Two-World Background of Special Relativity. Part I

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A new sheet of spacetime is isolated and added to the existing sheet, thereby yielding a pair of co-existing sheets of spacetimes, which are four-dimensional inversions of each other. The separation of the spacetimes by the special-relativistic event horizon compels an interpretation of the existence of a pair of symmetrical worlds (or universes) in nature. Further more, a flat two-dimensional intrinsic spacetime that underlies the flat four-dimensional spacetime in each universe is introduced. The four-dimensional spacetime is outward manifestation of the two-dimensional intrinsic spacetime, just as the Special Theory of Relativity (SR) on four-dimensional spacetime is mere outward manifestation of the intrinsic Special Theory of Relativity (ϕSR) on two-dimensional intrinsic spacetime. A new set of diagrams in the two-world picture that involves relative rotation of the coordinates of the two-dimensional intrinsic spacetime is drawn and intrinsic Lorentz transformation derived from it. The Lorentz transformation in SR is then written directly from intrinsic Lorentz transformation in ϕSR without any need to draw diagrams involving relative rotation of the coordinates of four-dimensional spacetime, as usually done until now. Indeed every result of SR can be written directly from the corresponding result of ϕSR. The non-existence of the light cone concept in the two-world picture is shown and good prospect for making the Lorentz group SO(3,1) compact in the two-world picture is highlighted.

1 Introduction

The concept of other universe(s) or world(s) is not new in physics. In 1898, Schuster contemplated a universe containing negative mass [1]. The discovery in particle physics of the existence of an anti-particle to every particle afterwards, led some physicists to suggest the existence of an anti-atom (composed of anti-particles) to every atom (composed of particles); an anti-molecule to every molecule and an anti-macroscopic-object to every macroscopic object. Then in order to explain the preponderance of particles and matter over anti-particles and anti-matter respectively in this our universe, the existence of an anti-universe containing a preponderance of anti-matter over matter was suggested, as discussed in [2, see p. 695], for instance. However it has remained unknown until now whether the speculated universe containing negative mass of Schuster and an anti-universe containing a preponderance of anti-matter exist or not.

The purpose of this article is to show formally that the Special Theory of Relativity rests on a background of a two-world picture, in which an identical partner universe in a different spacetime to this universe of ours in our spacetime co-exist, and to commence the development of the two-world picture thus introduced. The placement of the other universe relative to our universe, as well as the configuration of matter in it shall be derived. The symmetry of state and symmetry of laws between the two universes shall be established. The definite interaction between the two universes in relativistic phenomena shall also be shown.

This article may be alternatively entitled as *Isolating a Symmetry-Partner Universe to Our Universe in the Context of the Special Theory of Relativity*. Apart from the derivation of the Lorentz transformation (LT) and its inverse with the aid of a new set of spacetime/intrinsic spacetime diagrams on the combined spacetimes/intrinsic spacetimes of the two co-existing identical "anti-parallel" universes, there are no further implications on the other results of SR usually derived from the LT and its inverse in the existing one-world picture. However SR must be deemed to be tremendously expanded or made more complete by exposing its two-world background and by the addition of a parallel two-dimensional intrinsic Special Theory of Relativity (ϕSR) on a flat two-dimensional intrinsic spacetime that underlies the flat four-dimensional spacetime of SR in each of the two universes.

There are several new implications of the two-world picture for SR as well, which include the non-existence of the light cone concept, good prospect for making SO(3,1) compact, a feat that has proved impossible in the existing one-world picture and inter-universe transitions of symmetry-partner particles between the two universes (at super-high energy regimes), on which the prospect for experimental test ultimately of the two-world picture rests. This initial articles goes as far as a single article can on the vast subject of two-world symmetry that lies at the foundation of the Special Theory of Relativity and possibly the whole of physics.

2 Two schemes towards the Lorentz boost

As can be easily demonstrated, the two schemes summarized in Table 1 both lead to the Lorentz boost, (which shall also

Scheme I	Scheme II
$x = x' \cosh\alpha + ct' \sinh\alpha$	$x = x' \sec\psi + ct' \tan\psi$
$ct = ct' \cosh\alpha + x' \sinh\alpha$	$ct = ct' \sec\psi + x' \tan\psi$
$y = y'\ ;\ z = z'$	$y = y'\ ;\ z = z'$
$\cosh\alpha = \dfrac{1}{\sqrt{1 - v^2/c^2}} = \gamma$	$\sec\psi = \dfrac{1}{\sqrt{1 - v^2/c^2}} = \gamma$
$\sinh\alpha = \dfrac{v/c}{\sqrt{1 - v^2/c^2}} = \beta\gamma$	$\tan\psi = \dfrac{v/c}{\sqrt{1 - v^2/c^2}} = \beta\gamma = \beta\gamma$
$\tanh\alpha = v/c = \beta$	$\sin\psi = v/c = \beta$

Table 1: Two schemes towards the derivation of the Lorentz boost graphically.

Fig. 1: a) All values of the number α generate the positive half-plane in Scheme I and b) all values of the angle ψ in the first cycle generate the positive and negative half-planes in Scheme II.

be referred to as the Lorentz transformation (LT)) and the Lorentz invariance (LI). Although the $\gamma = \cosh\alpha$ parametrization of the LT in Scheme I is more familiar, the $\gamma = \sec\psi$ parametrization in Scheme II is also known.

Now by letting $v/c = 0$ in Table 1 we obtain the following:

$$\cosh\alpha = 1;\ \sinh\alpha = \tanh\alpha = 0 \ \Rightarrow\ \alpha = 0,$$
$$\sec\psi = 1;\ \tan\psi = \sin\psi = 0 \ \Rightarrow\ \psi = 0.$$

By letting $v/c = 1$ we have

$$\cosh\alpha = \sinh\alpha = \infty;\ \tanh\alpha = 1 \ \Rightarrow\ \alpha = \infty,$$
$$\sec\psi = \tan\psi = \infty;\ \sin\psi = 1 \ \Rightarrow\ \psi = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$$

And by letting $v/c = -1$ we have

$$\cosh\alpha = \infty;\ \sinh\alpha = -\infty;\ \tanh\alpha = -1 \ \Rightarrow\ \alpha = -\infty,$$
$$\sec\psi = \infty;\ \tan\psi = -\infty;\ \sin\psi = -1 \ \Rightarrow\ \psi = -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \dots$$

Thus there are the following equivalent ranges of values of the parameter α and the angle ψ between the two schemes:

$$0 \leqslant \alpha \leqslant \infty \text{ (Scheme I)} \ \equiv\ 0 \leqslant \psi \leqslant \frac{\pi}{2} \text{ (Scheme II)}$$
$$-\infty \leqslant \alpha \leqslant \infty \text{ (Scheme I)} \ \equiv\ -\frac{\pi}{2} \leqslant \psi \leqslant \frac{\pi}{2} \text{ (Scheme II)}$$

The second range, which is $-\infty \leqslant \alpha \leqslant \infty$ (Scheme I) or $-\frac{\pi}{2} \leqslant \psi \leqslant \frac{\pi}{2}$ (Scheme II), generates the positive half-plane shown shaded in Figs. 1a and 1b.

If we consider Scheme I, then clearly there is only the positive half-plane as illustrated in Fig. 1a. This is so since the range $-\infty \leqslant \alpha \leqslant \infty$ generates the positive half-plane only, and there are no other values of α outside this range. Thus going to the negative half-plane is impossible in the context of SR in Scheme I.

If we consider Scheme II, on the other hand, then the range $-\frac{\pi}{2} \leqslant \psi \leqslant \frac{\pi}{2}$, which generates the positive half-plane in Fig. 1b is not exhaustive of the values of angle ψ in the first cycle. There is also the range $\frac{\pi}{2} \leqslant \psi \leqslant \frac{3\pi}{2}$, which generates the negative half-plane. Thus going into the negative half-plane is possible in SR in the context of Scheme II. There
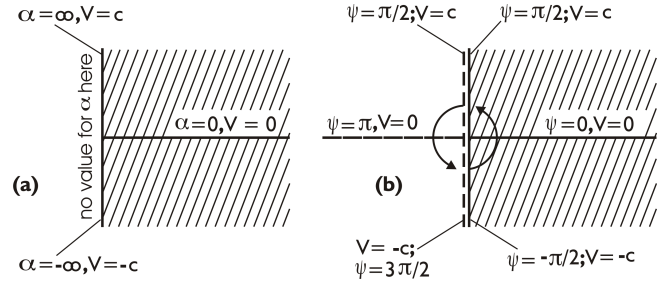
is actually no gap between the solid line and the broken line along the vertical as appears in Fig. 1b.

It must quickly be pointed out that there has not seemed to be any need to consider the second range $\frac{\pi}{2} \leqslant \psi \leqslant \frac{3\pi}{2}$ (or the negative half-plane) in Fig. 1b in physics until now because the parity inversion and time reversal associated with it can be achieved by reflection of coordinates of 3-space in the first range $-\frac{\pi}{2} \leqslant \psi \leqslant \frac{\pi}{2}$ (or in the positive half-plane) that also includes time reversal. However we consider it worthy of investigation whether the range $\frac{\pi}{2} \leqslant \psi \leqslant \frac{3\pi}{2}$ and the parity inversion it implies exist naturally apart from the possibility of parity inversion by coordinate reflection in the positive half-plane. Reasoning that parity inversion and time reversal will not be the only physical significance of the second range $\frac{\pi}{2} \leqslant \psi \leqslant \frac{3\pi}{2}$ (or the negative half-plane) in Fig. 1b, should it exist in nature, we deem it judicious to carry both ranges $-\frac{\pi}{2} \leqslant \psi \leqslant \frac{\pi}{2}$ and $\frac{\pi}{2} \leqslant \psi \leqslant \frac{3\pi}{2}$ along in the present development with the hope that the theory shall ultimately justify the existence of the second range or otherwise.

In translating Figs. 1a and 1b into spacetime diagrams, the positive horizontal lines along which, $v = 0$, $\alpha = 0$ and $\psi = 0$, in the figure, correspond to the 3-dimensional Euclidean space Σ with mutually orthogonal dimensions x, y and z in the Cartesian system of coordinates; the positive vertical lines along which, $v = c$, $\alpha = \infty$ and $\psi = \frac{\pi}{2}$, correspond to the positive time dimension ct, while the negative vertical lines along which $v = -c$, $\alpha = -\infty$ and $\psi = -\frac{\pi}{2}$, correspond to the negative time dimension (or the time reversal dimension) $-ct^*$. In addition, the horizontal line in the negative half-plane in Fig. 1b corresponds to a negative 3-dimensional Euclidean space (not known in physics until now) to be denoted by $-\Sigma^*$ with mutually orthogonal dimensions $-x^*$, $-y^*$ and $-z^*$ in the rectangular system. Thus Figs. 1a and 1b translate into the space-time diagrams of Figs. 2a and 2b respectively. Representation of the Euclidean 3-spaces by lines along the horizontal and the time dimensions by vertical normal lines to the "space axes", as done in Figs. 2a and 2b, is a well known practice in the graphical representation of four-dimensional spacetime, exemplified by the modern Minkowski diagrams [3].

Figure 2a pertains to Scheme I in Table 1. The four-dimensional spacetime with dimensions x, y, z and ct is the
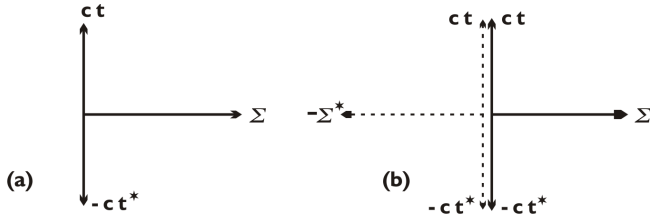
Fig. 2: The spacetime domains generated by a) all values of the number α in Scheme I and b) all values of the angle ψ in the first cycle in Scheme II.

Minkowski space as known. In addition, there is the negative time dimension $-ct^*$ that serves the role of time reversal dimension, (which is different from the *past time* axis in the past light cone). There are no second and third quadrants in Fig. 2a, since the negative half-plane is inaccessible in Scheme I.

Figure 2b pertains to Scheme II in Table 1. There are two "anti-parallel" Minkowski spaces in Fig. 2b namely, the one with positive dimensions, $(\Sigma, ct) \equiv (x, y, z, ct)$, generated by the range of angles $0 \leqslant \psi \leqslant \frac{\pi}{2}$ in the first quadrant in Fig. 1b, to be referred to as the positive Minkowski space, and the other with all negative dimensions, $(-\Sigma^*, -ct^*) \equiv (-x^*, -x^*, -y^*, -ct^*)$, generated by the range of angles $\pi \leqslant \psi \leqslant \frac{3\pi}{2}$ in the third quadrant, to be referred to as the negative Minkowski space. There are in addition the negative time dimension $-ct^*$ that serves the role of the time reversal dimension to the positive Minkowski space, while the positive time dimension ct serves the role of time reversal dimension to the negative Minkowski space.

It shall again be quickly added that the spacetime dimensions of the negative Minkowski space constitute parity inversion and time reversal with respect to the spacetime dimensions of the positive Minkowski space and conversely. Figure 2b says that this situation exists naturally, quite apart from the fact that parity inversion (by coordinate reflection), $x \rightarrow -x$; $y \rightarrow y$; $z \rightarrow z$ or $x \rightarrow -x$; $y \rightarrow -y$; $z \rightarrow -z$ and time reversal $t \rightarrow -t$ are achievable within the positive half-plane, that is within the positive Minkowski space (fist quadrant) plus the fourth quadrant in Figs. 2a and 2b. Schemes I and II have been known to imply the existence of the positive half plane only in physics until now. The investigation of the implications of the existence naturally of the negative half-plane in parallel with the positive half-plane in Figs. 1b and 2b shall be started in this paper.

3 Minkowski's diagrams as graphical representation of Lorentz transformation in Scheme I

There is essentially nothing new in this section. Its inclusion is necessary so that the derivation newly of the LT and its inverse graphically in the context of Scheme II from the next section can be compared with the known derivation of the LT and its inverse graphically in the context of Scheme I, which

shall be re-presented in this section.

For the relative motion of two frames, (which involves positive time dimension), the time reversal dimension $-ct^*$ is irrelevant, leaving only the first quadrant in Fig. 2a, (in the context of Scheme I). Thus relative rotations of the spacetime coordinates of the particle's (or primed) frame and the observer's (or unprimed) frame, for every pair of frames in relative motion, are limited to the interior of the first quadrant in Scheme I, which corresponds to the first quadrant in Figs. 1a and 2a. As is clear from Fig. 2a, Scheme I pertains to a one-world picture, including the time reversal dimension.

