Two-World Background of Special Relativity. Part II

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The two-world background of the Special Theory of Relativity started in part one of this article is continued in this second part. Four-dimensional inversion is shown to be a special Lorentz transformation that transforms the positive spacetime coordinates of a frame of reference in the positive universe into the negative spacetime coordinates of the symmetry-partner frame of reference in the negative universe in the two-world picture, contrary to the conclusion that four-dimensional inversion is impossible as actual transformation of the coordinates of a frame of reference in the existing one-world picture. By starting with the negative spacetime dimensions in the negative universe derived in part one, the signs of mass and other physical parameters and physical constants in the negative universe are derived by application of the symmetry of laws between the positive and negative universes. The invariance of natural laws in the negative universe is demonstrated. The derived negative sign of mass in the negative universe is a conclusion of over a century-old effort towards the development of the concept of negative mass in physics.

1 Introduction

A brief summary of the new geometrical representation of Lorentz transformation and its inverse in the two-world picture and the other associated issues presented in part one of this article [1], is appropriate at the beginning of this second part.

Having deduced from the $\gamma = \sec\psi$ parametrization of the Lorentz boost that a pair of flat four-dimensional spacetimes (or a pair of Minkowski's spaces), which are four-dimensional inversions of each other namely, $(\Sigma, ct) \equiv (x^1, x^2, x^3, ct)$ and $(-\Sigma^*, -ct^*) \equiv (-x^{1*}, -x^{2*}, -x^{3*}, -ct^*)$, co-exist in nature and that this implies the co-existence in nature of a pair of symmetrical worlds (or universes), referred to as our (or positive) universe and negative universe, a pair of two-dimensional intrinsic spacetimes denoted respectively by $(\phi\rho, \phi c\phi t)$ and $(-\phi\rho^*, -\phi c\phi t^*)$, which underlie the flat four-dimensional spacetimes (Σ, ct) of the positive universe and $(-\Sigma^*, -ct^*)$ of the negative universe respectively, were introduced (as *ansatz*) in [1]. The derived graphical representation of the larger spacetime/intrinsic spacetime of the co-existing "anti-parallel" worlds (or universes) was then derived and presented as Fig. 7 of [1].

A new set of intrinsic spacetime diagrams that involve rotations of the primed affine intrinsic spacetime coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ relative to the unprimed affine intrinsic spacetime coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ of a pair of frames in relative motion in the positive universe, which are united with the symmetrical rotations of the primed affine intrinsic spacetime coordinates $-\phi\tilde{x}'^*$ and $-\phi c\,\phi\tilde{t}'^*$ relative to the unprimed affine intrinsic spacetime coordinates $-\phi\tilde{x}^*$ and $-\phi c\phi\tilde{t}^*$ of the symmetry-partner pair of frames in simultaneous identical relative motion in the negative universe, are then drawn on the larger spacetime/intrinsic spacetime of combined positive and negative universes, as Figs. 8a, 8b, 9a and 9b of [1]. The intrinsic Lorentz transformations (ϕLT) and its inverse are derived from the set of intrinsic spacetime diagrams and intrinsic Lorentz invariance (ϕLI) validated in the context of the intrinsic Special Theory of Relativity (ϕSR) on each of the flat two-dimensional intrinsic spacetimes $(\phi\rho, \phi c\phi t)$ of the positive universe and $(-\phi\rho^*, -\phi c\phi t^*)$ of the negative universe.

The flat four-dimensional spacetimes (Σ, ct) and $(-\Sigma^*, -ct^*)$ being the outward (or physical) manifestations of their underlying flat two-dimensional intrinsic spacetimes $(\phi\rho, \phi c\phi t)$ and $(-\phi\rho^*, -\phi c\phi t^*)$ respectively and the Special Theory of Relativity (SR) on each of the spacetimes (Σ, ct) and $(-\Sigma^*, -ct^*)$ being mere outward manifestations of the intrinsic Special Theory of Relativity (ϕSR) on each of $(\phi\rho, \phi c\phi t)$ and $(-\phi\rho^*, -\phi c\phi t^*)$ respectively, the Lorentz transformation (LT) and its inverse are written directly and Lorentz invariance (LI) validated on each of the flat four-dimensional spacetimes (Σ, ct) and $(-\Sigma^*, -ct^*)$, as outward manifestations of intrinsic Lorentz transformation (ϕLT) and its inverse and intrinsic Lorentz invariance (ϕLI) derived graphically on each of $(\phi\rho, \phi c\phi t)$ and $(-\phi\rho^*, -\phi c\phi t^*)$.

There is consequently no need to draw spacetime diagrams involving relative rotations of the primed affine spacetime coordinates $\tilde{x}'$ and $c\tilde{t}'$ relative to the unprimed affine spacetime coordinates $\tilde{x}$ and $c\tilde{t}$ of a pair of frames in relative motion along their collinear $\tilde{x}'-$ and $\tilde{x}-$ axes in the positive universe, which would be united with the symmetrical rotations of the primed affine spacetime coordinates $-\tilde{x}'^*$ and $-c\tilde{t}'^*$ relative to the unprimed affine spacetime coordinates $-\tilde{x}^*$ and $-c\tilde{t}^*$ of the symmetry-partner pair of frames in simultaneous identical relative motion in the negative universe, on the larger spacetime of combined positive and negative universes, in deriving LT and its inverse and in validating LI in the positive and negative universes. Indeed such diagrams do not exist and if drawn, they must be understood that they are intrinsic (that is, non-observable) or hypothetical diagrams only, as noted in [1].

The fact that the derived intrinsic Lorentz transformation represents rotation of intrinsic spacetime coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of a particle's frame relative to intrinsic spacetime coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ respectively of the observer's frame at intrinsic angle $\phi\psi$, where $\phi\psi$ can vary continuously in the entire range $[0, 2\pi]$, except that $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$ must be avoided, are shown in [1]. The non-existence of the light cone concept and good prospect for making SO(3,1) compact in the two-world picture are also shown in [1].

The next natural step in the theoretical justification of the two-world background of the Special Theory of Relativity started in part one of this article, to which this second part is devoted, is the derivations of the signs of mass and other physical parameters and physical constants and investigation of Lorentz invariance of natural laws in the negative universe. The matter arising from [1] namely, the formal derivation (or isolation) of the flat two-dimensional intrinsic spacetimes $(\phi\rho, \phi c\phi t)$ and $(-\phi\rho^*, -\phi c\phi t^*)$ that underlie the flat four-dimensional spacetimes (Σ, ct) and $(-\Sigma^*, -ct^*)$ respectively, which were introduced (as *ansatz*) in [1], requires further development of the two-world picture than in this second part of this article to resolve.

2 Four-dimensional inversion as special Lorentz transformation of the coordinates of a frame of reference in the two-world picture

The intrinsic Lorentz transformation (ϕLT) and its inverse in the two-world picture have been written in the generalized forms of equations (44) and (45) of part one of this article [1]. They can be applied for all intrinsic angles $\phi\psi$ in the first cycle, while avoiding $\phi\psi = -\frac{\pi}{2}$, $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$, of relative rotation of the affine intrinsic spacetime coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the intrinsic particle's (or primed) frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ relative to the affine intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ of the intrinsic observer's (or unprimed) frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$ on the larger two-dimensional intrinsic spacetime of combined positive and negative universes. They are reproduced here as follows

$$\left.\begin{aligned} \phi c\phi\tilde{t}' &= \sec\phi\psi(\phi c\phi\tilde{t} - \phi\tilde{x}\sin\phi\psi) \\ \phi\tilde{x}' &= \sec\phi\psi(\phi\tilde{x} - \phi c\phi\tilde{t}\sin\phi\psi) \end{aligned}\right\} \tag{1}$$

and

$$\left.\begin{aligned} \phi c\phi\tilde{t} &= \sec\phi\psi(\phi c\phi\tilde{t}' + \phi\tilde{x}'\sin\phi\psi) \\ \phi\tilde{x} &= \sec\phi\psi(\phi\tilde{x}' + \phi c\phi\tilde{t}'\sin\phi\psi) \end{aligned}\right\}, \tag{2}$$

where, as mentioned above, the intrinsic angle $\phi\psi$ can take on values in the range $[0, 2\pi]$, while avoiding $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$.

Systems (1) and (2) on the flat two-dimensional intrinsic spacetime (or in the intrinsic Minkowski space) $(\phi\rho, \phi c\phi t)$ of intrinsic Special Theory of Relativity (ϕSR) are made manifest outwardly (or physically) on the flat four-dimensional spacetime (the Minkowski space) (Σ, ct) of the Special Theory of Relativity (SR) in the positive universe respectively as follows, as developed in [1]

$$\left.\begin{aligned} c\tilde{t}' &= \sec\psi(c\tilde{t} - \tilde{x}\sin\psi) \\ \tilde{x}' &= \sec\psi(\tilde{x} - c\tilde{t}\sin\psi), \quad \tilde{y}' = \tilde{y}, \quad \tilde{z}' = \tilde{z} \end{aligned}\right\} \tag{3}$$

and

$$\left.\begin{aligned} c\tilde{t} &= \sec\psi(c\tilde{t}' + \tilde{x}'\sin\psi), \\ \tilde{x} &= \sec\psi(\tilde{x}' + c\tilde{t}'\sin\psi), \quad \tilde{y} = \tilde{y}', \quad \tilde{z} = \tilde{z}' \end{aligned}\right\}, \tag{4}$$

where, again, the angle ψ can take on values in $[0, 2\pi]$, excluding $\psi = \frac{\pi}{2}$ and $\psi = \frac{3\pi}{2}$.

