \right\}} - 1 \right) + \left(\frac{1 + \frac{\lvert \mathrm{v} \rvert}{c}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} - 1 \right)$	$z \simeq \frac{H_0 r}{c} \left[1 - \gamma \frac{\lvert \mathrm{v} \rvert}{c} \left(\frac{H_0 r}{2c} - 1 \right) \right] + \frac{\lvert \mathrm{v} \rvert}{c}$
Constant speed compression	$z = \left(e^{\frac{H_0 r}{c} \left\{ 1 - \gamma \frac{\lvert \mathrm{v} \rvert}{c} + \gamma \frac{H_0 r \lvert \mathrm{v} \rvert}{2c^2} \left(1 - \frac{\lvert \mathrm{v} \rvert}{c} \right) \right\}} - 1 \right) + \left(\frac{1 - \frac{\lvert \mathrm{v} \rvert}{c}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} - 1 \right)$	$z \simeq \frac{H_0 r}{c} \left[1 + \gamma \frac{\lvert \mathrm{v} \rvert}{c} \left(\frac{H_0 r}{2c} - 1 \right) \right] - \frac{\lvert \mathrm{v} \rvert}{c}$

Table 1: The redshift law and its linear approximation, obtained in the framework of a globally non-holonomic universe, where the isotropic space (the "home space" of isotropic trajectories and massless light-like particles, e.g. photons) rotates with the velocity of light and at an angular velocity equal to the Hubble constant.

effect, while the redshift in the spectra of far galaxies must be much larger than the Doppler redshift, and approaching exponential growth at large distances. We however do not observe any systematic redshift on the stars of our Galaxy and the near galaxies (moreover the Andromeda Galaxy is blueshifted). Edwin Hubble had discovered a systematic redshift on only far galaxies. Meanwhile, a systematic Doppler redshift must be observed on the near objects, if our Universe expands. Therefore, the expanding scenario does not seem to properly characterize our Universe.

6. In a compressing universe, according to the redshift law, the near objects must be blueshifted (on the average) due to the Doppler-effect, while far galaxies must be redshifted due to the always positive exponential (growing up with distance) term in the redshift law. Meanwhile, we do not observe any average blueshift on the objects both within our Galaxy or near it (the blueshift of the Andromeda Galaxy can be explained by the relative motion of it toward our Galaxy). Therefore, the compressing scenario also does not seem to properly characterize our Universe.

Consequently, the empirical Hubble law, which is a result of astronomical observations, is completely explained by the theoretical redshift law we have deduced in a static spherical universe, while the expansion or compression of space would lead to the unbelievable changes of the redshift law, never registered in astronomical observations. Therefore, I conclude that we have enough reasons to mean the space of our Universe static as a whole.

On the other hand, this conclusion does not exclude expansion or compression of local volumes of space. According to Zelmanov's theory of an inhomogeneous anisotropic universe [2, 3], a local volume element of a universe can evolve in another way than the universe as a whole. Thus, the local redshift or blueshift anomalies, which differ from the redshift law (1.9, 1.10) we have deduced for a static universe, manifest the fact that the space of our universe, static as a whole, is evolving (expanding or compressing) in its local volume elements.

For instance, supernova explosions lead to the rapid expansion of the surrounding (local) volume of space. An observer near a supernova should register the redshift effect according to the expansion. In contrast, the process of collapse leads to compression of the local space surrounding a collapsing object. Therefore, an observer near a collapsing object should register the blueshift effect which manifests the fact that the surrounding space compresses. Thus, collapsing bodies in the

Universe can be indicated by not only accretion of the near matter onto such a body, but also by the blueshift in the compressing local space of it. Note that, according to the redshift law we have deduced, the blueshift effect of a compressing space is valid at only small distances where the redshift due to the global non-holonomity of the Universe is small. Therefore, in searching for a blueshift effect in a compressing volume (actually, in look for the collapsing bodies in the Universe), we should limit the area of our search by the distance to the Andromeda Galaxy or by a distance which is not much larger.

In this row, bizarre should seem the result of observation produced near a Cepheid, because its local space experiences periodical expansions and compressions, i.e. oscillates, with a short period equal to the period of pulsation of the star itself (days).

October 31, 2008

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