Now the Lorentz transformation (LT) is usually derived analytically in the Special Theory of Relativity (SR), following Albert Einstein's 1905 paper [4]. In his paper, Einstein inferred from two principles of relativity, the LT and its inverse for motion along the x'-direction of the coordinate system (ct', x', y', z') attached to a particle moving at speed v relative to an observer's frame (ct, x, y, z), where the coordinates x' and x are taken to be collinear, respectively as follows:

$$t' = \gamma\left(t - \frac{v}{c^2}x\right); \quad x' = \gamma(x - vt); \quad y' = y; \quad z' = z \quad (1)$$

and

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right); \quad x = \gamma(x' + vt'); \quad y = y'; \quad z = z', \quad (2)$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. As demonstrated in Einstein's paper, each of systems (1) and (2) satisfies the Lorentz invariance,

$$c^2t'^2 - x'^2 - y'^2 - z'^2 = c^2t^2 - x^2 - y^2 - z^2. \quad (3)$$

Somewhat later, Minkowski explored the graphical (or coordinate- geometrical) implication of the LT and its inverse [5]. In the graphical approach, the first two equations of the inverse LT, system (2), is interpreted as representing rotations of the coordinates x' and ct' of the particle's (or primed) frame relative to the coordinates x and ct respectively of the observer's (or unprimed) frame, while the last two equations are interpreted as representing no special-relativistic rotations of coordinates y' and z' relative to y and z respectively (since relative motion of SR does not occur along these coordinates).

The Minkowski spacetime diagrams from which the LT and its inverse have sometimes been derived for two frames in relative motion along their collinear x'- and x-axes, are shown as Figs. 3a and 3b, where the surface of the future light cone is shown by the broken lines.

The coordinates y' and z' of the particle's frame, as well as the coordinates y and z of the observer's frame remain not rotated from the horizontal, and have not been shown in Figs. 3a and 3b. The net coordinate projection along the horizontal in Fig. 3a, which in ordinary Euclidean geometry would be $x' \cos\phi + ct' \sin\phi$, is given in the Minkowski geometry as $x' \cosh\alpha + ct' \sinh\alpha$. This is the net coordinate projection to be denoted by x, along the X-axis of the observer's frame.

Similarly the net coordinate projection along the vertical in Fig. 3a is $ct' \cosh\alpha + x' \sinh\alpha$ in the Minkowski geometry. This is the net coordinate projection, to be denoted by ct, along the cT-axis of the observer's frame. Thus the following familiar transformation of coordinates has been derived from Fig. 3a:

$$\left. \begin{aligned} ct &= ct' \cosh\alpha + x' \sinh\alpha; \\ x &= x' \cosh\alpha + ct' \sinh\alpha; \quad y = y'; \quad z = z' \end{aligned} \right\}, \quad (4)$$

where the trivial transformations, $y = y'$ and $z = z'$ of the coordinates along which relative motion of SR does not occur have been added.

The inverse of system (4) that can be similarly derived from Fig. 3b is the following:

$$\left. \begin{aligned} ct' &= ct \cosh\alpha - x \sinh\alpha; \\ x' &= x \cosh\alpha - ct \sinh\alpha; \quad y' = y; \quad z' = z \end{aligned} \right\}, \quad (5)$$

System (5) can be presented in a matrix form as follows:

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\alpha & -\sinh\alpha & 0 & 0 \\ -\sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \quad (6)$$

which of the form $\mathbf{x}' = L\,\mathbf{x}$.

By considering the spatial origin, $x' = y' = z' = 0$, of the primed frame, system (4) reduces as follows:

$$x = ct' \sinh\alpha \quad \text{and} \quad ct = ct' \cosh\alpha. \quad (7)$$

Division of the first into the second equation of system (7) gives

$$\frac{x}{ct} = \frac{v}{c} = \tanh\alpha, \quad (8)$$

where, $x/t = v$, is the speed of the primed frame relative to the unprimed frame.

Using (8) along with $\cosh^2\alpha - \sinh^2\alpha = 1$ gives the following:

$$\cosh\alpha = \frac{1}{\sqrt{1 - v^2/c^2}} \equiv \gamma, \quad (9a)$$

$$\sinh\alpha = \frac{v/c}{\sqrt{1 - v^2/c^2}} \equiv \beta\gamma. \quad (9b)$$

Substitution of equations (9a) and (9b) into systems (4) and (5) gives the LT and its inverse in the usual forms of systems (1) and (2).

The transformation from the usual trigonometric ratios, cosine and sine, of the angle ϕ in Figs. 3a and 3b, where $\tan\phi = v/c$; $-\frac{\pi}{4} < \phi < \frac{\pi}{4}$ (the light-cone), to hyperbolic functions, cosh and sinh of a number α in expressing coordinate projections on spacetime, in order to reproduce the Lorentz transformation in the Minkowski graphical approach, is compelled by the need for the parameter α to take on values in
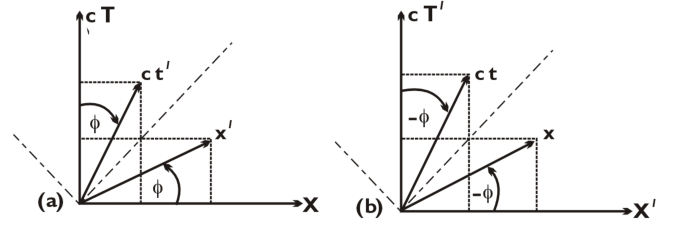

Fig. 3: The Minkowski diagrams sometimes used to derive the Lorentz transformation and its inverse in the existing one-world picture.

the unbounded range $(-\infty, \infty)$ (in Fig. 1a) of Scheme I, as the speed v of the particle relative to the observer takes on values in the unbounded range $(-c, c)$. In other words, the need to transform from the trigonometric ratios, cosine and sine, of the angle ϕ in Figs. 3a and 3b to hyperbolic functions, cosh and sinh, of a number α is compelled by the need to restrict to the positive half-plane of Fig. 1a or to the one-world picture in Special Relativity until now.

There is also a known mathematical significance to the LT system (5) or (6) and its inverse system (4) derived from the Minkowski diagrams of Figs. 3a and 3b. This is the fact that the 4×4 matrix L that generates the Lorentz boost (6), which contains the parameter α in the unbounded range $(-\infty, \infty)$, is a member of the pseudo-orthogonal Lorentz group SO(3,1), which is a non-compact Lie group with an unbounded parameter space [6]. Moreover the matrix L is non-singular for any finite value of α as required for all group SO(3,1) matrices. This implies that non-physical discontinuities do not appear in the Minkowski space generated. Singularities appear in systems (4) and (5) for the extreme values of α namely, $\alpha = \infty$ and $\alpha = -\infty$ only, which are not included in the range of α. These extreme values of α correspond to speeds $v = c$ and $v = -c$ respectively, which material particles cannot attain in relative motion.

The Lorentz boost is just a special Lorentz transformation. The general Lorentz transformation Λ is written in the factorized form [6] as follows:

$$\Lambda = R(\gamma, \beta, 0)\, L_3(\alpha)\, R(\phi, \theta, \varphi)^{-1}, \quad (10)$$

where $L_3(\alpha)$ is the Lorentz boost along the z-axis with speed $v = c \tanh\alpha$; $0 \leqslant \alpha < \infty$, and the Euler angles for rotation in the Euclidean 3-space have their usual finite ranges.

Since the group SO(3) matrices are closed and bounded, and are hence compact, the compactness or otherwise of Λ is determined by the Lorentz boost. Thus since the Lorentz boost is non-compact, the Lorentz group SO(3,1) is non-compact as known. There is no way of making SO(3,1) compact within the Minkowski one-world picture since the parameter α naturally lies within the unbounded range $-\infty < \alpha < \infty$ in this picture. Thus the Minkowski diagrams of Figs. 3a and 3b and the LT and its inverse of systems (5) and (4) or the implied transformation matrix L in Eq. (6) derived from them, have been seen as physical significance of

the Lorentz group in mathematics, or perhaps the other way round.

From the point of view of physics, on the other hand, one observes that the coordinates x' and ct' of the primed frame are non-orthogonal (or are skewed) in Fig. 3a, and the coordinates x and ct of the unprimed frame are skewed in Fig. 3b. These coordinates are orthogonal in the absence of relative motion of the frames. Even in relative motion, an observer at rest relative to the primed frame could not detect the uniform motion of his frame. Hence the primed frame is stationary relative to an observer at rest relative to it with or without the motion of the primed frame relative to the unprimed frame. Yet Fig. 3a shows that the coordinates of the primed frame are skewed with respect to an observer at rest relative to it while it is in uniform motion relative to the unprimed frame. This skewness of the spacetime coordinates of a frame is then an effect of the uniform motion of the frame, which an observer at rest relative to it could detect. This contradicts the fact that an observer cannot detect any effect of the uniform motion of his frame. Skewness of rotated coordinates cannot be avoided in Minkowski's diagrams because relative rotation of coordinates must be restricted to the first quadrant in Scheme I (or in the one-world picture), as deduced earlier.

Skewness of spacetime coordinates of frames of reference is not peculiar to the Minkowski diagrams. It is a general feature of all the existing spacetime diagrams (in the one-world picture) in Special Relativity. There are at least two other spacetime diagrams in Special Relativity, apart from the Minkowski diagrams namely, the Loedel diagram [7] and the Brehme diagram [8]. The spacetime coordinates of two frames in relative motion are skewed in the Loedel and Brehme diagrams shown as Figs. 4a and 4b respectively, for two frames in relative motion along their collinear x'- and x-axes.

Skewness of the coordinates of a frame of reference in uniform relative motion is undesirable because it is an effect of uniform motion of a frame which an observer at rest relative to the frame could detect, which negates the fundamental principle that no effect of uniform motion is detectable, as mentioned earlier. Moreover it gives apparent preference for one of two frames of reference in uniform relative motion, which, again, is a contradiction of a tenet of Special Relativity.

4 Geometric representation of Lorentz transformation in Scheme II

Having discussed the existing geometric representation of the Lorentz transformation and its inverse in Special Relativity in the context of Scheme I in Table 1 (or in the one-world picture) in the preceding section, we shall develop a new set of spacetime diagrams that are compatible with the Lorentz transformation and its inverse in the context of Scheme II in Table 1 in the rest of this paper. We shall, in particular, watch out for the possibility of making the Lorentz group SO(3,1)
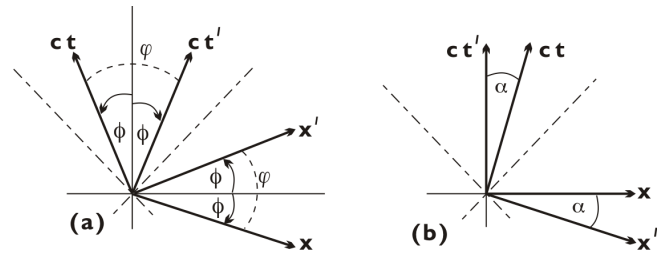

Fig. 4: a) The Loedel diagram and b) the Brehme diagram for two frames in uniform relative motion.

compact and for removing the skewness of rotated spacetime coordinates of frames of reference in the existing spacetime diagrams of Special Relativity (in the one-world picture or in the context of Scheme I).

4.1 Co-existence of two identical universes in the context of Scheme II

As shall be sufficiently justified with progress in this article, the co-existence of two anti-parallel Minkowski spaces in Fig. 2b implies the co-existence of two "anti-parallel" worlds (or universes) in nature. The dimensions x, y, z and ct of the positive Minkowski space, which are accessible to us by direct experience, are the dimensions of our universe (or world). The dimensions $-x^*, -y^*, -z^*$ and $-ct^*$ of the negative Minkowski space, which are inaccessible to us by direct experience, and hence, which have remained unknown until now, are the dimensions of another universe. Dummy star label has been put on the dimensions of the other universe, which are non-observable to us in our universe, in order to distinguish them from the dimensions of our universe.

The negative spacetime dimensions $-x^*, -y^*, -z^*$ and $-ct^*$ are inversions in the origin (or four-dimensional inversion) of the positive spacetime dimensions x, y, z and ct. Thus the spacetime dimensions of the universe with negative dimensions, to be referred to as the negative universe for brevity, and the spacetime dimensions of our universe, (to sometimes be referred to as the positive universe), have an inversion-in-the-origin symmetry. There is a one-to-one mapping of points in spacetimes between the positive (or our) universe and the negative universe. In other words, to every point in spacetime in our universe, there corresponds a unique symmetry-partner point in spacetime in the negative universe.

In addition to the inversion in the origin relationship between the spacetime dimensions of the positive and negative universes, we shall prescribe a reflection symmetry of spacetime geometry between the two universes. In other words, if we denote the spacetime manifold of the positive universe by $\mathbf{M}$ and that of the negative universe by $-\mathbf{M}^*$, then spacetime geometry at a point in spacetime in the positive universe shall be prescribed by $\mathbf{M}$ and the metric tensor $g_{\mu\nu}$ at that point, that is, by $(\mathbf{M}, g_{\mu\nu})$, while spacetime geometry shall be prescribed at the symmetry-partner point in the negative universe by $(-\mathbf{M}^*, g_{\mu\nu})$, where it must be remembered that the metric

tensor is invariant with reflections of coordinates. Symmetry of spacetime geometry between the two universes can only be prescribed at this point of development of the two-world picture.

Now Mach's principle is very fundamental. We shall make recourse to the principle here for the purpose of advancing our argument for the symmetry of state between the positive and negative universes, while knowing that the principle in itself has noting to do with Special Relativity. Essentially the Mach's principle states that the geometry of a space is determined by the distribution of mass - energy in that space [9, see p. 400]. It follows from the foregoing paragraph and Mach's principle that there is a reflection symmetry of the distribution of mass-energy in spacetimes between the two universes. Actually this is also a prescription at this point since the symmetry of spacetime geometry is a prescription.

Reflection symmetry of geometry of spacetime and of the distribution of mass-energy in spacetime also imply reflection symmetry of motions of particles and objects, natural or caused by animate object, between the two universes. In other words, corresponding to an event, natural or man-made, taking place within a local region of spacetime in our universe, there is an identical event within the symmetry-partner local region of spacetime in the negative universe. (This is the symmetry of state between the two universes). The two universes are perfectly identical in state at all times. The perfect symmetry of natural and man-made events (or perfect symmetry of state) between the two universes is a prescription at this point.