However, it must be noted, as discussed in [1], that while the intrinsic angle $\phi\psi$ measures actual rotation of the affine intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the intrinsic particle's frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ relative to the intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ respectively of the intrinsic observer's frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$ in system (1), the angle ψ refers to intrinsic (i.e. non-observable) or hypothetical rotation of the coordinates $\tilde{x}'$ and $c\tilde{t}'$ of the particle's frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ relative to the coordinates $\tilde{x}$ and $c\tilde{t}$ of the observer's frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ respectively in system (3). The affine spacetime coordinates $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$, $c\tilde{t}'$ of the particle's frame are not rotated relative to the coordinates $\tilde{x}$, $\tilde{y}$, $\tilde{z}$, $c\tilde{t}$ of the observer's frame and conversely in the present geometrical representation of Lorentz transformation and its inverse in the two-world picture started in [1].

We shall for now assume the possibility of continuous rotation of the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the intrinsic particle's frame by intrinsic angle $\phi\psi = \pi$, while avoiding $\phi\psi = \frac{\pi}{2}$, relative to the intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ of the intrinsic observer's frame in the two-world picture, as developed in [1]. As also mentioned in [1], the explanation of how rotation through all angles ψ in $[0, \pi]$ while avoiding $\psi = \frac{\pi}{2}$ can be achieved shall not be of concern in this paper.

Then by letting $\phi\psi = \pi$ we have $\sec\phi\psi = -1$, $\sin\phi\psi = 0$ and system (1) simplifies as follows

$$\phi c\phi\tilde{t}' = -\phi c\phi\tilde{t} \quad \text{and} \quad \phi\tilde{x}' = -\phi\tilde{x}. \tag{5}$$

The meaning of system (5) is that upon rotation through intrinsic angle $\phi\psi = \pi$ of the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the intrinsic particle's frame relative to the intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ respectively of the intrinsic observer's frame in the positive universe, the rotated intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ transform into (or become) intrinsic coordinates of an observer's frame with negative sign $-\phi\tilde{x}$ and $-\phi c\phi\tilde{t}$ respectively.

The outward manifestation on flat four-dimensional spacetime of system (5) is the following

$$c\tilde{t}' = -c\tilde{t}, \quad \tilde{x}' = -\tilde{x}, \quad \tilde{y}' = -\tilde{y}, \quad \tilde{z}' = -\tilde{z}. \tag{6}$$

System (6) is valid because the intrinsic space coordinates $\phi\tilde{x}'$ and $-\phi\tilde{x}$ are made manifest in the coordinates $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$ of 3-space $\tilde{\Sigma}'$ and the coordinates $-\tilde{x}$, $-\tilde{y}$, $-\tilde{z}$ of 3-space $-\tilde{\Sigma}$ respectively, as explained in [1].

Although the coordinates $c\tilde{t}'$ and $\tilde{x}'$ of the particle's frame are not rotated relative to the coordinates $c\tilde{t}$ and $\tilde{x}$ of the observer's frame, once the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the intrinsic particle's frame are rotated by intrinsic angle $\phi\psi = \pi$ relative to the intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ of the intrinsic observer's frame, thereby giving rise to system (5), then system (6) will arise automatically as the outward manifestation of system (5). It may be observed that system (6) cannot be derived by letting $\psi = \pi$ in system (3).

According to system (5), the intrinsic particle's frame whose intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ are inclined at intrinsic angle $\phi\psi = \pi$ relative to the respective intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ of the intrinsic observer's frame in the positive universe, although is at rest relative to the observer's frame, since $\sin\phi\psi = \phi v/\phi c = 0 \Rightarrow \phi v = 0$ for $\phi\psi = \pi$, it possesses negative intrinsic spacetime coordinates relative to the intrinsic observer's frame in the positive universe. This implies that the intrinsic particle's frame has made transition into the negative universe. As confirmation of this fact, letting $\phi\psi = \pi$ in Fig. 8a of [1] causes the inclined intrinsic coordinate $\phi\tilde{x}'$ to lie along $-\phi\tilde{x}^*$ along the horizontal in the third quadrant and the inclined intrinsic coordinate $\phi c\phi\tilde{t}'$ to lie along $-\phi c\phi\tilde{t}^*$ along the vertical in the third quadrant in that figure.

The negative intrinsic coordinates $-\phi\tilde{x}$ and $-\phi c\phi\tilde{t}$ in system (5) are clearly the intrinsic coordinates of the symmetry-partner intrinsic observer's frame in the negative universe. Then by putting a dummy star label on the unprimed negative intrinsic coordinates in system (5) as our conventional way of denoting the coordinates/intrinsic coordinates and parameters/intrinsic parameters of the negative universe, in order to differentiate them from those of the positive universe we have

$$\phi c\phi\tilde{t}' = -\phi c\phi\tilde{t}^*, \quad \phi\tilde{x}' = -\phi\tilde{x}^*. \tag{7}$$

Likewise, by putting dummy star label on the negative spacetime coordinates in system (6), since they are the coordinates if the symmetry-partner observer's frame in the negative universe we have

$$c\tilde{t}' = -c\tilde{t}^*, \quad \tilde{x}' = -\tilde{x}^*, \quad \tilde{y}' = -\tilde{y}^*, \quad \tilde{z}' = -\tilde{z}^*. \tag{8}$$

System (8) is the outward manifestation on flat four-dimensional spacetime of system (7). System (7) is the form taken by the generalized intrinsic Lorentz transformation (1) for $\phi\psi = \pi$ and system (8) is the form taken by the generalized Lorentz transformation (3) for $\psi = \pi$.

Since the intrinsic particle's frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ is at rest relative to the symmetry-partner intrinsic observer's frame $(-\phi\tilde{x}^*, -\phi c\phi\tilde{t}^*)$ in the negative universe in system (7), which is so since $\sin\phi\psi = \phi v/\phi c = 0 \Rightarrow \phi v = 0$, as mentioned earlier, the intrinsic coordinates $-\phi\tilde{x}^*$ and $-\phi c\phi\tilde{t}^*$ of the intrinsic "stationary" observer's frame are identical to the coordinates $-\phi\tilde{x}'^*$ and $-\phi c\phi\tilde{t}'^*$ of the symmetry-partner intrinsic particle's frame in the negative universe. Consequently system (7) is equivalent to the following transformation of the primed intrinsic coordinates of the intrinsic particle's frame in the positive universe into the primed intrinsic coordinates of the symmetry-partner intrinsic particle's frame in the negative universe:

$$\phi c\phi\tilde{t}' = -\phi c\phi\tilde{t}'^*, \quad \phi\tilde{x}' = -\phi\tilde{x}'^*$$

or

$$\phi c\phi\tilde{t}' \to -\phi c\phi\tilde{t}'^*, \quad \phi\tilde{x}' \to -\phi\tilde{x}'^*. \tag{9}$$

This is inversions in the origin (or intrinsic two-dimensional inversions) of the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the intrinsic particle's frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ in the positive universe, which arises by virtue of actual rotations of the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ by intrinsic angle $\phi\psi = \pi$ relative to the intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ respectively of the intrinsic observer's frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$ in the positive universe. The intrinsic two-dimensional inversion (9) is still the generalized intrinsic Lorentz transformation (1) for $\phi\psi = \pi$.

The outward manifestation on the flat four-dimensional spacetime of system (9), which also follows from system (8), is the following

$$c\tilde{t}' = -c\tilde{t}'^*, \;\; \tilde{x}' = -\tilde{x}'^*, \;\; \tilde{y}' = -\tilde{y}'^*, \;\; \tilde{z}' = -\tilde{z}'^*$$

or

$$c\tilde{t}' \to -c\tilde{t}'^*, \;\; \tilde{x}' \to -\tilde{x}'^*, \;\; \tilde{y}' \to -\tilde{y}'^*, \;\; \tilde{z}' \to -\tilde{z}'^*. \tag{10}$$

This is the corresponding inversions in the origin (or four-dimensional inversions) of the coordinates $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$ and $c\tilde{t}'$ of the particle's frame in the positive universe, which arises as outward manifestation of system (9). The four-dimensional inversion (10) is still the generalized Lorentz transformation of system (3) for $\psi = \pi$. It shall be reiterated for emphasis that the coordinates $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$ and $c\tilde{t}'$ of the particle's frame in the positive universe are not actually rotated by angle $\psi = \pi$ relative to the coordinates $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and $c\tilde{t}$ of the observer's frame in the positive universe, but that system (10) arises as a consequence of system (9) that arises from actual rotation of intrinsic coordinates.