There is also a perfect symmetry of laws between the two universes, which implies that natural laws take on perfectly identical forms in the two universes. Symmetry of laws between the two universes is simply the extension of the invariance of laws found in our universe to the negative universe, which follows partly from the validity of local Lorentz invariance in the negative universe to be demonstrated shortly. The two universes could not possess symmetry of state if the laws that guide events and phenomena in them are different. The perfect symmetry of laws between the two universes shall be demonstrated with the advancement of the two-world picture.

The negative spacetime dimensions of the negative universe implies that distance in space, which is a positive scalar quantity in our (positive) universe, is a negative scalar quantity in the negative universe, and that interval of time, which is a positive quantity in the positive universe is a negative quantity in the negative universe; (it does not connote going to the past in our time dimension). This can be easily ascertained from the definition of distance, which is given in 3-space in the negative universe as, $d = \sqrt{(-x^*)^2 + (-y^*)^2 + (-z^*)^2}$. If we consider motion along the dimension $-x^*$ solely, then we must let $-y^* = -z^* = 0$, to have $d = \sqrt{(-x^*)^2} = -x^*$. Likewise the distance element of Special Relativity in the negative universe is, $ds^* = \sqrt{(-ct^*)^2 - (-x^*)^2 - (-y^*)^2 - (-z^*)^2}$. If we
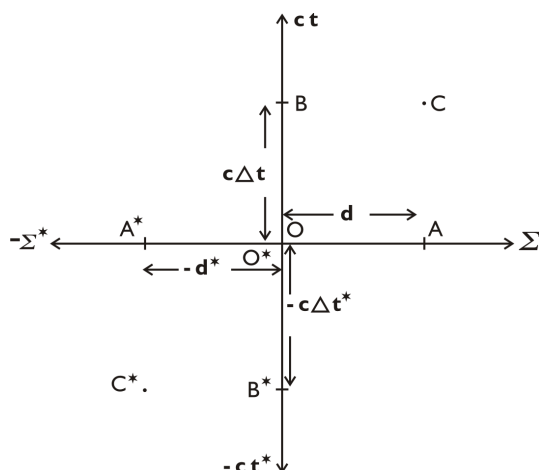

Fig. 5: Combined positive and negative Minkowski's spaces of the positive and negative universes.

let $-x^* = -y^* = -z^* = 0$, for propagation in time only, then $ds^* = \sqrt{(-ct^*)^2} = -ct^*$. Interestingly the negative worldline element ($ds^* < 0$) in the negative universe is the negative root ($-ds$) of the quadratic line element ds^2, which is usually discarded since it conveys nothing to us from the point of view of experience in the positive universe.

4.2 Non-separation of symmetry-partner points in spacetimes in the positive and negative universes

It shall be shown here that a point in spacetime in our (or positive) universe is effectively not separated in space or in time dimension from its symmetry-partner point in spacetime in the negative universe, for every pair of symmetry-partner points in spacetimes in the two universes. Now let us consider the larger spacetime of combined positive and negative universes, Fig. 2b, which is re-illustrated as Fig. 5.

Point A* in the negative Euclidean 3-space $-\Sigma^*$ of the negative universe is the symmetry-partner to point A in the positive Euclidean 3-space Σ of the positive universe. Point B* in the negative time dimension $-ct^*$ of the negative universe is the symmetry-partner to point B in the positive time dimension ct of the positive universe. Hence points C* and C are symmetry-partner points on four-dimensional spacetimes in the two universes.

Now let points A and O in the positive 3-space Σ of the positive universe be separated by a positive distance d, say, since distances in space are positive scalar quantities in the positive universe. Then the symmetry-partner points A* and O* in the negative 3-space $-\Sigma^*$ of the negative universe are separated by negative distance $-d^*$, since distances in space are negative scalar quantities in the negative universe. Hence the distance in 3-space between point A in the positive universe and its symmetry-partner point A* in the negative universe is, $d - d^* = 0$, since d and $-d^*$ are equal in magnitude. This implies that the symmetry-partner points A and A* are effectively separated by zero distance in space with respect to

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observers (or people) in the positive and negative universes.

Likewise, if the interval of positive time dimension ct between point O and point B is the positive quantity $c\Delta t$, then the interval of the negative time dimension $-ct^*$ between point O* and point B* is the negative quantity $-c\Delta t^*$, since intervals of time are negative quantities in the negative universe. Hence the interval of time dimension between point B in ct in the positive universe and its symmetry-partner point B* in $-ct^*$ in the negative universe is, $c\Delta t - c\Delta t^* = 0$. This implies that the symmetry-partner points B and B* in the time dimensions are effectively separated by zero interval of time dimension with respect to observers (or people) in the positive and negative universes. It then follows that the time t of an event in the positive universe is effectively separated by zero time interval from the time $-t^*$ of the symmetry-partner event in the negative universe. Thus an event in the positive universe and its symmetry-partner in the negative universe occur simultaneously.

It follows from the foregoing two paragraphs that symmetry-partner points C and C* in spacetimes in the positive and negative universes are not separated in space or time, and this is true for every pair of symmetry-partner points in spacetimes in the two universes. Although symmetry-partner points in spacetimes in the positive and negative universes coincide at the same point, or are not separated, they do not touch because they exist in different spacetimes.

One consequence of the foregoing is that local spacetime coordinates, $(\Sigma, ct) \equiv (x, y, z, ct)$, originating from a point O in the positive universe and the symmetry-partner local spacetime coordinates, $(-\Sigma^*, -ct^*) \equiv (-x^*, -y^*, -z^*, -ct^*)$, originating from the symmetry-partner point O* in spacetime in the negative universe can be drawn from the same point on paper, as done in Fig. 5, and geometrical construction whose predictions will conform with observation or experiment in each of the two universes can be based on this in the two-world picture, as shall be done in the rest of this section.

4.3 Introducing a flat two-dimensional intrinsic spacetime underlying the flat four-dimensional spacetime

Since it is logically required for this article to propagate beyond this point and since space limitation in this paper does not permit the presentation of its derivation, which shall be presented elsewhere, we shall present (as *ansatz*) at this point certain flat two-dimensional intrinsic spacetime with dimensions to be denoted by $\phi\rho$ and $\phi c\phi t$, where $\phi\rho$ is intrinsic space dimension (actually a one-dimensional intrinsic space) and $\phi c\phi t$ is intrinsic time dimension, which underlies the flat four-dimensional spacetime (the Minkowski space) of the Special Relativity, usually denoted by (x^0, x^1, x^2, x^3); $x^0 = ct$, but which shall be denoted by (Σ, ct) in this article for convenience, where Σ is the Euclidean 3-space with dimensions x^1, x^2 and x^3.

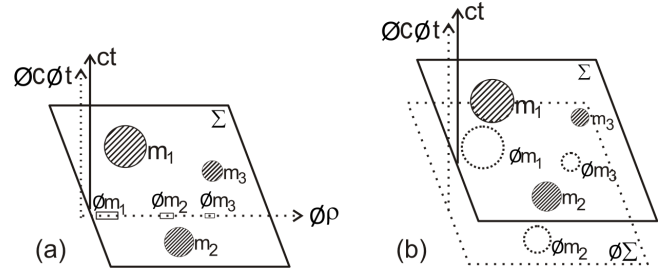Every particle or object with a three-dimensional inertial

Fig. 6: a) The flat 4-dimensional spacetime and its underlying flat 2-dimensional intrinsic spacetime with the inertial masses of three objects scattered in the Euclidean 3-space and their one-dimensional intrinsic inertial masses aligned along the isotropic one-dimensional intrinsic space with respect to observers in spacetime. b) The flat 2-dimensional intrinsic spacetime with respect to observers in spacetime in a is a flat four-dimensional intrinsic spacetime containing 3-dimensional intrinsic inertial masses of particles and objects in 3-dimensional intrinsic space with respect to intrinsic-mass-observers in intrinsic spacetime.

mass m in the Euclidean 3-space Σ has its one-dimensional intrinsic mass to be denoted by ϕm underlying it in the one-dimensional intrinsic space $\phi\rho$. The one-dimensional intrinsic space $\phi\rho$ underlying the Euclidean 3-space Σ is an isotropic dimension with no unique orientation in Σ. This means that $\phi\rho$ can be considered to be orientated along any direction in Σ. The straight line intrinsic time dimension $\phi c\phi t$ likewise lies parallel to the straight line time dimension ct along the vertical in the graphical presentation of the flat spacetime of SR of Fig. 2 or Fig. 5.

If we temporarily consider the Euclidean 3-space Σ as an hyper-surface, $t = const$, represented by a plane-surface along the horizontal (instead of a line along the horizontal as in the previous diagrams) and the time dimension ct as a vertical normal line to the hyper-surface, then the graphical representation of the flat four-dimensional spacetime (Σ, ct) and its underlying flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the context of SR described in the foregoing paragraph is depicted in Fig. 6a.

Figure 6a is valid with respect to observers in the flat physical four-dimensional spacetime (Σ, ct). The one-dimensional intrinsic masses of all particles and objects are aligned along the singular isotropic one-dimensional intrinsic space $\phi\rho$, whose inertial masses are scattered arbitrarily in the physical Euclidean 3-space Σ with respect to these observers, in (Σ, ct), as illustrated for three such particles and objects in Fig. 6a.

On the other hand, the intrinsic space is actually a flat three-dimensional domain to be denoted by $\phi\Sigma$, with mutually orthogonal dimensions $\phi x^1, \phi x^2$ and ϕx^3, at leat in the small, with respect to intrinsic-mass-observers in $\phi\Sigma$. The intrinsic masses ϕm of particles and objects are likewise three-dimensional with respect to the intrinsic-mass-observers in $\phi\Sigma$. The intrinsic mass ϕm of a particle or object in the intrinsic space $\phi\Sigma$ lies directly underneath the inertial mass m

of the particle or object in the physical Euclidean 3-space Σ, as illustrated for three such particles or objects in Fig. 6b.

The flat four-dimensional physical spacetime (Σ, ct) containing the three-dimensional inertial masses m of particles and objects in the Euclidean 3-space Σ is the outward manifestation of the flat four-dimensional intrinsic spacetime $(\phi\Sigma, \phi c\phi t)$ containing the three-dimensional intrinsic masses ϕm of the particles and objects in $\phi\Sigma$ in Fig. 6b. It is due to the fact that the flat three-dimensional intrinsic space $\phi\Sigma$ is an isotropic space, that is, all directions in $\phi\Sigma$ are the same, with respect to observers in the physical Euclidean 3-space Σ that the dimensions $\phi x^1, \phi x^2$ and ϕx^3 of $\phi\Sigma$, which are mutually orthogonal, at least locally, with respect to the intrinsic-mass-observers in $\phi\Sigma$, are effectively directed along the same non-unique direction in $\phi\Sigma$, thereby effectively constituting a singular one-dimensional intrinsic space (or an intrinsic space dimension) $\phi\rho$ with no unique orientation in $\phi\Sigma$ and consequently with no unique orientation in the physical Euclidean 3-space Σ overlying $\phi\Sigma$ with respect to observers on the flat spacetime (Σ, ct), as illustrated in Fig. 6a.

As follows from the foregoing paragraph, Fig. 6a is the correct diagram with respect to observers in spacetime (Σ, ct). It is still valid to say that the flat four-dimensional spacetime (Σ, ct) is the outward (or physical) manifestation of the flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$ and that three-dimensional inertial mass m in Σ is the outward (or physical) manifestation of one-dimensional intrinsic mass ϕm with respect to observers in (Σ, ct) in Fig. 6a. Observers on the flat four-dimensional spacetime (Σ, ct) must formulate intrinsic physics in intrinsic spacetime as two-dimensional intrinsic theories on flat intrinsic spacetime $(\phi\rho, \phi c\phi t)$.

It is for convenience that the three-dimensional Euclidean space Σ shall be represented by a line along the horizontal as done in Figs. 2a and 2b and Fig. 5 and as shall be done in the rest of this article, instead of a plane surface along the horizontal in Figs. 6a and 6b. Thus the flat four-dimensional spacetime and its underlying flat two-dimensional intrinsic spacetime shall be presented graphically in the two-world picture as Fig. 7. The origins O and O* are not actually separated contrary to their separation in Fig. 7.

Figure 7 is Fig. 5 modified by incorporating the flat two-dimensional intrinsic spacetimes underlying the flat four-dimensional spacetimes of the positive and negative universes into Fig. 5. Figure 7 is a fuller diagram than Fig. 5. As mentioned earlier, the intrinsic spacetime and intrinsic parameters in it along with their properties and notations shall be derived elsewhere.

The intrinsic spacetime dimensions $\phi\rho$ and $\phi c\phi t$ and one-dimensional intrinsic masses ϕm of particles and objects in the intrinsic space $\phi\rho$ are hidden (or non-observable) to observers on the flat four-dimensional spacetime (Σ, ct). The symbol ϕ attached to the intrinsic dimensions, intrinsic coordinates and intrinsic masses is used to indicate their intrinsic (or hidden) natures with respect to observers in spacetime.
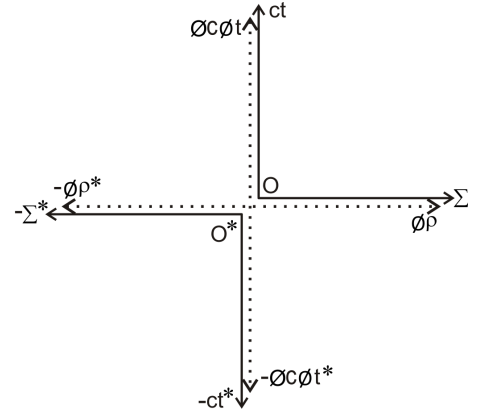

Fig. 7: Combined flat four-dimensional spacetimes and combined underlying flat two-dimensional intrinsic spacetimes of the positive and negative universes.

When the symbol ϕ is removed from the flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$ we obtain the observed flat four-dimensional spacetime (Σ, ct) and when ϕ is removed from the one-dimensional intrinsic mass ϕm in $\phi\rho$ we obtain the observed three-dimensional inertial mass m in the Euclidean 3-space Σ.

As the inertial mass m moves at velocity $\vec{v}$ in the Euclidean 3-space Σ of the flat four-dimensional spacetime (Σ, ct) relative to an observer in (Σ, ct), the intrinsic mass ϕm performs intrinsic motion at intrinsic speed ϕv in the one-dimensional intrinsic space $\phi\rho$ of the flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$ relative to the observer in (Σ, ct), where $|\phi v| = |\vec{v}|$. The inertial mass m of a particle in Σ and its intrinsic mass ϕm in $\phi\rho$ are together always in their respective spaces, irrespective of whether m is in motion or at rest relative to the observer.

Finally in the *ansatz* being presented in this sub-section, the intrinsic motion of the intrinsic rest mass ϕm_0 of a particle at intrinsic speed ϕv in an intrinsic particle's frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ relative to the observer's intrinsic frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$ on flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$ pertains to two-dimensional intrinsic Special Theory of Relativity to be denoted by ϕSR, while the corresponding motion of the rest mass m_0 of the particle at velocity $\vec{v}$ in the particle's frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ relative to the observer's frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ on the flat four-dimensional spacetime (Σ, ct), pertains to the Special Theory of Relativity (SR) as usual. The SR on flat four-dimensional spacetime (Σ, ct) is mere outward manifestation of ϕSR on the underlying flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$.

The intrinsic motion at intrinsic speed ϕv of the intrinsic rest mass ϕm_0 of a particle in the particle's intrinsic frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ relative to the observer's intrinsic frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$, gives rise to rotation of the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ relative to the intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ on the vertical intrinsic spacetime plane (which are on the $(\phi\rho, \phi c\phi t)$-plane) in Fig. 7. It must be observed that rotation

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of the intrinsic coordinate $\phi\tilde{x}'$ can take place on the vertical intrinsic spacetime plane only in Fig. 6a or Fig. 7.

Two-dimensional intrinsic spacetime diagram and its inverse must be drawn on the vertical $(\phi\rho, \phi c\phi t)$-plane in the two-world picture and intrinsic Lorentz transformation (ϕLT) and its inverse derived from them in the context of ϕSR. The intrinsic Lorentz invariance (ϕLI) on the flat two-dimensional intrinsic spacetime must be validated and every result in the context of the two-dimensional intrinsic Special Theory of Relativity (ϕSR), each of which has its counterpart in SR, must be derived from the ϕLT and its inverse in the manner the results of SR are derived from the LT and its inverse.

Once ϕSR has been formulated as described above, then SR being mere outward (or physical) manifestation on the flat four-dimensional spacetime (Σ, ct) of ϕSR on the flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$, the results of SR namely, the LT and its inverse, the Lorentz invariance (LI) on the flat four-dimensional spacetime and every other results of SR can be written directly from the corresponding results of ϕSR, without having to draw spacetime diagrams involving the rotation of the coordinates $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ of the primed frame relative to the coordinates $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ of the unprimed frame on the flat four-dimensional spacetime (Σ, ct) in the context of SR. This procedure shall be demonstrated in the next sub-section.

4.4 New spacetime/intrinsic spacetime diagrams for derivation of Lorentz transformation/intrinsic Lorentz transformation in the two-world picture

Consider two frames of reference with extended unprimed straight line affine coordinates $\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t}$ and extended primed straight line affine coordinates $\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}'$ respectively on the flat metric four-dimensional spacetime (Σ, ct). Let a three-dimensional observer (or a 3-observer), Peter, say, be located in 3-space of the unprimed frame and another 3-observer, Paul, say, be located in 3-space of the primed frame.

Corresponding to the 3-dimensional observer Peter in the 3-space of the unprimed frame, there is the one-dimensional observer (or 1-observer) in the time dimension of the unprimed frame to be denoted by $\tilde{P}$eter. Likewise corresponding to the 3-observer Paul in 3-space of the primed frame is the one-dimensional observer (or 1-observer) $\tilde{P}$aul in the time dimension of the primed frame. Thus there is the 4-observer (Peter, $\tilde{P}$eter) in the unprimed frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ and the 4-observer (Paul, $\tilde{P}$aul) in the primed frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ in the positive universe. There is the symmetry-partner 4-observer (Peter*, $\tilde{P}$eter*) in the symmetry-partner unprimed frame $(-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*, -c\tilde{t}^*)$ and symmetry-partner 4-observer (Paul*, $\tilde{P}$aul*) in the symmetry-partner primed frame $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$ in the negative universe.

Before proceeding further, let us shine some light on the concepts of metric spacetime and affine spacetime that have been introduced in the preceding two paragraphs. As well

known, the metric spacetime (Σ, ct) is the physical four-dimensional spacetime, which is flat with constant Lorentzian metric tensor in the context of SR (and is postulated to be curved with Riemannian metric tensor in the context of the General Theory of Relativity, GR). The matter (or mass) of particles and objects are contained in the metric 3-space Σ (with Euclidean metric tensor in the context of SR). Thus particles and objects exist and move in the four-dimensional metric spacetime in the theories of relativity. The coordinates or dimensions of the metric spacetime shall be denoted by x, y, z and ct without label (in the Cartesian system of coordinates of 3-space) in this article.

On the other hand, the coordinates of an affine spacetime shall be differentiated from those of a metric spacetime by an over-head tilde label as $\tilde{x}, \tilde{y}, \tilde{z}$ and $c\tilde{t}$. These are mere mathematical entities without physical (or metrical) quality used to identify the positions and to track the motion of material points relative to a specified origin in a metric spacetime. The affine coordinates $\tilde{x}, \tilde{y}, \tilde{z}$ and $c\tilde{t}$ are straight line coordinates that can be of any extensions in the flat metric spacetime of SR. Just as it is said that "the path of a fish in water cannot be known", so is the path (i.e. the locus of the affine coordinates) of a material point through a metric spacetime non-discernible or without metrical quality. An affine spacetime can be described as mere mathematical scaffolding without physical (or metrical) significance for identifying possible positions of material particles in the metric spacetime. The extended three-dimensional affine space constituted by the affine coordinates $\tilde{x}, \tilde{y}$ and $\tilde{z}$ cannot hold matter (or mass of particles and objects).

Now corresponding to the unprimed frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ of the 4-observer (Peter, $\tilde{P}$eter) prescribed on the flat four-dimensional metric spacetime (Σ, ct) earlier, is the unprimed intrinsic frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$ of intrinsic 2-observer (ϕPeter, $\phi\tilde{P}$eter) in the two-dimensional metric intrinsic spacetime $(\phi\rho, \phi c\phi t)$ underlying (Σ, ct) in the first quadrant in Fig. 7 and corresponding to the primed frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ of the 4-observer (Paul, $\tilde{P}$aul) prescribed in the metric spacetime (Σ, ct) is the primed intrinsic frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ of intrinsic 2-observer (ϕPaul, $\phi\tilde{P}$aul) in the two-dimensional metric intrinsic spacetime $(\phi\rho, \phi c\phi t)$ underlying (Σ, ct) in Fig. 7. The intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ of the unprimed intrinsic frame in $(\phi\rho, \phi c\phi t)$ are extended straight line affine intrinsic coordinates like the coordinates $\tilde{x}, \tilde{y}, \tilde{x}$ and $c\tilde{t}$ of the unprimed frame in (Σ, ct). The intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the primed intrinsic frame in $(\phi\rho, \phi c\phi t)$ are likewise extended straight line affine intrinsic coordinates like the coordinates $\tilde{x}', \tilde{y}', \tilde{x}'$ and $c\tilde{t}'$ of the primed frame in (Σ, ct).

The summary of all of the foregoing is that we have prescribed a pair of frames with extended straight line affine coordinates namely, $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ of 4-observer (Peter, $\tilde{P}$eter) and $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ of 4-observer (Paul, $\tilde{P}$aul) on the flat four-dimensional metric spacetime (Σ, ct) and underlying pair of intrinsic frames with extended straight line affine intrinsic co-

ordinates namely, $(\phi\tilde{x}, \phi c\phi\tilde{t})$ of intrinsic 2-observer (ϕPeter, $\phi\tilde{\text{Peter}}$) and $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ of intrinsic 2-observer (ϕPaul, $\phi\tilde{\text{Paul}}$) on the flat two-dimensional metric intrinsic spacetime $(\phi\rho, \phi c\phi t)$ that underlies (Σ, ct) in the first quadrant (or in our universe) in Fig. 7.

The perfect symmetry of state between the positive and negative universes requires that there are identical symmetry-partner pair of frames with extended straight line affine coordinates $(-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*, -c\tilde{t}^*)$ of symmetry-partner 4-observer (Peter*, $\tilde{\text{Peter}}$*) and $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$ of symmetry-partner 4-observer (Paul*, $\tilde{\text{Paul}}$*) on the flat four-dimensional metric spacetime $(-\Sigma^*, -ct^*)$ and underlying pair of intrinsic frames with extended straight line affine intrinsic coordinates namely, $(-\phi\tilde{x}^*, -\phi c\phi\tilde{t}^*)$ of intrinsic 2-observer (ϕPeter*, $\phi\tilde{\text{Peter}}$*) and $(-\phi\tilde{x}'^*, -\phi c\phi\tilde{t}'^*)$ of intrinsic 2-observer (ϕPaul*, $\phi\tilde{\text{Paul}}$*) on the flat two-dimensional metric intrinsic spacetime $(-\phi\rho^*, -\phi c\phi t^*)$ that underlies $(-\Sigma^*, -ct^*)$ in the third quadrant (or in negative universe) in Fig. 7.

As done at the beginning of section 2, let us consider the propagation at a constant speed v of the rest mass m_0 of a particle along the coordinate $\tilde{x}'$ of the particle (or primed) frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ relative to the 3-observer Peter in the 3-space $\tilde{\Sigma}(\tilde{x}, \tilde{y}, \tilde{z})$ of the observer's frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ in the positive universe (or our universe), where the coordinates $\tilde{x}'$ and $\tilde{x}$ shall be taken to be collinear. Correspondingly, the intrinsic rest mass ϕm_0 of the particle is in intrinsic motion at intrinsic speed ϕv along the intrinsic coordinate $\phi\tilde{x}'$ of the particle's intrinsic frame (or the primed intrinsic frame) $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ relative to the intrinsic observer's frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$ with respect to the intrinsic 1-observer ϕPeter in the one-dimensional intrinsic space $(\phi\tilde{x})$ of the observer's frame and hence with respect to the 3-observer Peter in $\tilde{\Sigma}(\tilde{x}, \tilde{y}, \tilde{z})$ overlying $\phi\tilde{x}$, where the intrinsic coordinates $\phi\tilde{x}'$ and $\phi\tilde{x}$ are necessarily collinear since they are affine intrinsic coordinates in the singular isotropic one-dimensional metric intrinsic space $\phi\rho$.

The intrinsic motion at intrinsic speed ϕv of the intrinsic rest mass ϕm_0 of the particle along the intrinsic coordinate $\phi\tilde{x}'$ of the particle's intrinsic frame $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ relative to the observer's intrinsic frame $(\phi\tilde{x}, \phi c\phi\tilde{t})$ described in the foregoing paragraph, will cause the anti-clockwise rotation of the extended straight line affine intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the primed intrinsic frame at equal intrinsic angle $\phi\psi$ relative to the extended straight line affine intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ respectively of the unprimed intrinsic frame.

The perfect symmetry of state between the positive and negative universes discussed earlier, implies that the rest mass of the symmetry-partner particle (its sign is yet to be determined), is in simultaneous motion at constant speed v along the coordinate $-\tilde{x}'^*$ of the particle's frame $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$ relative to the symmetry-partner 3-observer* Peter* in the 3-space $-\tilde{\Sigma}^*(-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*)$ of the observer's frame $(-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*, -c\tilde{t}^*)$ in the negative universe. Correspondingly, the intrinsic rest mass of the symmetry-partner particle is in intrinsic motion at constant intrinsic speed ϕv along

the intrinsic coordinate $-\phi\tilde{x}'^*$ of the particle's intrinsic frame $(-\phi\tilde{x}'^*, -\phi c\phi\tilde{t}'^*)$ relative to the intrinsic observer's frame $(-\phi\tilde{x}^*, -\phi c\phi\tilde{t}^*)$, with respect to the intrinsic 1-observer* ϕPeter* in the intrinsic space $-\phi\tilde{x}^*$ of the intrinsic observer's frame and consequently with respect to the 3-observer* Peter* in the 3-space $-\tilde{\Sigma}^*(-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*)$ of the observer's frame overlying $-\phi\tilde{x}^*$ in the negative universe. Consequently the extended affine intrinsic coordinates $-\phi\tilde{x}'^*$ and $-\phi c\phi\tilde{t}'^*$ of the particle's frame will be rotated anti-clockwise at equal intrinsic angle $\phi\psi$ relative to the extended straight line affine intrinsic coordinates $-\phi\tilde{x}^*$ and $-\phi c\phi\tilde{t}^*$ respectively of the observer's intrinsic frame.

Now on the larger spacetime/intrinsic spacetime of combined positive universe and negative universe depicted in Fig. 7, the extended straight line affine intrinsic time coordinate $\phi c\phi\tilde{t}'$ of the primed intrinsic frame in the first quadrant can rotate into the second quadrant with respect to the 3-observer (Peter) in the 3-space $\tilde{\Sigma}(\tilde{x}, \tilde{y}, \tilde{z})$ along the horizontal in the first quadrant in Fig. 7. This is so since the intrinsic angle $\phi\psi$ has values in the negative half-plane in Fig. 1b, which correspond to the second and third quadrants in Fig. 7. Similarly the extended straight line affine intrinsic time coordinate $-\phi c\phi\tilde{t}'^*$ of the primed intrinsic frame in the third quadrant can rotate into the fourth quadrant with respect to 3-observer* (Peter*) in the 3-space $-\tilde{\Sigma}^*$ along the horizontal in the third quadrant, since $\phi\psi$ has value in the positive half-plane in Fig. 1b, which corresponds to the fourth and first quadrants in Fig. 7, with respect to 3-observers* in $-\Sigma^*$ along the horizontal in the third quadrant in Fig.7. Thus the rotation of the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ relative to $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ respectively in Fig. 8a is possible (or will ensue) in the two-world picture.