Corresponding to system (9) expressing inversions in the origin of intrinsic coordinates of the intrinsic particle's frame, derived from the intrinsic Lorentz transformation (1) for $\phi\psi = \pi$, is the following inversions in the origin of the unprimed intrinsic coordinates of the intrinsic observer's frame, which can be derived from the inverse intrinsic Lorentz transformation (2) for $\phi\psi = \pi$:

$$\phi c\phi\tilde{t} = -\phi c\phi\tilde{t}^*, \quad \phi\tilde{x} = -\phi\tilde{x}^*$$

or

$$\phi c\phi\tilde{t} \to -\phi c\phi\tilde{t}^*, \quad \phi\tilde{x} \to -\phi\tilde{x}^*. \tag{11}$$

And the outward manifestation on flat four-dimensional spacetime of system (11) is the following four-dimensional inversions of the coordinates of the observer's frame

$$c\tilde{t} = -c\tilde{t}^*, \quad \tilde{x} = -\tilde{x}^*, \quad \tilde{y} = -\tilde{y}^*, \quad \tilde{z} = -\tilde{z}^*$$

or

$$c\tilde{t} \to -c\tilde{t}^*, \quad \tilde{x} \to -\tilde{x}^*, \quad \tilde{y} \to -\tilde{y}^*, \quad \tilde{z} \to -\tilde{z}^*. \quad (12)$$

We have thus shown that intrinsic two-dimensional inversion is the special intrinsic Lorentz transformation (1) or its inverse (2) for $\phi\psi = \pi$. It transforms the intrinsic spacetime coordinates of a frame in the positive universe into the intrinsic spacetime coordinates of the symmetry-partner frame in the negative universe or conversely. Four-dimensional inversion is likewise the special Lorentz transformation (3) or its inverse (4) for $\psi = \pi$, which transforms the spacetime coordinates of a frame in the positive universe into the spacetime coordinates of the symmetry-partner frame in the negative universe or conversely.

On the other hand, it has been concluded in the context of the existing one-world background of the Special Theory of Relativity (or in the one-world picture) that four-dimensional inversion in impossible as actual transformation of the coordinates of a frame of reference. This, as discussed in [2, see p.39], for example, is due to the fact four-dimensional inversion carries the time axis from the future light cone into the past light cone, which is impossible without going through regions of spacelike geodesics that requires the introduction of imaginary spacetime coordinates in the one-world picture.

The light cone concept does not exist in the two-world picture, as deduced in sub-section 4.7 of [1]. Consequently continuous relative rotation of intrinsic spacetime coordinates of two frames through all intrinsic angles $\phi\psi$ in $[0, 2\pi]$, while avoiding $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$, is possible, (granting that how $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$ are avoided shall be explained,) without going into regions of spacelike geodesics in the two-world picture. Four-dimensional inversion, (which does not involve actual relative rotation of spacetime coordinates of two frames), being mere outward manifestation of intrinsic two-dimensional inversion that involves actual relative rotation of intrinsic spacetime coordinates of two frames, is therefore possible as transformation of the coordinates of a frame of reference in the two-world picture.

3 Sign of mass in the negative universe derived from generalized mass expression in Special Relativity in the two-world picture

Now the intrinsic particle's frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ contains the intrinsic rest mass ϕm_0 of the particle at rest relative to it and the particle's frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ contains the rest mass m_0 of the particle at rest relative to it in the positive universe. The question arises; what are the signs of the intrinsic rest mass and rest mass of the symmetry-partner particle contained in the symmetry-partner intrinsic particle's frame $(-\phi\tilde{x}'^*, -\phi c\phi\tilde{t}'^*)$ and symmetry-partner particle's frame $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$ respectively in the negative universes? The answer to this question shall be sought from the the generalized intrinsic mass relation in the context of the intrinsic Special Theory of Relativity (ϕSR) and from the corresponding generalized mass relation in the context of the Special Theory of Relativity (SR) in the two-world picture in this section and by requiring the symmetry of laws between the positive and negative universes in the next section.

The well known mass relation on flat four-dimensional spacetime (Σ, ct) in the context of SR is the following

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}\,. \quad (13)$$

The corresponding intrinsic mass relation on the flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the context of the intrinsic Special Theory of Relativity (ϕSR) is

$$\phi m = \frac{\phi m_0}{\sqrt{1 - \phi v^2/\phi c^2}}\,. \quad (14)$$

The three-dimensional masses m_0 and m in the three-dimensional Euclidean space are the outward manifestation of the one-dimensional intrinsic masses ϕm_0 and ϕm respectively in the one-dimensional intrinsic space, as illustrated in Fig. 6a of [1].

Then by using the relation, $\sec\phi\psi = (1 - \phi v^2/\phi c^2)^{-\frac{1}{2}}$ and $\sec\psi = (1 - v^2/c^2)^{-\frac{1}{2}}$ derived and presented as Eqs. (19) and (32) respectively in [1], Eqs. (14) and (13) can be written respectively as follows

$$\phi m = \phi m_0 \sec\phi\psi \quad (15)$$

and

$$m = m_0 \sec\psi\,. \quad (16)$$

Eqs. (15) and (16) are the generalized forms in the two-world picture of the intrinsic mass relation in the context of ϕSR and mass relation in the context of SR respectively. They can be applied for all intrinsic angle $\phi\psi$ and all angles ψ in the range $[0, 2\pi]$, except that $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$ must be avoided.

By letting $\phi\psi = \pi$ in Eq. (15) and $\psi = \pi$ in Eq. (16) we have

$$\phi m = -\phi m_0 \equiv -\phi m_0^* \quad (17)$$

and

$$m = -m_0 \equiv -m_0^*\,. \quad (18)$$

However the intrinsic particle's frame is stationary relative to the intrinsic observer's frame for $\phi\psi = \pi$, since then $\sin\phi\psi = \phi v/\phi c = 0 \Rightarrow \phi v = 0$, as noted earlier. Consequently the intrinsic special-relativistic mass $\phi m = \phi m_0(1 - \phi v^2/\phi c^2)^{-\frac{1}{2}}$ must be replaced by the intrinsic rest mass ϕm_0 in (17) and the special-relativistic mass $m = m_0(1 - v^2/c^2)^{-\frac{1}{2}}$ must be replaced by the rest mass m_0 in (18) to have respectively as follows

$$\phi m_0 = -\phi m_0^* \text{ or } \phi m_0 \to -\phi m_0^* \quad (19)$$

and

$$m_0 = -m_0^* \text{ or } m_0 \to -m_0^*\,. \quad (20)$$

Just as the positive intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the intrinsic particle's frame in the positive universe transform into the negative intrinsic coordinates $-\phi\tilde{x}'^*$ and $-\phi c\phi\tilde{t}'^*$ of the symmetry-partner intrinsic particle's frame in the negative universe expressed by system (9), by virtue of the generalized intrinsic Lorentz transformation (1) for $\phi\psi = \pi$, the positive intrinsic rest mass ϕm_0 of the particle contained in the intrinsic particle's frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ in the positive universe, transforms into negative intrinsic rest mass $-\phi m_0^*$ contained in the intrinsic particle's frame $(-\phi\tilde{x}'^*, -\phi c\phi\tilde{t}'^*)$ in the negative universe, by virtue of the generalized intrinsic mass relation (15) for $\phi\psi = \pi$. The negative intrinsic rest mass $-\phi m_0^*$ is certainly the intrinsic rest mass of the symmetry-partner particle in the negative universe.

Likewise as the positive coordinates $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$ and $c\tilde{t}'$ of a particle's frame in the positive universe transform into negative coordinates $-\tilde{x}'^*$, $-\tilde{y}'^*$, $-\tilde{z}'^*$ and $-c\tilde{t}'^*$ of the symmetry-partner particle's frame in the negative universe, expressed by system (10), by virtue of the generalized Lorentz transformation (3) for $\psi = \pi$, the positive rest mass m_0 of the particle contained in the particle's frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ in the positive universe, transforms into negative rest mass $-m_0^*$ contained in the symmetry-partner particle's frame $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$ in the negative universe, by virtue of the generalized relativistic mass relation (16) for $\psi = \pi$. Again the negative rest mass $-m_0^*$ is certainly the rest mass of the symmetry-partner particle in the negative universe.

It follows from the foregoing two paragraphs that the intrinsic particle's frame containing positive intrinsic rest mass of the particle in the positive universe, to be denoted by $(\phi\tilde{x}', \phi c\phi\tilde{t}'; \phi m_0)$, corresponds to the symmetry-partner intrinsic particle's frame containing negative intrinsic rest mass $(-\phi\tilde{x}'^*, -\phi c\phi\tilde{t}'^*; -\phi m_0^*)$ in the negative universe. The particle's frame containing the positive rest mass of the particle $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0)$ in the positive universe, likewise corresponds to the symmetry-partner particle's frame containing negative rest mass $(-\tilde{x}', -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*; -m_0^*)$ in the negative universe.