The intrinsic coordinate $\phi\tilde{x}$ is the projection along the horizontal of the inclined $\phi\tilde{x}'$ in Fig. 8a. That is, $\phi\tilde{x} = \phi\tilde{x}' \cos\phi\psi$. Hence we can write,

$$\phi\tilde{x}' = \phi\tilde{x} \sec\phi\psi \,.$$

This transformation of affine intrinsic space coordinates is all that should have been possible with respect to the intrinsic 1-observer ϕPeter in the intrinsic space $\phi\tilde{x}$ of the intrinsic observer's frame along the horizontal and consequently with respect to 3-observer (Peter) in the 3-space $\tilde{\Sigma}(\tilde{x}, \tilde{y}, \tilde{z})$ of the observer's frame from Fig. 8a, but for the fact that the negative intrinsic time coordinate $-\phi c\phi\tilde{t}'^*$ of the negative universe rotated into the fourth quadrant also projects component $-\phi c\phi\tilde{t}' \sin\phi\psi$ along the horizontal, which must be added to the right-hand side of the last displayed equation yielding,

$$\phi\tilde{x}' = \phi\tilde{x} \sec\phi\psi - \phi c\phi\tilde{t}' \sin\phi\psi \,.$$

The dummy star label used to differentiate the coordinates and parameters of the negative universe from those of the positive universe has been removed from the component $-\phi c\phi t'^* \sin\phi\psi$ projected along the horizontal by the coordi-
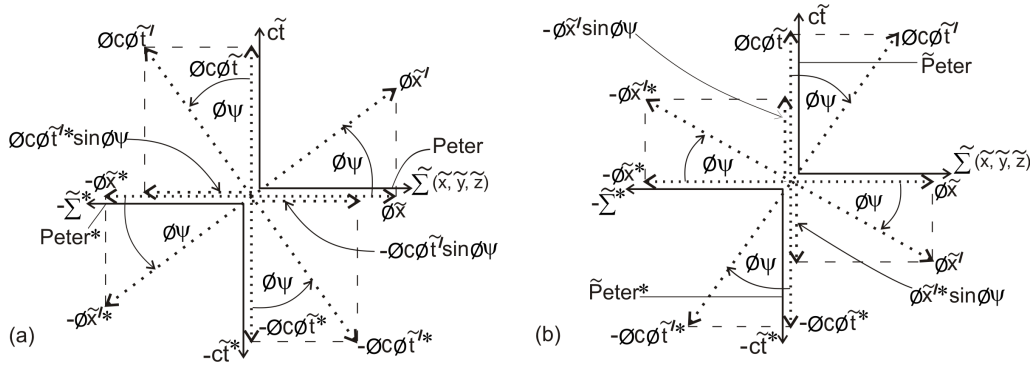
Fig. 8: a) The diagram used to derive partial intrinsic Lorentz transformations / partial Lorentz transformations with respect to 3-observers in the 3-spaces in the positive and negative universes. b) The complementary diagram to a used to derive partial intrinsic Lorentz transformations / partial Lorentz transformations with respect to 1-observers in the time dimensions in the positive and negative universes.

nate $-\phi c\phi\tilde{t}'^*$ of the negative universe rotated into the fourth quadrant in Fig. 8a, since the projected component is now an intrinsic coordinate in the positive universe.

But the intrinsic coordinates $\phi c\phi\tilde{t}$ and $\phi c\phi\tilde{t}'$ are also related as, $\phi c\phi\tilde{t} = \phi c\phi\tilde{t}' \cos\phi\psi$ hence $\phi c\phi\tilde{t}' = \phi c\phi\tilde{t} \sec\phi\psi$, along the vertical in the same Fig. 8a. By replacing $\phi c\phi\tilde{t}'$ by $\phi c\phi\tilde{t} \sec\phi\psi$ in the last displayed equation we have

$$\phi\tilde{x}' = \phi\tilde{x} \sec\phi\psi - \phi c\phi\tilde{t} \tan\phi\psi \qquad (11)$$

(w.r.t. 3-observer Peter in $\tilde{\Sigma}$).

Likewise the affine intrinsic time coordinate $\phi c\phi\tilde{t}$ is the projection along the vertical of the inclined affine intrinsic coordinate $\phi c\phi\tilde{t}'$ in Fig. 8b. Hence $\phi c\phi\tilde{t} = \phi c\phi\tilde{t}' \cos\phi\psi$ or

$$\phi c\phi\tilde{t}' = \phi c\phi\tilde{t} \sec\phi\psi .$$

This affine intrinsic time coordinate transformation is all that should have been possible with respect to the 1-observer P̌eter in the time dimension $c\tilde{t}$ of the observer's frame from Fig. 8b, but for the fact that the inclined negative intrinsic space coordinate $-\phi\tilde{x}'^*$ of the negative universe rotated into the second quadrant also projects component $-\phi\tilde{x}' \sin\phi\psi$ along the vertical, which must be added to the right-hand side of the last displayed equation yielding,

$$\phi c\phi\tilde{t}' = \phi c\phi\tilde{t} \sec\phi\psi - \phi\tilde{x}' \sin\phi\psi .$$

The dummy star label has again been removed from the component $-\phi\tilde{x}'^* \sin\phi\psi$ projected along the vertical in the second quadrant by the inclined intrinsic coordinate $-\phi\tilde{x}'^*$ of the negative universe rotated into the second quadrant, since the projected component is now an intrinsic coordinate in the positive universe.

But the intrinsic coordinate $\phi\tilde{x}$ is related to $\phi\tilde{x}'$ along the horizontal in the same Fig. 8b as, $\phi\tilde{x} = \phi\tilde{x}' \cos\phi\psi$ or $\phi\tilde{x}' = \phi\tilde{x} \sec\phi\psi$ along the horizontal in Fig. 8b. Then by replacing $\phi\tilde{x}'$ by $\phi\tilde{x} \sec\phi\psi$ in the last displayed equation we have

$$\phi c\phi\tilde{t}' = \phi c\tilde{t} \sec\phi\psi - \phi\tilde{x} \tan\phi\psi \qquad (12)$$

(w.r.t. 1-observer P̌eter in $c\tilde{t}$).

The concept of 1-observer in the time dimension added to 3-observer in 3-space to have 4-observer in four-dimensional spacetime introduced above is in agreement with the known four-dimensionality of particles and bodies in 4-geometry of relativity. Anti-clockwise (or positive) rotation of the intrinsic space coordinate $\phi\tilde{x}'$ by intrinsic angle $\phi\psi$ towards the intrinsic time coordinate $\phi c\phi\tilde{t}$ along the vertical with respect to the 3-observer (Peter) in the 3-space $\tilde{\Sigma}(\tilde{x}, \tilde{y}, \tilde{z})$ of the observer's frame in Fig. 8a, corresponds to clockwise (or positive) rotation of the intrinsic time coordinate $\phi c\phi\tilde{t}'$ by equal intrinsic angle $\phi\psi$ towards the intrinsic space coordinate $\phi\tilde{x}$ along the horizontal with respect to the 1-observer (P̌eter) in the time dimension $c\tilde{t}$ of the observer's frame in Fig. 8b. The explanation of the fact that anti-clockwise rotation of the primed intrinsic spacetime coordinates relative to unprimed intrinsic spacetime coordinates is positive rotation with respect to 3-observers in 3-spaces in Fig. 8a, while clockwise rotation of primed intrinsic spacetime coordinates relative to unprimed intrinsic spacetime coordinates is positive rotation with respect to 1-observers in the time dimensions in Fig. 8b, requires further development of the two-world picture than in this paper. It shall be presented elsewhere.

The partial intrinsic Lorentz transformation of affine intrinsic space coordinates (11) with respect to the 3-observer Peter in the 3-space $\tilde{\Sigma}(\tilde{x}, \tilde{y}, \tilde{z})$ of the observer's frame and the partial intrinsic Lorentz transformation of affine intrinsic time coordinates (12) with respect to the 1-observer P̌eter in the time dimension $c\tilde{t}$ of the observer's frame must be collected to obtain the intrinsic Lorentz transformation of extended straight line affine intrinsic spacetime coordinates with respect to 4-observers (Peter, P̌eter) in the observer's frame as follows:

$$\left.\begin{array}{l} \phi c\phi\tilde{t}' = \phi c\phi\tilde{t} \sec\phi\psi - \phi\tilde{x} \tan\phi\psi \\ \quad \text{(w.r.t. 1-observer P̌eter in } c\tilde{t}) \\[2mm] \phi\tilde{x}' = \phi\tilde{x} \sec\phi\psi - \phi c\phi\tilde{t} \tan\phi\psi \\ \quad \text{(w.r.t. 3-observer Peter in } \tilde{\Sigma}) \end{array}\right\}, \qquad (13)$$
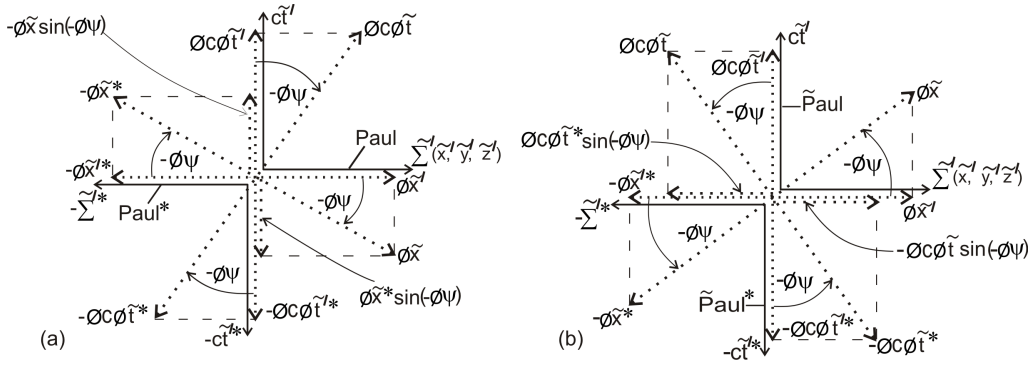
Fig. 9: The inverse diagrams to Figures 8a and 8b respectively, used to derive inverse intrinsic Lorentz transformations / inverse Lorentz transformations in the positive and negative universes.

where $-\frac{\pi}{2} < \phi\psi < \frac{\pi}{2}$ (temporarily).

The range $-\frac{\pi}{2} < \phi\psi < \frac{\pi}{2}$ of the intrinsic angles $\phi\psi$ in system (13) in the positive universe is temporary as indicated. This temporary range shall be modified later in this section. The fact that the intrinsic angle $\phi\psi$ can have values in the range $[0, \frac{\pi}{2})$ in the first quadrant in Figs. 8a and 8b in the two-world picture, instead of the range $[0, \frac{\pi}{4})$ of the angle ϕ in the Minkowski diagrams, (Figs. 3a and 3b in the one-world picture), is due to the non-existence of light-cones in the two-world picture, as shall be established shortly.

In order to obtain the inverses of equations (11) and (12) and hence the inverse to system (13), let us draw the inverses of Figs. 8a and 8b. The inverse to Fig. 8a obtained by rotating all intrinsic coordinates clockwise by negative intrinsic angle $-\phi\psi$ with respect to 3-observer in the 3-spaces $\tilde{\Sigma}$ and $-\tilde{\Sigma}^*$ in Fig. 8a is depicted in Fig. 9a and the the inverse to Fig. 8b obtained by rotating all intrinsic coordinates anti-clockwise by negative intrinsic angle $-\phi\psi$ with respect to 1-observer in the time dimensions $c\tilde{t}$ and $-c\tilde{t}^*$ in Fig. 8b is depicted in Fig. 9b.

The clockwise sense of negative rotation (i.e. by negative intrinsic angle) of intrinsic coordinates in Fig. 9a is valid with respect to the 3-observer (Paul) in the 3-space $\tilde{\Sigma}'$ of the primed (or particle's) frame with respect to whom positive rotation is anti-clockwise. Hence the transformation of intrinsic coordinates derived from Fig. 9a is valid with respect to the 3-observer (Paul) in $\tilde{\Sigma}'$. On the other hand, the anti-clockwise sense of negative rotation of intrinsic coordinates in Fig. 9b is valid relative to the 1-observer (P̃aul) in the time dimension $c\tilde{t}'$, with respect to whom positive rotation is clockwise. Hence the intrinsic coordinate transformation derived from Fig. 9b is valid relative to the 1-observer (Paul) in $c\tilde{t}'$.

Again the affine intrinsic time coordinate $\phi c\phi\tilde{t}'$ is the projection along the vertical of the inclined $\phi c\phi\tilde{t}$ in Fig. 9a. That is, $\phi c\phi\tilde{t}' = \phi c\phi\tilde{t}\cos(-\phi\psi) = \phi c\phi\tilde{t}\cos\phi\psi$. Hence we can write,

$$\phi c\phi\tilde{t} = \phi c\phi\tilde{t}'\sec\phi\psi .$$

This transformation of affine intrinsic time coordinates is

all that should have been possible along the vertical in Fig. 9a by the 3-observer (Paul) in $\tilde{\Sigma}'$ of the particle's frame, but for the fact that the unprimed negative intrinsic space coordinate $-\phi\tilde{x}^*$ of the negative universe rotated into the second quadrant projects component, $-\phi\tilde{x}\sin(-\phi\psi) = \phi\tilde{x}\sin\phi\psi$, along the vertical, which must be added to the right-hand side of the last displayed equation to have as follows:

$$\phi c\phi\tilde{t} = \phi c\phi\tilde{t}'\sec\phi\psi + \tilde{x}\sin\phi\psi .$$

The dummy star label has again been removed from the component $-\phi\tilde{x}^*\sin(-\phi\psi)$ projected along the vertical in the second quadrant by the negative intrinsic space coordinate $-\phi\tilde{x}^*$ of the negative universe rotated into the second quadrant in Fig. 9a, since the projected component is now an intrinsic coordinate in the positive universe.

But $\phi\tilde{x}$ and $\phi\tilde{x}'$ are related as $\phi\tilde{x}\cos(-\phi\psi) = \phi\tilde{x}'$ hence, $\phi\tilde{x} = \phi\tilde{x}'\sec\phi\psi$, along the horizontal in the same Fig. 9a. By using this in the last displayed equation we have

$$\phi c\phi\tilde{t} = \phi c\phi\tilde{t}'\sec\phi\psi + \phi\tilde{x}'\tan\phi\psi \qquad (14)$$

(w.r.t. 3-observer Paul in $\tilde{\Sigma}'$).

Likewise the affine intrinsic space coordinate $\phi\tilde{x}$ is related to $\phi\tilde{x}'$ and the component $-\phi c\phi\tilde{t}\sin(-\phi\psi)$ projected along the horizontal with respect to the 1-observer P̃aul in the time dimension $c\tilde{t}'$ of the particle's frame in Fig. 9b as

$$\phi\tilde{x} = \tilde{x}'\sec\phi\psi + \phi c\phi\tilde{t}\sin\phi\psi .$$

Then by using the relation, $\phi c\phi\tilde{t} = \phi c\phi\tilde{t}'\sec\phi\psi$, which also holds along the vertical in the same Fig. 9b in the last displayed equation, we have

$$\phi\tilde{x} = \phi\tilde{x}'\sec\phi\psi + \phi c\phi\tilde{t}'\tan\phi\psi \qquad (15)$$

(w.r.t. 1-observer P̃aul in $c\tilde{t}'$).

By collecting the partial intrinsic coordinate transformations (14) and (15) we obtain the inverse intrinsic Lorentz transformation to system (13) with respect to 4-observer

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(Paul, P̃aul) in the particle's (or primed) frame as follows:

$$\left.\begin{array}{c}\phi c\,\phi\tilde{t} = \phi c\,\phi\tilde{t}'\sec\phi\psi + \phi\tilde{x}'\tan\phi\psi \\ \text{(w.r.t. 3-observer Paul in }\tilde{\Sigma}') \\ \phi\tilde{x} = \phi\tilde{x}'\sec\phi\psi + \phi c\,\phi\tilde{t}'\tan\phi\psi \\ \text{(w.r.t. 1-observer P̃aul in }c\tilde{t}')\end{array}\right\}, \qquad (16)$$

where $-\frac{\pi}{2} < \phi\psi < \frac{\pi}{2}$ (temporarily).

Again the range $-\frac{\pi}{2} < \phi\psi < \frac{\pi}{2}$ of the intrinsic angles $\phi\psi$ in system (16) in the positive universe is temporary as indicated. It shall be modified shortly in this section.

By considering the origin $\phi\tilde{x}' = 0$ of the intrinsic space coordinate $\phi\tilde{x}'$ of the primed intrinsic frame, system (16) simplifies as follows:

$$\phi\tilde{x} = \phi c\,\phi\tilde{t}'\tan\phi\psi \quad \text{and} \quad \phi c\,\phi\tilde{t} = \phi c\,\phi\tilde{t}'\sec\phi\psi. \qquad (17)$$

Then by dividing the first into the second equation of system (17) we have

$$\frac{\phi\tilde{x}}{\phi c\,\phi\tilde{t}} = \sin\phi\psi.$$

But, $\phi\tilde{x}/\phi\tilde{t} = \phi v$, is the intrinsic speed of the primed intrinsic frame relative to the unprimed intrinsic frame. Hence,

$$\sin\phi\psi = \phi v/\phi c = \phi\beta \qquad (18)$$

$$\sec\phi\psi = \frac{1}{\sqrt{1 - \phi v^2/\phi c^2}} = \phi\gamma. \qquad (19)$$

By using relations (18) and (19) in systems (13) we have

$$\phi c\,\phi\tilde{t}' = \frac{1}{\sqrt{1 - \phi v^2/\phi c^2}}\left(\phi c\,\phi\tilde{t} - \frac{\phi v}{\phi c}\,\phi\tilde{x}\right)$$

(w.r.t. 1-observer P̃eter in $c\tilde{t}$),

$$\phi\tilde{x}' = \frac{1}{\sqrt{1 - \phi v^2/\phi c^2}}\left(\phi\tilde{x} - \frac{\phi v}{\phi c}\,\phi c\,\phi\tilde{t}\right)$$

(w.r.t. 3-observer Peter in $\tilde{\Sigma}$), or

$$\left.\begin{array}{c}\phi\tilde{t}' = \phi\gamma\left(\phi\tilde{t} - \frac{\phi v}{\phi c^2}\,\phi\tilde{x}\right) \\ \text{(w.r.t. 1-observer P̃eter in }c\tilde{t}) \\ \phi\tilde{x}' = \phi\gamma\left(\phi\tilde{x} - \phi v\,\phi\tilde{t}\right) \\ \text{(w.r.t. 3-observer Peter in }\tilde{\Sigma})\end{array}\right\}. \qquad (20)$$

And by using equations (18) and (19) in system (16) we have

$$\phi c\,\phi\tilde{t} = \frac{1}{\sqrt{1 - \phi v^2/\phi c^2}}\left(\phi c\,\phi\tilde{t}' + \frac{\phi v}{\phi c}\,\phi\tilde{x}'\right)$$

(w.r.t. 3-observer Peter in $\tilde{\Sigma}'$),

$$\phi\tilde{x} = \frac{1}{\sqrt{1 - \phi v^2/\phi c^2}}\left(\phi\tilde{x}' + \frac{\phi v}{\phi c}\,\phi c\,\phi\tilde{t}'\right)$$

(w.r.t. 1-observer P̃eter in $c\tilde{t}'$), or

$$\left.\begin{array}{c}\phi\tilde{t} = \phi\gamma\left(\phi\tilde{t}' + \frac{\phi v}{\phi c^2}\,\phi\tilde{x}'\right) \\ \text{(w.r.t. 3-observer Paul in }\tilde{\Sigma}') \\ \phi\tilde{x} = \phi\gamma\left(\phi\tilde{x}' + \phi v\,\phi\tilde{t}'\right) \\ \text{(w.r.t. 1-observer P̃aul in }c\tilde{t}')\end{array}\right\}. \qquad (21)$$

Systems (20) and (21) are the explicit forms of the intrinsic Lorentz transformation (ϕLT) of extended affine intrinsic coordinates and its inverse respectively on the flat two-dimensional metric intrinsic spacetime $(\phi\rho, \phi c\phi t)$ that underlies the flat four-dimensional metric spacetime (Σ, ct) in the positive universe in Fig. 7.

As can be easily verified, either system (13) or (16) or its explicit form (20) or (21) implies intrinsic Lorentz invariance (ϕLI) on $(\phi\rho, \phi c\phi t)$:

$$\phi c^2\phi\tilde{t}^2 - \phi\tilde{x}^2 = \phi c^2\phi\tilde{t}'^2 - \phi\tilde{x}'^2. \qquad (22)$$

Just as the 4-observer (Peter, P̃eter) in the unprimed frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$ derives system (13) given explicitly as system (20) from Figs. 8a and 8b and the 4-observer (Paul, P̃aul) in the primed frame derives the system (16) given explicitly as system (21) from Figs. 9a and 9b in the positive universe, the symmetry-partner 4-observer* (Peter*, P̃eter*) in the unprimed frame $(-\tilde{x}^*, -\tilde{y}^*, -\tilde{z}^*, -c\tilde{t}^*)$ in the negative universe derives the ϕLT and its inverse from Figs. 8a and 8b and the symmetry-partner observer* (Paul*, P̃aul*) in the primed frame $(-\tilde{x}'^*, -\tilde{y}'^*, -\tilde{z}'^*, -c\tilde{t}'^*)$ in the the negative universe derives the inverse ϕLT from Figs. 9a and 9b, and the 4-observers (Peter*, P̃eter*) and (Paul*, P̃aul*) write

$$\left.\begin{array}{c}-\phi c\,\phi\tilde{t}'^* = -\phi c\,\phi\tilde{t}^*\sec\phi\psi - (-\phi\tilde{x}^*)\tan\phi\psi \\ \text{(w.r.t. 1-observer* P̃eter* in }-c\tilde{t}^*) \\ -\phi\tilde{x}'^* = -\phi\tilde{x}^*\sec\phi\psi - (-\phi c\,\phi\tilde{t}^*)\tan\phi\psi \\ \text{(w.r.t. 3-observer* Peter* in }-\tilde{\Sigma}^*)\end{array}\right\} \qquad (23)$$

and

$$\left.\begin{array}{c}-\phi c\,\phi\tilde{t}^* = -\phi c\,\phi\tilde{t}'^*\sec\phi\psi + (-\phi\tilde{x}'^*)\tan\phi\psi \\ \text{(w.r.t. 3-observer* Paul* in }-\tilde{\Sigma}'^*) \\ -\phi\tilde{x}^* = -\phi\tilde{x}'^*\sec\phi\psi + (-\phi c\,\phi\tilde{t}'^*)\tan\phi\psi \\ \text{(w.r.t. 1-observer* P̃aul* in }-c\tilde{t}'^*)\end{array}\right\}, \qquad (24)$$

where $-\frac{\pi}{2} < \phi\psi < \frac{\pi}{2}$ (temporarily).

The range $-\frac{\pi}{2} < \phi\psi < \frac{\pi}{2}$ of the intrinsic angles $\phi\psi$ in systems (23) and (24) in the negative universe is temporary as indicated. It shall be modified shortly in this section.

Systems (23) and (24) can also be put in their explicit

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forms respectively as follows by virtue of Eqs. (18) and (19):

$$-\phi\tilde{t}'^* = \phi\gamma\left(-\phi\tilde{t}^* - \frac{\phi v}{\phi c^2}(-\phi\tilde{x}^*)\right)$$
$$\text{(w.r.t. 1-observer* } \tilde{P}\text{eter* in } -c\tilde{t}^*)$$
$$\left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} \quad (25)$$
$$-\phi\tilde{x}'^* = \phi\gamma(-\phi\tilde{x}^* - \phi v(-\phi\tilde{t}^*))$$
$$\text{(w.r.t. 3-observer* Peter* in } -\tilde{\Sigma}^*)$$

and

$$-\phi\tilde{t}^* = \phi\gamma\left(-\phi\tilde{t}'^* + \frac{\phi v}{\phi c^2}(-\phi\tilde{x}'^*)\right)$$
$$\text{(w.r.t. 3-observer* Paul* in } -\tilde{\Sigma}'^*)$$
$$\left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} . \quad (26)$$
$$-\phi\tilde{x}^* = \phi\gamma(-\phi\tilde{x}'^* + \phi v(-\phi\tilde{t}'^*))$$
$$\text{(w.r.t. 1-observer* } \tilde{P}\text{aul* in } -c\tilde{t}'^*)$$

Again system (23) or (24) or the explicit form (25) or (26) implies intrinsic Lorentz invariance on the flat two-dimensional intrinsic spacetime $(-\phi\rho^*, -\phi c\phi t^*)$ in the negative universe:

$$(-\phi c^2\phi\tilde{t}^*)^2 - (-\phi\tilde{x}^*)^2 = (-\phi c^2\phi\tilde{t}'^*)^2 - (-\phi\tilde{x}'^*)^2. \quad (27)$$

The intrinsic LT of system (13) and its inverse of system (16) or their explicit forms of systems (20) and (21) and the intrinsic Lorentz invariance (22) they imply, pertain to two-dimensional intrinsic Special Theory of Relativity (ϕSR) on the flat two-dimensional metric intrinsic spacetime $(\phi\rho, \phi c\phi t)$ that underlies the flat four-dimensional metric spacetime (Σ, ct) in the positive universe in Fig. 7. In symmetry, the intrinsic LT and its inverse of system (23) and (24) or their explicit forms (25) and (26) and the intrinsic Lorentz invariance (27) they imply pertain to the intrinsic Special Theory of Relativity (ϕSR) on flat two-dimensional metric intrinsic spacetime $(-\phi\rho^*, -\phi c\phi t^*)$ that underlies the flat four-dimensional metric spacetime $(-\Sigma^*, -ct^*)$ in the negative universe.

Having derived the intrinsic LT of system (13) on page 11 and its inverse of system (16) on page 13 and their explicit forms of systems (20) and (21) in the context of intrinsic 2-geometry ϕSR in the positive universe, we must now obtain their outward (or physical) manifestations on the flat four-dimensional spacetime in the context of 4-geometry Special Theory of Relativity (SR). We do not have to draw a new set of diagrams in the two-world picture in which extended straight line affine spacetime coordinates $\tilde{x}'$ and $c\tilde{t}'$ of the primed frame are rotated relative to the extended affine coordinates $\tilde{x}$ and $c\tilde{t}$ respectively of the unprimed frame on the vertical (x, ct)-plane, while the affine coordinates $\tilde{y}'$ and $\tilde{z}'$ of the primed frame along which relative motion of SR do not occur are not rotated on the vertical spacetime plane. Indeed such diagram does exist. Figures 8a and 8b and their inverses Figs. 9a and 9b, in which the intrinsic spacetime coordinates are rotated being the only diagrams of Special Relativity/intrinsic Special Relativity (SR/ϕSR) in the two-world picture.

As discussed earlier, the flat four dimensional metric spacetime $(\Sigma, ct) \equiv (x, y, z, ct)$ is the outward (or physical) manifestation of the flat two-dimensional metric intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in Fig. 7. Likewise the extended mutually orthogonal straight line affine coordinates $\tilde{x}, \tilde{y}$ and $\tilde{z}$ constitute a flat affine 3-space, shown as a straight line and denoted by $\tilde{\Sigma}(\tilde{x}, \tilde{y}, \tilde{z})$ along the horizontal in the first quadrant. It is the outward manifestation of the extended straight line affine intrinsic coordinate $\phi\tilde{x}$ underlying it in Figs. 8a and 8b. And the extended straight line affine time coordinate $c\tilde{t}$ is the outward (or physical) manifestation of the extended straight line affine intrinsic time coordinate $\phi c\phi\tilde{t}$ along the vertical in Figs. 8a and 8b. The extended straight line affine spacetime coordinates $\tilde{x}', \tilde{y}', \tilde{z}'$ and $c\tilde{t}$ are likewise the outward manifestations of the extended affine intrinsic spacetime coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ in Figs. 9a and 9b.

It follows by virtue of the foregoing paragraph that the LT and its inverse in the context of SR are the outward (or physical) manifestations of the intrinsic Lorentz transformation (ϕLT) of system (13) or (20) and its inverse of system (16) or (21). We must simply remove the symbol ϕ in systems (13) and (16) to have the LT and its inverse in SR respectively as follows:

$$c\tilde{t}' = c\tilde{t}\sec\psi - \tilde{x}\tan\psi$$
$$\text{(w.r.t. } \tilde{P}\text{eter in } c\tilde{t})$$
$$\left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \quad (28)$$
$$\tilde{x}' = \tilde{x}\sec\psi - c\tilde{t}\tan\psi, \quad \tilde{y}' = \tilde{y}, \quad \tilde{z}' = \tilde{z}$$
$$\text{(w.r.t. Peter in } \tilde{\Sigma})$$

and

$$c\tilde{t} = c\tilde{t}'\sec\psi + \tilde{x}'\tan\psi$$
$$\text{(w.r.t. Paul in } \tilde{\Sigma}')$$
$$\left.\begin{array}{c} \\ \\ \\ \end{array}\right\}, \quad (29)$$
$$\tilde{x} = \tilde{x}'\sec\psi + c\tilde{t}'\tan\psi, \quad \tilde{y} = \tilde{y}', \quad \tilde{z} = \tilde{z}'$$
$$\text{(w.r.t. } \tilde{P}\text{aul in } c\tilde{t}')$$

where $-\frac{\pi}{2} < \psi < \frac{\pi}{2}$ (temporarily).