The conclusion that follows from the foregoing is that intrinsic rest masses and rest masses of material particles and objects (that appear in classical, that is, in non-special-relativistic intrinsic physics and physics) are negative quantities in the negative universe. The special-relativistic intrinsic masses $\phi m = \gamma(\phi v)\phi m_0$ and special-relativistic masses $m = \gamma(v)m_0$ of material particles and objects that appear in special-relativistic intrinsic physics and special-relativistic physics respectively are therefore negative quantities in the negative universe.

4 Derivation of the signs of physical parameters and physical constants in the negative universe by application of symmetry of laws between the positive and negative universes

Four-dimensional inversion is the transformation of the positive spacetime coordinates of a frame in the positive universe into the negative spacetime coordinates of the symmetry-partner frame in the negative universe, as systems (10) and (12) show. Thus the simultaneous negation of spacetime coordinates in the classical or special-relativistic form of a natural law amounts to writing that law in the negative universe.

Now the prescribed perfect symmetry of state between the positive and negative universes discussed in sub-section 4.1 of part one of this article [1], will be impossible unless there is also a perfect symmetry of laws between the two universes. That is, unless natural laws take on identical forms in the two universes. Perfect symmetry of laws between the positive and negative universes is immutable, as shall be demonstrated shortly in this article. It must be recalled that Lorentz invariance in the negative universe, (which is an important component of the invariance of laws in the negative universe), has been validated from the derived LT and its inverse in the negative universe of systems (38) and (39) of [1].

The simultaneous negation of space and time coordinates in a natural law in the positive universe in the process of writing it in the negative universe will change the form of that law in general unless physical quantities and constants, such as mass, electric charge, temperature, flux, etc, which also appear in the law (usually as differential coefficients in the instantaneous differential laws) are given the appropriate signs. By combining the simultaneous negation of space and time dimensions with the invariance of laws, the signs of physical quantities and constants in the negative universe can be derived. The derivations of the signs of the fundamental quantities namely, mass, electric charge and absolute temperature in the negative universe shall be done below. The signs of all derived (or non-fundamental) physical quantities and physical constants can then be inferred from their dimensions, as shall be demonstrated.

Consider a body of constant mass m being accelerated by a force $\vec{F}$ directed along the positive X-axis of the frame attached to it. In the positive universe, Newton's second law of motion for this body is the following

$$\vec{F} = \left(m\,\frac{d^2x}{dt^2}\right)\hat{\imath}\,. \tag{21}$$

Since the dimensions of 3-space of the negative universe is inversion in the origin of the dimensions of 3-space of the positive universe, the dimensions, unit vector and force, $(x, y, z, t; \hat{\imath}; \vec{F})$, in the positive universe correspond to $(-x^*, -y^*, -z^*, -t^*; -\hat{\imath}^*; -\vec{F}^*)$ in the negative universe. Thus in the negative universe, we must let $x \to -x^*$, $t \to -t^*$, $\hat{\imath} \to -\hat{\imath}^*$ and $\vec{F} \to -\vec{F}^*$, while leaving m unchanged meanwhile in (21) to have as follows

$$-\vec{F}^* = \left(m\,\frac{d^2(-x^*)}{d(-t^*)^2}\right)(-\hat{\imath}^*) = \left(m\,\frac{d^2x^*}{dt^{*2}}\right)\hat{\imath}^*. \tag{22}$$

While Eq. (21) states that a body pushed towards the positive x-direction by a force $\vec{F}$, moves along the positive x-direction, (away from the force), in the positive universe, Eq. (22) states that a body pushed in the $-x^*$-direction in the negative universe by a force $-\vec{F}^*$, moves in the $+x^*$-direction, with unit vector $+\hat{\imath}^*$, (towards the force), in the negative universe. This implies that Newton's second law of motion is different in the negative universe, contrary to the required invariance of natural laws in that universe.

In order for (22) to retain the form of (21), so that Newton's second law of motion remains unchanged in the negative universe, we must let $m \to -m^*$ in it to have as follows

$$-\vec{F}^* = \left(-m^* \frac{d^2 x^*}{dt^{*2}}\right)(\hat{\imath}^*) = \left(m^* \frac{d^2 x^*}{dt^{*2}}\right)(-\hat{\imath}^*)\,, \qquad (23)$$

which is of the form of (21) upon cancelling the signs. The fact that we must let $m \to -m^*$ in (22) to arrive at (23) implies that mass is a negative quantity in the negative universe.

Newton's second law has been chosen because it involves spacetime coordinates and mass and no other physical quantity or constant. However the negation of mass in the negative universe does not depend on the natural law adopted, it follows from any chosen law once the signs in the negative universe of other physical quantities and physical constants that appear in that law have been correctly substituted, in addition to the simultaneous negation of space and time coordinates in the law.

The negation of mass also follows from the required invariance of the metric tensor with the reflection of spacetime dimensions. For if we consider the Schwarzschild metric in empty space at the exterior of a spherically symmetric gravitational field source, for example, then the non-trivial components of the metric tensor are, $g_{00} = -g_{11}^{-1} = 1 - 2GM/rc^2$. By letting $r \to -r^*$, we must also let $M \to -M^*$ in order to preserve the metric tensor in the negative universe. It can be verified that this is true for all other metric tensors in General Relativity.

Thus negative mass in the negative universe has again been derived from the symmetry of natural laws between the positive and negative universes, which has been derived from the generalized mass relation in the Special Theory of Relativity in the two-world picture in the preceding section.

For electric charge, the electrostatic field $\vec{E}$ emanating from a particle (assumed spherical in shape) with net electric charge q in the positive universe is given at radial distance r from the centre of the particle as follows

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_0 r^3}\,. \qquad (24)$$

The symmetry-partner electrostatic field emanating from the symmetry-partner particle in the negative universe is inversion in the origin of the electrostatic field in the positive universe. Hence the electrostatic field in the negative universe points in opposite direction in space as its symmetry-partner field $\vec{E}$ of Eq. (24) in the positive universe. This implies that the symmetry-partner electrostatic field in the negative universe is $-\vec{E}^*$. By letting $r \to -r^*$, $\vec{r} \to -\vec{r}^*$ and $\vec{E} \to -\vec{E}^*$ in (24), while retaining q and ϵ_0 meanwhile we have

$$-\vec{E}^* = \frac{q(-\vec{r}^*)}{4\pi\epsilon_0(-r*)^3} = \frac{q\vec{r}^*}{4\pi\epsilon_0 r^{*3}} \qquad (25)$$

In order for (25) to retain the form of (24), so that Coulomb's law remains unchanged in the negative universe, we must let $q/\epsilon_0 \to -(q^*/\epsilon_0^*)$ to have

$$-\vec{E}^* = -\frac{q^*\vec{r}^*}{4\pi\epsilon_0^* r^{*3}}\,, \qquad (26)$$

which is of the form of Eq. (24) upon cancelling the signs. The negative sign of $-(q^*/\epsilon_0^*)$ is associated with the electric charge, while the electric permittivity of free space retains its positive sign in the negative universe. This can be ascertained from the relation for the divergence of electric field namely,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}\,. \qquad (27)$$

In the negative universe, we must let $\vec{\nabla} \to -\vec{\nabla}^*$, $\vec{E} \to -\vec{E}^*$, $\rho \to \rho^*$, (since $\rho = q/V \to -q^*/(-V^*) = q^*/V^* = \rho^*$), while retaining ϵ_0 meanwhile in (27) to have

$$-\vec{\nabla}^* \cdot (-\vec{E}^*) = \frac{\rho^*}{\epsilon_0}\,. \qquad (28)$$

In order for (28) to retain the form of (27), we must let $\epsilon_0 \to \epsilon_0^*$, which confirms the positivity of the electric permittivity of free space in the negative universe. The conclusion then is that the electric charge of a particle in the negative universe has opposite sign as the electric charge of its symmetry-partner in the positive universe.

We are now left to determine the sign in the negative universe of the last fundamental quantity namely, absolute temperature. It has been found impossible to determine the sign of absolute temperature in the negative universe in a unique manner from consideration of the equations of thermodynamics, kinetic theory of gases and transport phenomena. It has been necessary to make recourse to the more fundamental notions of the "arrow of entropy" and "arrow of time" in order to propagate. These notions have been made tangible by the works of Prigogine [3].

We know that entropy always increases or always "flows" along the positive direction of the "entropy axis" S in our (or the positive) universe, even as time always increases or always "flows" into the future direction, that is, along the positive time axis ct in our universe. Thus the arrow of time and the arrow of entropy lie parallel to each other in our universe. Or in the words of Prigogine, "a [positively directed] arrow of time is associated with a [positively directed] arrow of entropy". Thus absolute entropy is a positive quantity in our (or positive) universe, just as time is a positive quantity in our (or positive) universe. The arrow of time and the arrow of entropy likewise lie parallel to each other in the negative universe. We then infer from this that entropy is negatively directed and is hence a negative quantity in the negative universe, since time is negatively directed and is hence a negative quantity in the negative universe.

Having determined the sign of absolute entropy in the negative universe from the above reasoning, it is now an easy matter to determine the sign of absolute temperature in the negative universe. For let us write the following fundamental relation for absolute entropy in our universe:

$$S = k \ln W \,, \tag{29}$$

where k is the Boltzmann constant and W is the number of micro-states in an ensemble in the quantum-mechanical formulation [4]. In the negative universe, we must let $S \to -S^*$ and $W \to W^*$ while retaining k meanwhile to have as follows

$$-S^* = k \ln W^*. \tag{30}$$

In order for (30) to retain the form of (29) we must let $k \to -k^*$, in (30), to have as follows

$$-S^* = -\, k^* \ln W^*, \tag{31}$$

which is of the form of (29) upon cancelling the signs. Thus the Boltzmann constant is a negative quantity in the negative universe.

The average energy ε of a molecule, for one degree-of-freedom motion of a diatomic molecule in a gas maintained at thermal equilibrium at temperature T, is given as follows

$$\varepsilon = \frac{2}{3}\, kT \,, \tag{32}$$

where, again, k is the Boltzmann constant. In the negative universe, we must let $\varepsilon \to -\varepsilon^*$, (since the kinetic energy $\frac{1}{2}mv^2$ of molecules, like mass m, is a negative quantity in the negative universe), and $k \to -k^*$, in (32) while retaining T meanwhile to have as follows

$$-\varepsilon^* = \frac{2}{3}\,(-k^*)\, T \,, \tag{33}$$

which is of the form of (32) upon cancelling the signs. The transformation, $T \to T^*$, required to convert Eq. (33) into Eq. (32) implies that absolute temperature is a positive quantity in the negative universe.

In summary, the fundamental quantities namely, mass m, electric charge Q and absolute temperature T, transform between the positive and negative universes as, $m \to -m^*$, $Q \to -Q^*$ and $T \to T^*$.

By writing various natural laws in terms of negative spacetime dimensions, negative mass, negative electric charge and positive absolute temperature and requiring the laws to retain their usual forms in the positive universe, the signs of other physical quantities and constants in the negative universe can be derived. However a faster way of deriving the signs in the negative universe of derived physical quantities and constants is to check the signs of their dimensions in the negative universe, as demonstrated for a few quantities and constants below.

Let us consider the Boltzmann constant k and absolute entropy S, whose negative signs in the negative universe have been deduced above. They both have the unit, Joule/Kelvin, or dimension $ML^2/T^2\Theta$ in the positive universe, where M represents mass "dimension", L represents length dimension, T represents time dimension and Θ represents absolute temperature "dimension". In the negative universe, we must let $M \to -M^*$, $L \to -L^*$, $T \to -T^*$ and $\Theta \to \Theta^*$, to have the dimensions of Boltzmann constant and absolute entropy in the negative universe as $-M^*(-L^*)^2/(-T^*)^2\Theta^* = -M^*L^{*2}/T^{*2}\Theta^*$. The Boltzmann constant and absolute entropy are negative quantities in the negative universe, since their common dimension is negative in the negative universe.

The Planck constant has the unit Joule/second and dimension ML^2/T^3 in the positive universe. In the negative universe, it has dimension of $-M^*(-L^*)^2/(-T^*)^3)$, which is positive. Hence the Planck constant is a positive quantity in the negative universe.

The specific heat capacity c_p has the unit Joule/kg×Kelvin and dimension $L^2/T^2\Theta$ in the positive universe. In the negative universe it has dimension $(-L^*)^2/\,(-T^*)^2\Theta^*$, which is positive. Hence specific heat capacity is a positive quantity in the negative universe.

The electric permittivity of space ϵ has the unit of Joule×metre/Coulomb^2 and dimension ML^3/T^2C^2 in the positive universe, where C is used to represent the charge "dimension". In the negative universe, it has dimension $(-M^*)(-L^*)^3/(-T^*)^2(-C^*)^2 = M^*L^{*3}/T^{*2}C^{*2}$, which is positive. Hence the electric permittivity of space is a positive quantity in the negative universe. This fact has been derived earlier in the process of deriving the sign of electric charge in the negative universe. Likewise magnetic permeability of space μ has dimension ML/C^2 in the positive universe and dimension $-M^*(-L^*)/(-C^*)^2 = M^*L^*/C^{*2}$, in the negative universe. It is hence a positive quantity in both the positive and negative universes.

An angular measure in space in the positive universe has the same sign as the symmetry-partner angular measure in the negative universe. This follows from the fact that an arc length, $s = r\theta$ [metre], in the positive universe corresponds to a negative arc length, $s^* = -\,(r^*\theta^*)$ [−metre*], in the negative universe. In other words, an arc length in the positive universe and its symmetry-partner in the negative universe transform as, $r\theta \to -(r^*\theta^*)$. But the radii of the symmetry-partner arcs transform as, $r \to -r^*$. It follows from these two transformations that an angular measure in space in the positive universe has the same sign as its symmetry-partner in the negative universe, that is, $\pm\theta \to \pm\theta^*$ and $\pm\varphi \to \pm\varphi^*$, etc.

Finally, a dimensionless quantity or constant in the positive universe necessarily has the same sign as its symmetry-partner in the negative universe, as follows from the above. Examples of dimensionless constants are the dielectric constants, ϵ_r and μ_r.

Table 1 gives a summary of the signs of some physical

Physical quantity/constant	Symbol	Intrinsic quantity/ constant	Sign: positive universe	Sign: negative universe
Distance/dimension of space	$dx\,;\,x$	$d\phi x\,;\,\phi x$	+	−
Interval/dimension of time	$dt\,;\,t$	$d\phi t\,;\,\phi t$	+	−
Mass	m	ϕm	+	−
Electric charge	q	q	+ or −	− or +
Absolute entropy	S	ϕS	+	−
Absolute temperature	T	T	+	+
Energy (total, kinetic)	E	ϕE	+	−
Potential energy	U	ϕU	+ or −	− or +
Radiation energy	$h\nu$	$h\phi\nu$	+	−
Electrostatic potential	Φ_E	$\phi\Phi_E$	+ or −	+ or −
Gravitational potential	Φ_g	$\phi\Phi_g$	−	−
Electric field	$\vec{E}$	ϕE	+ or −	− or +
Magnetic field	$\vec{B}$	ϕB	+ or −	− or +
Planck constant	h	h	+	+
Boltzmann constant	k	ϕk	+	−
Thermal conductivity	k	ϕk	+	−
Specific heat capacity	c_p	ϕc_p	+	+
Speed	v	ϕv	+	+
Electric permittivity	ϵ_0	$\phi\epsilon_0$	+	+
Magnetic permeability	μ_0	$\phi\mu_0$	+	+
Angle	θ, φ	$\phi\theta, \phi\varphi$	+ or −	+ or −
Parity	Π	$\phi\Pi$	+ or −	− or +
⋮	⋮	⋮	⋮	⋮

Table 1: The signs of physical parameters/intrinsic parameters and physical constants/intrinsic constants in the positive and negative universes.

quantities and physical constants in the positive and negative universes. The signs in the positive and negative universes of other physical quantities and constants that are not included in Table 1 can be easily determined from the signs of their dimensions in the negative universe. The appropriateness of the names positive universe and negative universe is made clearer by Table 1.

5 Demonstrating the invariance of the natural laws in the negative universe

It shall be shown in this section that the simultaneous negations of spacetime dimensions and mass, along with simultaneous reversal of the sign of electric charge, retention of the positive sign of absolute temperature and substitution of the signs of other physical quantities and physical constants in the negative universe summarized in column 5 of Table 1 in its complete form, render all natural laws unchanged. However only the invariance of a few laws in the negative universe namely, mechanics (classical and special-relativistic), quantum mechanics, electromagnetism and propagation of light, the theory of gravity, cosmology and fundamental interactions in elementary particle physics shall be demonstrated for examples.

5.1 Further on the invariance of classical mechanics, classical gravitation and Special Relativity in the negative universe

Demonstrating the invariance of classical mechanics in the negative universe consists essentially in showing that Newton's laws of motion for a body under an impressed force and due to interaction of the body with an external force field are invariant under the simultaneous operations of inversion of all coordinates (or dimensions) of 3-space (parity inversion), time reversal and mass negation. The laws are given respectively as follows in the positive universe:

$$\vec{F}_{\text{mech}} = m\,\frac{d^2r}{dt^2}\,\hat{r} \tag{34}$$

and

$$\vec{F}_{\text{field}} = m\,(-\nabla\Phi)\,\hat{k}\,, \tag{35}$$

where $\hat{r}$ and $\hat{k}$ are unit vectors in the directions of the forces $\vec{F}_{\text{mech}}$ and $\vec{F}_{\text{field}}$ respectively.

In the negative universe, we must let $\vec{F}_{\text{mech}} \to -\vec{F}^*_{\text{mech}}$, $\vec{F}_{\text{field}} \to -\vec{F}^*_{\text{field}}$, $m \to -m^*$, $r \to -r^*$, $t \to -t^*$, $\nabla \to -\nabla^*$, $\Phi \to \Phi^*$ (for gravitational and elastic potentials), $\hat{r} \to -\hat{r}^*$ and $\hat{k} \to -\hat{k}^*$ in (34) and (35) to have as follows

$$-\vec{F}^*_{\text{mech}} = -\,m^*\,\frac{d^2(-r^*)}{d(-t^*)^2}\,(-\hat{r}^*) = m^*\,\frac{d^2r^*}{dt^{*2}}\,(-\hat{r}^*) \tag{36}$$

and

$$
\begin{aligned}
-\vec{F}^{*}_{\text{field}} &= -m^{*}\Big(-(-\nabla^{*})(\Phi^{*})\Big)(-\hat{k}^{*}) \\
&= m^{*}(-\nabla^{*}\Phi^{*})(-\hat{k}^{*}) \qquad (37)
\end{aligned}
$$

Equations (36) and (37) are the same as Eqs. (34) and (35) respectively upon cancelling the signs.