The trivial transformations $\tilde{y} = \tilde{y}'$ and $\tilde{z} = \tilde{z}'$ of the coordinates along which relative motion of SR does not occur have been added to the first and second equations of systems (28) obtained by simply removing symbol ϕ from system (13) on page 11 and to the first and second equations of system (29) obtained by simply removing symbol ϕ from system (16) on page 13, thereby making the resulting LT of system (28) and its inverse of system (29) consistent with the 4-geometry of SR. The angle ψ being the outward manifestation in spacetime of the intrinsic angle $\phi\psi$ in intrinsic spacetime, has the same temporary range in systems (28) and (29) as does $\phi\psi$ in systems (13) and (16). This temporary range of ψ shall also be modified shortly in this section.

System (28) indicates that the affine spacetime coordinates $\tilde{x}'$ and $c\tilde{t}'$ are rotated at equal angle ψ relative to the affine spacetime coordinates $\tilde{x}$ and $c\tilde{t}$ respectively, while $\tilde{y}$ is not rotated relative $\tilde{y}$ and $\tilde{z}'$ is not rotated relative to $\tilde{z}$ by angle ψ in the context of SR and system (29) indicates that $\tilde{x}$

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and $c\tilde{t}$ are rotated by equal negative angle $-\psi$ relative to $\tilde{x}'$ and $c\tilde{t}'$ respectively. However the relative rotations of the affine coordinates of the four-dimensional spacetime do not exist in reality, as discussed earlier. The indicated rotations in systems (28) and (29) may be referred to as intrinsic (i.e. non-observable or hypothetical) relative rotations of affine spacetime coordinates only, which is what the actual relative rotations of affine intrinsic spacetime coordinates in Figs. 8a and 8b and Figs. 9a and 9b represent.

By considering the spatial origin $\tilde{x}' = \tilde{y}' = \tilde{z}' = 0$ of the primed frame, system (29) reduces as follows:

$$c\tilde{t} = c\tilde{t}' \sec\psi \quad \text{and} \quad \tilde{x} = \tilde{x}' \tan\psi. \qquad (30)$$

And by dividing the second equation into the first equation of system (30) we have

$$\frac{\tilde{x}}{c\tilde{t}} = \sin\psi.$$

But, $\tilde{x}/\tilde{t} = v$, is the speed of the primed frame $(\tilde{x}', \tilde{y}', \tilde{z}', c\tilde{t}')$ frame relative to the unprimed frame $(\tilde{x}, \tilde{y}, \tilde{z}, c\tilde{t})$, for relative motion along the collinear $\tilde{x}$ and $\tilde{x}'$ coordinates of the frames. Hence

$$\sin\psi = v/c = \beta, \qquad (31)$$

$$\sec\psi = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma. \qquad (32)$$

Relations (31) and (32) on flat four-dimensional spacetime corresponds to relations (18) and (19) respectively on flat two-dimensional intrinsic spacetime. By using Eqs. (31) and (32) in systems (28) and (29) we obtain the LT and its inverse in their usual explicit forms respectively as follows:

$$\left.\begin{array}{c} \tilde{t}' = \gamma\left(\tilde{t} - \dfrac{v}{c^2}\,\tilde{x}\right) \\ \text{(w.r.t. P̃eter in } c\tilde{t}) \\[2mm] \tilde{x}' = \gamma\left(\tilde{x} - v\tilde{t}\right), \quad \tilde{y}' = \tilde{y}, \quad \tilde{z}' = \tilde{z} \\ \text{(w.r.t. Peter in } \tilde{\Sigma}) \end{array}\right\} \qquad (33)$$

and

$$\left.\begin{array}{c} \tilde{t} = \gamma\left(\tilde{t}' + \dfrac{v}{c^2}\,\tilde{x}'\right) \\ \text{(w.r.t. Paul in } \tilde{\Sigma}') \\[2mm] \tilde{x} = \gamma\left(\tilde{x}' + v\tilde{t}'\right), \quad \tilde{y} = \tilde{y}', \quad \tilde{z} = \tilde{z}' \\ \text{(w.r.t. P̃eter in } c\tilde{t}') \end{array}\right\}. \qquad (34)$$

Systems (33) and (34) are the outward (or physical) manifestations on flat four-dimensional spacetime (Σ, ct) in the context of SR of systems (20) and (21) respectively on the flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the context of ϕSR in the positive universe.

Systems (28) and (29) or the explicit form (33) or (34) implies Lorentz invariance (LI) in SR in the positive universe:

$$c^2\tilde{t}^2 - \tilde{x}^2 - \tilde{y}^2 - \tilde{z}^2 = c^2\tilde{t}'^2 - \tilde{x}'^2 - \tilde{y}'^2 - \tilde{z}'^2. \qquad (35)$$

This is the outward manifestation on flat four-dimensional spacetime of SR of the intrinsic Lorentz invariance (ϕLI) (22) on page 13 on flat two-dimensional intrinsic spacetime of ϕSR. Just as the intrinsic LT and its inverse of system (13) on page 11 and (16) on page 13 in the context of ϕSR are made manifest in systems (28) and (29) respectively in SR in the positive universe, the intrinsic LT and its inverse of systems (23) and (24) in ϕSR are made manifest in LT and its inverse in SR in the negative universe respectively as follows:

$$\left.\begin{array}{c} -c\tilde{t}'^* = -c\tilde{t}^* \sec\psi - (-\tilde{x}^*) \tan\psi \\ \text{(w.r.t. P̃eter}^* \text{ in } -c\tilde{t}^*) \\[2mm] -\tilde{x}'^* = -\tilde{x}^* \sec\psi - (-c\tilde{t}^*) \tan\psi, \\ -\tilde{y}'^* = -\tilde{y}^*, \quad -\tilde{z}'^* = -\tilde{z}^* \\ \text{(w.r.t. Peter}^* \text{ in } -\tilde{\Sigma}^*) \end{array}\right\} \qquad (36)$$

and

$$\left.\begin{array}{c} -c\tilde{t}^* = -c\tilde{t}'^* \sec\psi + (-\tilde{x}'^*) \tan\psi \\ \text{(w.r.t. Paul}^* \text{ in } -\tilde{\Sigma}'^*) \\[2mm] -\tilde{x}^* = -\tilde{x}'^* \sec\psi + (-c\tilde{t}'^*) \tan\psi, \\ -\tilde{y}^* = -\tilde{y}'^*, \quad -\tilde{z}^* = -\tilde{z}'^* \\ \text{(w.r.t. P̃aul}^* \text{ in } -c\tilde{t}'^*) \end{array}\right\}. \qquad (37)$$

And by using equations (31) and (32) in systems (36) and (37) we obtain the LT and it inverse in their usual explicit forms in the negative universe as follows:

$$\left.\begin{array}{c} -\tilde{t}'^* = \gamma\left(-\tilde{t}^* - \dfrac{v}{c^2}\,(-\tilde{x}^*)\right) \\ \text{(w.r.t. P̃eter}^* \text{ in } -c\tilde{t}^*) \\[2mm] -\tilde{x}'^* = \gamma\left(-\tilde{x}^* - v(-\tilde{t}^*)\right), \quad -\tilde{y}'^* = -\tilde{y}^*, \quad -\tilde{z}'^* = -\tilde{z}^* \\ \text{(w.r.t. Peter}^* \text{ in } -\tilde{\Sigma}^*) \end{array}\right\} \qquad (38)$$

and

$$\left.\begin{array}{c} -\tilde{t}^* = \gamma\left(-\tilde{t}'^* + \dfrac{v}{c^2}\,(-\tilde{x}'^*)\right) \\ \text{(w.r.t. Paul}^* \text{ in } -\tilde{\Sigma}'^*) \\[2mm] -\tilde{x}^* = \gamma\left(-\tilde{x}'^* + v(-\tilde{t}'^*)\right), \quad -\tilde{y}^* = -\tilde{y}'^*, \quad -\tilde{z}^* = -\tilde{z}'^* \\ \text{(w.r.t. P̃aul}^* \text{ in } -c\tilde{t}'^*) \end{array}\right\}. \qquad (39)$$

Systems (38) and (39) are the outward manifestations on flat four-dimensional spacetime $(-\Sigma^*, -ct^*)$ of SR of systems (25) and (26) respectively on flat two-dimensional intrinsic spacetime $(-\phi\rho^*, -\phi c\phi t^*)$ of ϕSR in the negative universe. Either the LT (36) or its inverse (37) or the explicit form (38) or (39) implies Lorentz invariance in SR in the negative universe:

$$(-c\tilde{t}^*)^2 - (-\tilde{x}^*)^2 - (-\tilde{y}^*)^2 - (-\tilde{z}^*)^2 =$$
$$= (-c\tilde{t}'^*)^2 - (-\tilde{x}'^*)^2 - (-\tilde{y}'^*)^2 - (-\tilde{z}'^*)^2. \qquad (40)$$

This is the outward manifestation on the flat four-dimensional spacetime of SR of the intrinsic Lorentz invariance (27)
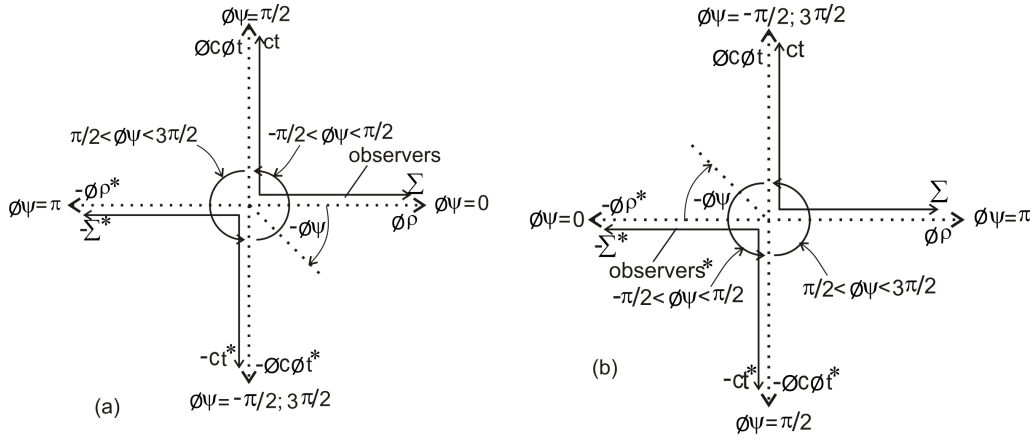
Fig. 10: The concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ within which the intrinsic angle $\phi\psi$ could take on values: a) with respect to 3-observers in the positive universe and b) with respect to 3-observers in the negative universe.

on page 14 on flat two-dimensional intrinsic spacetime of ϕSR in the negative universe. The restriction of the values of the intrinsic angle $\phi\psi$ to a half-plane $(-\frac{\pi}{2} < \phi\psi < \frac{\pi}{2})$ with respect to observers in the positive universe in systems (13) and (16) and with respect to observers* in the negative universe in systems (23) and (24) is a temporary measure as indicated in those systems. The intrinsic angle $\phi\psi$ actually takes on values on the entire plane $[-\frac{\pi}{2} \leqslant \phi\psi \leqslant \frac{3\pi}{2}]$ with respect to observers in the positive and negative universes, except that certain values of $\phi\psi$ namely, $-\frac{\pi}{2}$, $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, must be excluded, as shall be discussed more fully shortly. The values of $\phi\psi$ in the first cycle as well as negative senses of rotation (by negative intrinsic angle $-\phi\psi$) with respect to 3-observers in the 3-spaces in the positive and negative universes are shown in Figs. 10a and 10b respectively.

We have thus obtained a (new) set of spacetime/intrinsic spacetime diagrams namely, Figs. 8a and 8b and their inverses Figs. 9a and 9b in the context of Scheme II in Table 1 or in the two-world picture, for deriving intrinsic Lorentz transformation (ϕLT) and its inverse in terms of extended straight line affine intrinsic spacetime coordinates $\phi\tilde{x}'$, $\phi c\phi\tilde{t}'$ and $\phi\tilde{x}$, $\phi c\phi\tilde{t}$ on the flat two-dimensional metric intrinsic spacetime $(\phi\rho, \phi c\phi t)$ of the two-dimensional intrinsic Special Theory of Relativity (ϕSR) in both the positive and negative universes and for deriving the Lorentz transformation (LT) and its inverse in terms of extended straight line affine spacetime coordinates $\tilde{x}$, $\tilde{y}$, $\tilde{z}$, $c\tilde{t}$ and $\tilde{x}'$, $\tilde{y}'$, $\tilde{z}'$, $c\tilde{t}'$, as outward (or physical) manifestations on the flat four-dimensional spacetime of SR of the intrinsic Lorentz transformation (ϕLT) and its inverse of ϕSR in both the positive and negative universes. Figures 8a and 8b and their inverses Figs. 9a and 9b must replace the Minkowski diagrams of Figs. 3a and 3b in the context of Scheme I in Table 1 or in the one-world picture.

The skewness of the rotated spacetime coordinates in the Minkowski diagrams of Figs. 3a and 3b (and in the Loedel and Brehme diagrams of Figs. 4a and 4b), from which the LT and its inverse have sometimes been derived until now in

the existing one-world picture, has been remarked to be undesirable earlier in this paper because the observer at rest with respect to the frame with rotated spacetime coordinates could detect the skewness of the coordinates of his frame as an effect of the uniform motion of his frame. Moreover the skewness of the rotated coordinates of the "moving" frame vis-a-vis the non-skewed coordinates of the "stationary" frame (in the Minkowski diagrams) gives apparent preference to one of two frames in uniform relative motion. On the other hand, neither the skewness of the rotated intrinsic spacetime coordinates of the "moving" frame nor of the "stationary" frame occurs in Figs. 8a, 8b, 9a and 9b. The diagrams of Figs. 8a, 8b, 9a and 9b in the two-world picture do not give apparent preference for any one of the pair of intrinsic frames in relative intrinsic motion and consequently do not give apparent preference for any one of the pair of frames on four-dimensional spacetime in relative motion, since both intrinsic frames have mutually orthogonal intrinsic spacetime coordinates in each of those figures.

Although the negative universe is totally elusive to people in our (or positive) universe, just as our universe is totally elusive to people in the negative universe, from the point of view of direct experience, we have now seen in the above that the intrinsic spacetime coordinates of the two universes unite in prescribing intrinsic Lorentz transformation and intrinsic Lorentz invariance on the flat two-dimensional intrinsic spacetime and consequently in prescribing Lorentz transformation and Lorentz invariance on flat four-dimensional spacetime in each of the two universes. It can thus be said that there is intrinsic (or non-observable) interaction of four-dimensional spacetime coordinates of the two universes in Special Relativity.