The invariance in the negative universe of the classical laws of motion (34) and (35) in the positive universe implies that a body of negative mass $-m^{*}$ in the negative universe moves along a trajectory, when impressed upon by an external mechanical force $-\vec{F}^{*}_{\text{mech}}$, or when it is moving within a force field with potential function Φ^{*} in the negative universe, which is identical to the trajectory followed by the symmetry-partner body of positive mass m in the positive universe, which is impressed upon by an external symmetry-partner mechanical force $\vec{F}_{\text{mech}}$ or which is moving within a symmetry-partner force field with potential function Φ in the positive universe.

The invariance in the negative universe of trajectories of a body implied by the invariance in the negative universe of the differential classical laws of motion (34) and (35) for the body, established above can be alternatively formulated as the invariance in the negative universe of the variational formula of Maupertuis. In the positive universe, this is given as follows

$$
\delta \int_{p_1}^{P_2} \left(\frac{2}{m}(E-U)\right)^{1/2} dt = 0 \,. \qquad (38)
$$

In the negative universe, we must let $m \to -m^{*}$, $E \to -E^{*}$, $U \to -U^{*}$ and $dt \to -dt^{*}$ in (38) to have as follows

$$
\begin{aligned}
&\delta \int_{p_1^{*}}^{p_2^{*}} \left(\frac{2}{-m^{*}}(-E^{*}-(-U^{*}))\right)^{1/2} (-dt^{*}) = \\
&= \delta \int_{p_1^{*}}^{p_2^{*}} \left(\frac{2}{m^{*}}(E^{*}-U^{*})\right)^{1/2} dt^{*} = 0 \,. \qquad (39)
\end{aligned}
$$

The summary of the above is that although inertial mass, kinetic energy, distances in space and periods of time are negative in the negative universe, material particles in the negative universe perform identical motions under impressed forces and external force fields as their symmetry-partners perform under symmetry-partner impressed forces and external force fields in the positive universe. Thus outward external forces lead to outward motions of bodies both in the positive and negative universes. Attractive gravitational field in the positive universe correspond to symmetry-partner repulsive gravitational field in the negative universe, but they both give rise to attractive motions of particles (towards the field sources) in both universes. In brief, the transformation of classical mechanics in the positive universe into the negative universe does not give rise to strange motions and associated strange phenomena.

Demonstrating the invariance of classical gravitation (or classical gravitational interaction) in the negative universe consists in showing the invariance in the negative universe of the Newtonian law of gravity in differential form and the implied Newtonian law of universal gravity,

$$
\vec{\nabla}\cdot\vec{g} = -4\pi G\varrho \qquad (40)
$$

or

$$
\nabla^{2}\Phi = 4\pi G\varrho \qquad (41)
$$

and

$$
\vec{F} = m\vec{g} = -\frac{GMm\vec{r}}{r^{3}}\,, \qquad (42)
$$

where

$$
\begin{aligned}
\varrho &= m/V \;\; (\text{mass}-\text{density}), \qquad (43)\\
\Phi &= -GM/r\,, \qquad (44)\\
\vec{g} &= -GM\vec{r}/r^{3}. \qquad (45)
\end{aligned}
$$

In writing equations (43)–(45) in the negative universe, we must let $m \to -m^{*}$; $M \to -M^{*}$; $r \to -r^{*}$ and $V \to -V^{*}$ (volume of m) to have

$$
\frac{m}{V} \to \frac{-m^{*}}{-V^{*}} = \frac{m^{*}}{V^{*}} \Rightarrow \varrho \to \varrho^{*} \qquad (46)
$$

$$
-\frac{GM}{r} \to -\frac{G(-M^{*})}{-r^{*}} = -\frac{GM^{*}}{r^{*}} \Rightarrow \Phi \to \Phi^{*} \qquad (47)
$$

and

$$
-\frac{GM\vec{r}}{r^{3}} \to -\frac{G(-M)(-\vec{r}^{*})}{(-r^{*})^{3}} = \frac{GM^{*}\vec{r}^{*}}{r^{*3}} \Rightarrow \vec{g} \to -\vec{g}^{*}. \qquad (48)
$$

By using the transformations (46)–(48) along with $\vec{\nabla} \to -\vec{\nabla}^{*}$ in equations (40)–(42) we have

$$
(-\vec{\nabla}^{*})\cdot(-\vec{g}^{*}) = -4\pi G\varrho^{*}
$$

or

$$
\vec{\nabla}^{*}\cdot\vec{g}^{*} = -4\pi G\varrho^{*}, \qquad (49)
$$

$$
(-\nabla^{*})^{2}\Phi^{*} = 4\pi G\varrho^{*}
$$

or

$$
\nabla^{*2}\Phi^{*} = 4\pi G\varrho^{*} \qquad (50)
$$

and

$$
\vec{F}^{*} = (-m^{*})(-\vec{g}^{*}) = -\frac{G(-M^{*})(-m^{*})(-\vec{r}^{*})}{(-r^{*})^{3}}
$$

or

$$
\vec{F}^{*} = m^{*}\vec{g}^{*} = -\frac{GM^{*}m^{*}\vec{r}^{*}}{r^{*3}}\,. \qquad (51)
$$

A comparison of equations (40)–(42) in the positive universe with the corresponding equations (49)–(51) in the negative universe, shows that the Newtonian law of gravity in differential form and the implied Newtonian law of universal gravity are invariant in the negative universe. The invariance of classical gravitation (or classical gravitational interaction) in the negative universe has thus been demonstrated. This is true despite the fact that gravitational potential does not change sign while gravitational field (or gravitational acceleration) changes sign in the negative universe according to equations (47) and (48).

Demonstrating the invariance of Special Relativity in the negative universe consists in showing the invariance of Lorentz transformation, time dilation and length contraction formulae and the special-relativistic expressions for mass and other quantities in that universe. Now in the positive universe, for motion at speed v of a particle of rest mass m_0 along the x-axis of the coordinate system attached to it relative to an observer, the Lorentz transformation of the coordinates $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ of the primed (or particle's) frame into the coordinates $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ of the unprimed (or observer's) frame has been written as system (3). The special-relativistic mass is given in the positive universe by the usual expression (13), which shall be re-written here as

$$m = \gamma m_0 \,. \tag{52}$$

In the negative universe, we must let $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'; m_0) \to (-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*; -m_0^*)$, and also $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t}; m) \to (-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*, -c\tilde{t}^*; -m^*)$, yielding the Lorentz transformation of the coordinates of the frame of reference attached to the symmetry-partner particle in motion relative to the symmetry-partner observer in the negative universe written as system (38) in [1], which shall be re-written here as follows

$$\left.\begin{aligned} -\tilde{x}'^* &= \gamma\left(-\tilde{x}^* - v(-\tilde{t}^*)\right) \\ -\tilde{t}'^* &= \gamma\left(-\tilde{t}^* - \frac{v}{c^2}(-\tilde{x}^*)\right) \\ -\tilde{y}'^* &= -\tilde{y}^*, \quad -\tilde{z}'^* = -\tilde{z}^* \end{aligned}\right\}, \tag{53}$$

while the expression for special-relativistic mass in the negative universe becomes the following

$$-m^* = -\gamma m_0^*. \tag{54}$$

The expressions for time dilation and length contraction in the negative universe are similarly given respectively as follows

$$\Delta(-\tilde{t}^*) = \gamma\,\Delta(-\tilde{t}'^*)\,, \tag{55}$$

$$\Delta(-\tilde{x}^*) = \gamma^{-1}\,\Delta(-\tilde{x}'^*)\,. \tag{56}$$

Although the negative signs must be retained in (53), (54), (55) and (56) in the negative universe, mathematically the signs cancel, thereby making Lorentz transformation and the other equations of Special Relativity to retain their usual forms in the negative universe. Thus Lorentz invariance, (and local Lorentz invariance in gravitational fields), hold in the negative universe.

5.2 Invariance of quantum mechanics in the negative universe

The time-dependent Schrödinger wave equation is the following in the positive universe

$$H(\vec{r}, t, m, q)\,|\Psi(\vec{r}, t, m, q)\rangle = i\hbar\,\frac{\partial}{\partial t}\,|\Psi(\vec{r}, t, m, q)\rangle\,. \tag{57}$$

By writing (57) in the negative universe, while leaving Ψ unchanged meanwhile, we have

$$\begin{aligned} -H^*(-\vec{r}^*, -t^*, -m^*, -q^*)\,&|\Psi(\vec{r}, t, m, q)\rangle \\ &= i\hbar^*\frac{\partial}{\partial(-t^*)}\,|\Psi(\vec{r}, t, m, q)\rangle\,, \end{aligned} \tag{58}$$

where the fact that the Boltzmann constant transforms as $\hbar \to \hbar^*$ between the positive and negative universes in Table 1 has been used.

Now the wave function should transform between the positive and negative universes either as

$$\begin{aligned} \Psi(\vec{r}, t, m, q) \to \Psi^*(-\vec{r}^*, -t^*, -m^*, -q^*) &= \\ = \Psi^*(\vec{r}^*, t^*, m^*, q^*)& \end{aligned} \tag{59}$$

or as

$$\begin{aligned} \Psi(\vec{r}, t, m, q) \to -\Psi^*(-\vec{r}^*, -t^*, -m^*, -q^*) &= \\ = -\Psi^*(\vec{r}^*, t^*, m^*, q^*)\,.& \end{aligned} \tag{60}$$

The parity of the wave function is conserved in (59) and inverted in (60).

Let us consider the following wave function in the positive universe,

$$\Psi(\vec{r}, t) = A\sin\left(\vec{k}\cdot\vec{r} - \omega t\right) \tag{61}$$

The symmetry-partner wave function in the negative universe is obtained by letting $\vec{r} \to -\vec{r}^*$, $\vec{k} \to -\vec{k}^*$, $\omega \to -\omega^*$, $t \to -t^*$ and $A \to -A^*$ in (61) to have

$$\begin{aligned} \Psi^*(\vec{r}, t) &= -A^*\sin\left(-\vec{k}^*\cdot(-\vec{r}^*) - (-\omega^*)(-t^*)\right) \\ &= -A^*\sin\left(\vec{k}^*\cdot\vec{r}^* - \omega^* t^*\right)\,. \end{aligned} \tag{62}$$

The transformation $A \to -A^*$ is necessary since inversion in the origin of the coordinates of a Euclidean 3-space inverts the amplitude of a wave in that space. On the other hand, the phase of a wave function, being a dimensionless number, does not change sign in the negative universe. Thus the transformation (60) and not (59) is the correct transformation of the wave function between the positive and negative universes. This is obviously so since (60) is a parity inversion situation, which is in agreement with the natural parity inversion of a wave, $\Pi \to -\Pi$, between the positive and negative universes included in Table 1. By incorporating the transformation (60) into (58) we obtain the following

$$\begin{aligned} -H^*(-\vec{r}^*, -t^*, -m^*, -q^*)\,&|-\Psi^*(-\vec{r}^*, -t^*, -m^*, -q^*)\rangle = \\ &= -i\hbar^*\,\frac{\partial}{\partial t^*}\,|-\Psi^*(-\vec{r}^*, -t^*, -m^*, -q^*)\rangle \end{aligned}$$

or

$$\begin{aligned} H^*(\vec{r}^*, t^*, m^*, q^*)\,&|\Psi^*(\vec{r}^*, t^*, m^*, q^*)\rangle = \\ &= i\hbar^*\frac{\partial}{\partial t^*}\,|\Psi^*(\vec{r}^*, t^*, m^*, q^*)\rangle\,. \end{aligned} \tag{63}$$

This is of the form of Eq. (57). The invariance of the Schrödinger wave equation in the negative universe has thus been established. It is straight forward to demonstrate the invariance in the negative universe of the Dirac's equation for the electron and of Gordon's equation for bosons.

5.3 Invariance of Maxwell equations in the negative universe

The Maxwell equations in a medium with electric charge density ρ and electric current density $\vec{J}$ are given in the positive universe as follows

$$\left.\begin{array}{ll}\vec{\nabla}\cdot\vec{E}=\dfrac{\rho}{\epsilon}\,, & \vec{\nabla}\cdot\vec{B}=0\\ \vec{\nabla}\times\vec{B}=\mu\vec{J}+\epsilon\mu\,\dfrac{\partial\vec{E}}{\partial t}\,, & \vec{\nabla}\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\end{array}\right\}. \tag{64}$$

Now, $\rho=\frac{charge}{volume}$, is the electric charge density of the medium in the positive universe. The charge density of the symmetry-partner medium in the negative universe is the positive quantity, $\frac{-charge^*}{-volume^*}=\frac{charge^*}{volume^*}=\rho^*$. The magnitude of an electric current is, $I=\frac{charge}{time}$ or $I=\rho vA$, in the positive universe and the magnitude of its symmetry-partner in the negative universe is the positive quantity, $\frac{-charge^*}{-time^*}=\frac{charge^*}{time^*}=I^*$ or $\rho^* vA^*=I^*$, since speed v and area A do not change sign in the negative universe. Similarly the magnitude of an electric current density of a medium in the positive universe is, $J=\frac{current}{area}$, and the magnitude of the current density of the symmetry-partner medium in the negative universe is, $\frac{current^*}{area^*}=J^*$. Thus in obtaining the Maxwell equations in the negative universe, we must let $\vec{E}\to-\vec{E}^*$, $\vec{B}\to-\vec{B}^*$, $\rho\to\rho^*$, $\vec{J}\to\vec{J}^*$, $\vec{\nabla}\to-\vec{\nabla}^*$, $\epsilon\to\epsilon^*$, $\mu\to\mu^*$ and $t\to-t^*$ in system (65) to have as follows

$$\left.\begin{array}{l}-\vec{\nabla}^*\cdot(-\vec{E}^*)=\dfrac{\rho^*}{\epsilon^*}\,,\quad -\vec{\nabla}^*\cdot(-\vec{B}^*)=0\\ -\vec{\nabla}^*\times(-\vec{B}^*)=\mu^*\vec{J}^*+\epsilon^*\mu^*\,\dfrac{\partial(-\vec{E}^*)}{\partial(-t^*)}\\ -\vec{\nabla}^*\times(-\vec{E}^*)=-\dfrac{\partial(-\vec{B}^*)}{\partial(-t^*)}\end{array}\right\}. \tag{65}$$

System (65) with the negative signs is the form the Maxwell equations are written by physicists* in the negative universe. The signs cancel mathematically thereby making system (65) to retain the form of system (64) and thereby establishing the invariance of Maxwell equations in the negative universe.

The law of propagation of electromagnetic waves derived from the Maxwell equations remain invariant in the negative universe as a consequence of the above. The equations are given in the positive universe as follows

$$\nabla^2\vec{E}=\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}\,,\qquad \nabla^2\vec{B}=\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}\,, \tag{66}$$

while in the negative universe, the electromagnetic wave equations are given as follows

$$\left.\begin{array}{l}(-\nabla^*)^2(-\vec{E}^*)=\dfrac{1}{c^2}\dfrac{\partial^2(-\vec{E}^*)}{\partial(-t^*)^2}\\ (-\nabla^*)^2(-\vec{B}^*)=\dfrac{1}{c^2}\dfrac{\partial^2(-\vec{B}^*)}{\partial(-t^*)^2}\end{array}\right\}. \tag{67}$$

Thus as the perpendicular electric field and magnetic field $\vec{E}$ and $\vec{B}$ propagate as electromagnetic wave at the speed of light in the positive universe, the symmetry-partner perpendicular fields $-\vec{E}^*$ and $-\vec{B}^*$ propagate as the identical symmetry-partner electromagnetic wave at the speed of light in the negative universe.

The foregoing shows that although electric charge as well as electric field and magnetic field change signs in the negative universe, the laws of propagation of electric and magnetic fields and electromagnetic waves remain invariant in the negative universe.

5.4 Invariance of General Relativity and cosmology in the negative universe

Since system of coordinates does not enter the covariant tensor formulation of Einstein's field equations, the equations are equally valid for the negative dimensions of the negative universe. The most general form of Einstein's field equations in the positive universe is the following

$$R^\nu_\mu-\frac{1}{2}\,R\,g^\nu_\mu+\Lambda\,g^\nu_\mu=-\frac{8\pi G}{c^2}\,T^\nu_\mu\,, \tag{68}$$

where the energy-momentum tensor T^ν_μ is defined as follows

$$T^\nu_\mu=(p+\rho)\,u^\nu u_\mu-p\,g^\nu_\mu\,, \tag{69}$$

Λ is the cosmological constant, p and ρ are the pressure and density of the universe respectively, while the other quantities in (68) and (69) are as defined in the theory. Λ is usually set to zero in General Relativity when considering local gravitational problems but retained in cosmological problems.

For the static exterior field of a spherical body, we must let $\Lambda=T^\nu_\mu=0$ in (68) and require the vanishing of the Ricci tensor to have as follows

$$R_{\mu\nu}=0 \tag{70}$$

Adopting a metric with signature $(+\,-\,-\,-)$, the Schwarzschild solution to the field equation (70) is the following

$$ds^2=c^2dt^2\left(1-\frac{2GM}{rc^2}\right)-\frac{dr^2}{\left(1-\frac{2GM}{rc^2}\right)}-\\ -r^2\left(d\theta^2+\sin^2\theta\,d\varphi^2\right). \tag{71}$$

By letting $t\to-t^*$, $r\to-r^*$, $\theta\to\theta^*$, $\varphi\to\varphi^*$ and $M\to-M^*$ in (71) we find that the Schwarzschild line element or metric tensor remains invariant in the negative universe. Other forms of exterior line elements or metric tensors, such as Kerr's line element, as well as interior metric tensors remain invariant in the negative universe as well. This is so because ds^2 is quadratic in intervals cdt, dr, $rd\theta$ and $r\sin\theta d\varphi$, and the components of the metric tensor are dimensionless. This concludes the invariance of general relativity in the negative universe.

Now the metric of spatially homogeneous universe in comoving coordinates is the Robertson-Walker metric

$$ds^2 = c^2dt^2 - R(t)^2 \left(\frac{du^2 + u^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right)}{\left(1 + \frac{k}{4} u^2 \right)^2} \right), \quad (72)$$

where $u = r/r_0$ and the constant k is -1, 0 or $+1$, corresponding to spherical space, Euclidean space or pseudo-spherical space. Assuming that the universe is filled with perfect fluid, the field equation (68) along with the energy-momentum tensor (69) have been cast in the following forms, from which various models of the universe have been derived in General Relativity, as can be found in the standard texts on General Relativity

$$\frac{8\pi G\rho}{c^2} = -\Lambda + \left[\frac{3k}{R(t)^2} + \frac{3\dot{R}(t)^2}{c^2R(t)^2} \right], \quad (73)$$

$$\frac{8\pi G}{c^2}\left(\frac{p}{c^2}\right) = \Lambda - \left[\frac{k}{R(t)^2} + \frac{\dot{R}(t)^2}{c^2R(t)^2} + \frac{2\ddot{R}(t)}{c^2R(t)} \right], \quad (74)$$

$$R(t) = R_0 \exp(Ht), \quad R_0 = R(t=0), \quad (75)$$

where $R(t)$ is the "radius" of the universe, H is the Hubble constant given by

$$H = \frac{\dot{R}(t)}{R(t)} = \frac{1}{R(t)} \frac{dR(t)}{dt} \quad (76)$$

and the cosmological constant Λ is related to the Hubble constant H as follows

$$\Lambda = \frac{3H^2}{c^2}. \quad (77)$$

The parameters that appear in cosmological model, that is, in Eqs. (73) through (75), are the global time t, the "radius" of the universe $R(t)$, the mass-density of the universe ρ, the pressure of the universe p, the Hubble constant H, and the cosmological constant Λ. Also the rate of expansion $\dot{R}(t)$, as well as the acceleration $\ddot{R}(t)$, of the expanding universe enter into the equations. In the negative universe, we must let $t \to -t^*$, $R(t) \to -R^*(-t^*)$, $p \to p^*$, $H \to -H^*$, $\Lambda \to \Lambda^*$, $\dot{R}(t) \to \dot{R}^*(-t^*)$ and $\ddot{R}(t) \to -\ddot{R}^*(-t^*)$ in (73) through (75). Doing this, we find that the equations remain unchanged, so that physicists* in the negative universe formulate identical cosmological models as those in the positive universe. Consequently observers* in the negative universe make observation of that universe that are identical to the observation made of the positive universe by observers in the positive universe at all epochs.

It is easy and straight forward to demonstrate the invariance of the kinetic theory of gas, the laws of propagation of heat (conduction, convection and radiation) in continuous media, transport phenomena and other macroscopic laws of physics by following the procedure used to demonstrate the invariance of some macroscopic natural laws above with the aid of the complete form of Table 1.

5.5 Invariance of fundamental interactions in the negative universe

In a formal sense, the invariance in the negative universe of quantum chromodynamics, quantum electrodynamics, the electro-weak theory and quantum gravity must be demonstrated with the aid of the complete form of Table 1 in order to show the invariance in the negative universe of strong, electromagnetic, weak and gravitational interactions among elementary particles, as has been done for the macroscopic natural laws in this section. However we shall not attempt this. Rather we shall make recourse to the CPT theorem to demonstrate the invariance of the strong, electromagnetic and weak interactions in this section.

The CPT theorem, in a simplified form in [5, see p. 712], for instance, states that any hermitian interaction relativistically invariant, commutes with all products of the three operators C (charge conjugation), P (parity inversion), and T (time reversal) in any order. Even if an interaction is not invariant under one or two of the three operations, it must be invariant under CPT. The invariance of strong, weak and electromagnetic interactions under CPT is a well established fact in elementary particle physics [5].

Now the spacetime dimensions $-x^*$, $-y^*$, $-z^*$ and $-ct^*$ (in the Cartesian system of the dimensions of 3-space) of the third quadrant (or of the negative universe) are the products of natural parity inversion operation (P) and time reversal operation (T), (or of natural operation PT), on the spacetime dimensions x, y, z and ct of the first quadrant (or of the positive universe) in Fig. 5 or Fig. 7 of [1]. This implies, for instance, that the parity of a Schrodinger wave in the negative universe is natural inversion of parity of the symmetry-partner Schrodinger wave in the positive universe. The natural parity inversion of classical quantum-mechanical waves between the positive and negative universes equally applies to intrinsic parties of relativistic quantum mechanics and quantum field theories.

As also derived earlier in this paper and included in Table 1, the electric charge Q of a particle in the positive universe corresponds to an electric charge of equal magnitude but of opposite sign $-Q^*$ of the symmetry-partner particle in the negative universe. Thus the electric charge of a particle in the negative universe is the product of natural charge conjugation operation (C) on the electric charge of its symmetry-partner particle in the positive universe.

It follows from the foregoing two paragraphs that strong, weak and electromagnetic interactions among elementary particles in the negative universe are the products of natural operations of parity inversion (P), time reversal (T) and charge conjugation (C), in any order, (or of natural operation CPT), on strong, weak and electromagnetic interactions among elementary particles in the positive universe. The invariance of strong, weak and electromagnetic interactions among elementary particles in the negative universe follow from this and the CPT theorem.

The invariance of classical gravitation and the General Theory of Relativity (or of gravitational interaction) at the macroscopic level in the negative universe has been demonstrated earlier in this section. The invariance in the negative universe of gravitational interaction among elementary particles follow from this. This section shall be ended with a remark that all natural laws, including the fundamental interactions among elementary particles, take on the same forms in the positive and negative universes and this is perfect symmetry of laws between the positive and negative universes.

6 On the concept of negative mass in physics

The concept of negative mass is not new in physics. The earliest speculations include the elaborate theory of negative mass by Föppl in 1897 and Schuster's contemplation of a universe with negative mass in 1898 [6] . However, as mentioned in [6], the fundamental modern paper on negative mass can be deemed to begin with Bondi [7]. As also stated in [6], Bondi pointed out that the mass in classical mechanics actually consists of three concepts namely, inertial mass, m_i, passive gravitational mass m_p, and active gravitational mass m_a. In Newton's theory of gravity, $m_i = m_p = m_a$. Also in the General Theory of Relativity, the principle of equivalence requires that, $m_i = m_p = m_a$. Although all three mass concepts are usually taken to be positive in physics, the theories do not compel this, as noted in [6].

Several papers on negative mass listed in [6] have appeared after Bondi's paper [7]. As noted in [6], most of those papers investigate the interaction and possible co-existence of particles with masses of both signs. The paper by Bonnor [6] is an important reappraisal of the concept of negative mass in the more recent time. In his analysis, Bonnor starts with the assumption m_i, $m_p > 0$, $m_a < 0$. He arrives at the result that either $m_i < 0$, $m_p < 0$ and $m_a < 0$ for all particles and bodies or $m_i > 0$, $m_p > 0$ and $m_a > 0$ for all particles and bodies. He then chooses to work with the former case, that is, all three mass concepts negative in an hypothetical universe. He substitutes negative mass into mechanics, relativity, gravitation as well as cosmology and finds that observers located in the hypothetical universe would observe strange phenomena, such as pebbles or sand falling on a stretched membrane producing tension and not compression of the membrane, and a push on a trolley causing it to accelerate towards the person who pushed it, etc. It is certain that this our universe is not the hypothetical universe containing negative mass in [6].

The hypothetical universe containing negative mass in [6] is not the negative universe isolated in the two parts of this article either. This is so because only mass is made negative while space and time dimensions, as well as other physical quantities and constants retain their signs (in our universe) in the hypothetical universe of [6]. This proviso leads to the deduced observation of strange phenomena in the hypothetical universe. On the other hand, the negative universe of this article contains negative mass along with the negation of space and time dimensions, as well as the signs of other physical quantities and constants summarized in column 5 of Table 1. As demonstrated in the preceding section, the laws of physics retain their usual forms in the negative universe, and observers located in the negative universe observe phenomena in their universe that are identical to the phenomena observed in our (or positive) universe. There are no strange phenomena in the negative universe of the two parts of this article.

This section is perhaps the conclusion of over a century-old effort towards the development of the concept of negative mass in physics. Schuster's speculation one hundred and ten years ago of a universe containing negative mass must have now been realized. This second part of this article shall be ended at this point, while possible further development of the two-world background of Special Relativity (or the two-world picture) shall be investigated elsewhere.

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