The singularities at $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = -\frac{\pi}{2}$ or $\phi\psi = \frac{3\pi}{2}$ in systems (13) and (16), (of Scheme II in Table 1 or in the two-world picture), correspond to the singularities at $\alpha = \infty$ and $\alpha = -\infty$ in the coordinate transformation of systems (4) and (5) in the Minkowski one-world picture. Being smooth

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for all values of α, except for the extreme values, $\alpha = \infty$ and $\alpha = -\infty$, at its boundary represented by the vertical line in Fig. 1a, which corresponds to a line along the ct- and $-ct^*$-axes in Fig. 2a, the only (positive) Minkowski space including the time reversal dimension, (to be denoted by $(\Sigma, ct, -ct^*)$), in Fig. 2a in the one-world picture is usually considered to be sufficiently smooth. Similarly being smooth for all values of the intrinsic angle $\phi\psi$ in the first cycle, except for $\phi\psi = -\frac{\pi}{2}, \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$ along their interface in Fig. 2b, the positive Minkowski space including the time reversal dimension $(\Sigma, ct, -ct^*)$ and the negative Minkowski space including time reversal dimension $(-\Sigma^*, -ct^*, ct)$ of the two-world picture in Fig. 2b must be considered to be sufficiently smooth individually.

An attempt to compose the positive Minkowski space including the time reversal dimension $(\Sigma, ct, -ct^*)$ and the negative Minkowski space including time reversal dimension $(-\Sigma^*, -ct^*, ct)$ into a single space, over which $\phi\psi$ has values within the range $[-\frac{\pi}{2}, \frac{3\pi}{2}]$ or $[0, 2\pi]$, cannot work since the resultant space possesses interior (and not boundary) discontinuities at $\phi\psi = \frac{\pi}{2}$ in the case of the range $[-\frac{\pi}{2}, \frac{3\pi}{2}]$ and $\phi\psi = -\frac{\pi}{2}, \phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$ in the case of the range $[0, 2\pi]$, thereby making the single space generated non-smooth. This implies that the larger spacetime domain of combined positive and and negative universes cannot be considered as a continuum of event domain or as constituting a single world or universe. The lines of singularity $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = -\frac{\pi}{2}$ along the vertical ct- and and $-ct^*$-axes respectively represent event horizons, (the special-relativistic event horizons), to observers in 3-spaces Σ and $-\Sigma^*$ in the positive and negative universes respectively. These event horizons at $\phi\psi = \frac{\pi}{2}$ and $-\frac{\pi}{2}$ show up as singularities in the intrinsic Lorentz transformation (ϕLT) and its inverse of systems (13) and (16) and consequently in the LT and its inverse of systems (28) and (29) in the positive universe and in ϕLT and its inverse of systems (23) and (24) and consequently in the LT and its inverse of systems (36) and (37) in the negative universe.

The observers in 3-space on one side of the event horizons along the dimensions ct and $-ct^*$ in Fig. 5 and Fig. 7 cannot observe events taking place on the other side. This makes a two-world interpretation of Scheme II in Table 1 with the spacetime/intrinsic spacetime diagram of Fig. 7 mandatory.

4.5 Reduction of the LT and its inverse to length contraction and time dilation formulae from the point of view of what can be measured with laboratory rod and clock

Nature makes use of all the terms of the LT, system (28) or (33), and its inverse, system (29) or (34) to establish Lorentz invariance. However man could not detect all the terms of the LT and its inverse with his laboratory rod and clock. First of all, it is the last three equations of system (28) or (33) written by or with respect to the 3-observer (Peter) in 3-space in the unprimed frame with affine coordinates $\tilde{x}, \tilde{y}$ and $\tilde{z}$ and the first equation of system (29) or (34) written by or with respect to the 3-observer Paul in 3-space in the primed frame with affine coordinates $\tilde{x}', \tilde{y}'$ and $\tilde{z}'$ that are relevant for the measurements of distance in space by a rod in 3-space and of time duration by a clock kept in 3-space respectively of a special-relativistic event by 3-observers in 3-space. By collecting those equations we have the following:

$$\tilde{x}' = \tilde{x} \sec\psi - c\tilde{t} \tan\psi, \quad \tilde{y}' = \tilde{y}, \quad \tilde{z}' = \tilde{z} \qquad (41a)$$

(w.r.t. 3-observer Peter in $\tilde{\Sigma}$), and

$$c\tilde{t} = c\tilde{t}' \sec\psi + \tilde{x}' \tan\psi \qquad (41b)$$

(w.r.t. 3-observer Paul in $\tilde{\Sigma}'$).

Now when Peter picks his laboratory rod to measure length, he will be unable to measure the term $-c\tilde{t} \tan\psi$ of the first equation of system (41a) with his laboratory-rod. Likewise when Paul picks his clock to measure time duration, he will be unable to measure the term $\tilde{x}' \tan\psi$ in (41b) with his clock. Thus from the point of view of what can be measured by laboratory rod and clock by observers in 3-space, system (41a) and Eq. (41b) reduce as follows:

$$\tilde{x} = \tilde{x}' \cos\psi, \quad \tilde{y} = \tilde{y}', \quad \tilde{z} = \tilde{z}', \quad \tilde{t} = \tilde{t}' \sec\psi. \qquad (42)$$

System (42) becomes the following explicit form in terms of particle's speed relative to the observer by virtue of Eq. (32) on page 15:

$$\left. \begin{array}{l} \tilde{x} = \tilde{x}' \sqrt{1 - v^2/c^2}, \quad \tilde{y} = \tilde{y}', \quad \tilde{z} = \tilde{z}' \\ \tilde{t} = \dfrac{\tilde{t}'}{\sqrt{1 - v^2/c^2}} \end{array} \right\}. \qquad (43)$$

These are the well known length contraction and time dilation formulae for two frames in relative motion along their collinear $\tilde{x}$- and $\tilde{x}'$-axes in SR. Showing that they pertain to the measurable sub-space of the space of SR is the essential point being made here.

4.6 The generalized form of intrinsic Lorentz transformation in the two-world picture

Now let us rewrite the intrinsic Lorentz transformation (ϕLT) and its inverse of systems (13) on page 11 and (16) on page 13 in the positive universe in the generalized forms in which they can be applied for all values of $\phi\psi$ in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ in Fig. 10a by factorizing out $\sec\phi\psi$ to have respectively as follows:

$$\left. \begin{array}{l} \phi c\phi\tilde{t}' = \sec\phi\psi \, (\phi c\phi\tilde{t} - \phi\tilde{x} \sin\phi\psi) \\ \phi\tilde{x}' = \sec\phi\psi \, (\phi\tilde{x} - \phi c\phi\tilde{t} \sin\phi\psi) \end{array} \right\} \qquad (44)$$

and

$$\left. \begin{array}{l} \phi c\phi\tilde{t} = \sec\phi\psi \, (\phi c\phi\tilde{t}' + \phi\tilde{x}' \sin\phi\psi) \\ \phi\tilde{x} = \sec\phi\psi \, (\phi\tilde{x}' + \phi c\phi\tilde{t}' \sin\phi\psi) \end{array} \right\}. \qquad (45)$$

The 3-observers in the Euclidean 3-space Σ of the positive universe "observe" intrinsic Special Relativity (ϕSR) and consequently observe Special Relativity (SR) for intrinsic angles $\phi\psi$ in the range $(-\frac{\pi}{2}, \frac{\pi}{2})$. However as Fig. 10a shows, 3-observers in the positive universe could construct ϕSR and hence SR relative to themselves for all intrinsic angles $\phi\psi$ in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$, by using the generalized intrinsic Lorentz transformation (ϕLT) and its inverse of systems (44) and (45) and obtaining the LT and its inverse as outward manifestations on flat four-dimensional spacetime of the ϕLT and its inverse so derived, although they can observe Special Relativity for intrinsic angles $\phi\psi$ in $(-\frac{\pi}{2}, \frac{\pi}{2})$ in Fig. 10a only.

Likewise the ϕLT and its inverse in the negative universe of systems (25) on page 14 and (26) on page 14, shall be written in the generalized forms in which they can be applied for all intrinsic angles $\phi\psi$ in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ in Fig. 10b respectively as follows:

$$\left.\begin{aligned}-\phi c\,\phi\tilde{t}'^{*} &= \sec\phi\psi\,(-\phi c\,\phi\tilde{t}^{*} - (-\phi\tilde{x}^{*})\sin\phi\psi)\\-\phi\tilde{x}'^{*} &= \sec\phi\psi\,(-\phi\tilde{x}^{*} - (-\phi c\,\phi\tilde{t}^{*})\sin\phi\psi)\end{aligned}\right\} \quad (46)$$

and

$$\left.\begin{aligned}-\phi c\,\phi\tilde{t}^{*} &= \sec\phi\psi\,(-\phi c\,\phi\tilde{t}'^{*} + (-\phi\tilde{x}'^{*})\sin\phi\psi)\\-\phi\tilde{x}^{*} &= \sec\phi\psi\,(-\phi\tilde{x}'^{*} + (-\phi c\,\phi\tilde{t}'^{*})\sin\phi\psi)\end{aligned}\right\}. \quad (47)$$

The 3-observers* in the Euclidean 3-space $-\Sigma^{*}$ of the negative universe "observe" intrinsic Special Relativity (ϕSR) and hence observe Special Relativity (SR) for intrinsic angles $\phi\psi$ in the open interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ in Fig. 10b. Again as Fig. 10b shows, 3-observers* in the negative universe could construct ϕSR and hence SR relative to themselves for all intrinsic angles $\phi\psi$ in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$, by using the generalized ϕLT and its inverse of ϕSR of systems (46) and (47) and obtaining LT and its inverse of SR as outward manifestations on flat four-dimensional spacetime of the ϕLT and its inverse so constructed, although they can observe SR for intrinsic angles $\phi\psi$ in $(-\frac{\pi}{2}, \frac{\pi}{2})$ in Fig. 10b only.

The fact that the intrinsic Lorentz transformation (ϕLT) and its inverse represent continuous rotation of intrinsic spacetime coordinates $\phi\tilde{x}'$ and $\phi c\,\phi\tilde{t}'$ of the primed frame relative to the intrinsic spacetime coordinates $\phi\tilde{x}$ and $\phi c\,\phi\tilde{t}$ respectively of the unprimed frame through all intrinsic angles $\phi\psi$ in the closed range $[0, 2\pi]$, excluding rotation by $\phi\psi = -\frac{\pi}{2}, \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$, is clear from the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ of the intrinsic angle $\phi\psi$ in Figs. 10a and 10b over which the generalized ϕLT and its inverse of systems (44) and (45) in the positive universe and systems (46) and (47) in the negative universe could be applied. We shall not be concerned with the explanation of how the intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\,\phi\tilde{t}'$ of the particle's intrinsic frame can be rotated continuously relative to the intrinsic coordinates $\tilde{x}$ and $\phi c\,\phi\tilde{t}$ of the observer's intrinsic frame through intrinsic angles $\phi\psi$ in the range $[0, 2\pi]$, while avoiding $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$ in this paper.

4.7 Non-existence of light cones in the two-world picture

The concept of light-cone does not exist in the two-world picture. This follows from the derived relation, $\sin\phi\psi = \phi v/\phi c$, (Eq. (18) on page 13), which makes the intrinsic speed ϕv of relative intrinsic motion of every pair of intrinsic frames lower than the intrinsic light speed ϕc, ($\phi v < \phi c$), for all values of $\phi\psi$ in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ in Fig. 10a in the context of ϕSR and consequently speed v of relative motion of every pair of frames lower than the speed of light c, ($v < c$), for all intrinsic angles $\phi\psi$ in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ in Fig. 10a. The intrinsic angle $\phi\psi = \frac{\pi}{2}$ corresponds to intrinsic speed $\phi v = \phi c$ and $\phi\psi = -\frac{\pi}{2}$ or $\phi\psi = \frac{3\pi}{2}$ corresponds to $\phi v = -\phi c$, which are excluded from ϕSR. They correspond to speed $v = c$ and $v = -c$ respectively, which are excluded from SR.

We therefore have a situation where all intrinsic angles $\phi\psi$ in the closed range $[0, 2\pi]$, except $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$, (in Fig. 10a), are accessible to intrinsic Special Relativity (ϕSR) with intrinsic timelike geodesics and consequently to SR with timelike geodesics with respect to observers in the positive universe. All intrinsic angles $\phi\psi$ in the closed interval $[0, 2\pi]$, except $\phi\psi = \frac{\pi}{2}$ and $\phi\psi = \frac{3\pi}{2}$, (in Fig. 10b), are likewise accessible to ϕSR with intrinsic timelike geodesics and hence to SR with timelike geodesics with respect to observers* in the negative universe.

Intrinsic spacelike geodesics of for which $\phi v > \phi c$ and spacelike geodesics for which $v > c$ do not exist for any value of the intrinsic angle $\phi\psi$ in the four quadrants, that is, for $\phi\psi$ in the closed range $[0, 2\pi]$, on the larger spacetime/intrinsic spacetime domain of combined positive and negative universes in Fig. 7. Since the existence of light cones requires regions of spacelike geodesics outside the cones, the concept of light cones does not exist in the two-world picture.

4.8 Prospect for making the Lorentz group compact in the two-world picture

The impossibility of making the Lorentz group SO(3,1) compact in the context of the Minkowski geometry in the one-world picture has been remarked earlier in this paper. It arises from the fact that the unbounded parameter space $-\infty < \alpha < \infty$ of the Lorentz boost (the matrix L in (6) on page 4), in the one-world picture, is unavoidable. Compactification of the Lorentz group in the two-world picture would be interesting.

Now the new intrinsic matrix ϕL^{*} that generates the intrinsic Lorentz boost, $\phi\mathbf{x} \rightarrow \phi\mathbf{x}' = \phi L^{*}\phi\mathbf{x}$, on the flat two-dimensional intrinsic spacetime in Eq. (13) on page 11 in the positive universe or (23) on page 13 in the negative universe in the two-world picture is the following:

$$\phi L^{*} = \begin{pmatrix} \sec\phi\psi & -\tan\phi\psi \\ -\tan\phi\psi & \sec\phi\psi \end{pmatrix}, \quad (48)$$

where $\phi\psi$ takes on values in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ in the positive and negative universes, as

explained earlier and illustrated in Figs. 10a and 10b.

The corresponding new matrix L^* that generates the Lorentz boost, $\mathbf{x} \to \mathbf{x}' = L^*\mathbf{x}$, on flat four-dimensional spacetime in Eq. (28) on page 14 in the positive universe or (36) on page 15 in the negative universe in the two-world picture is the following

$$L^* = \begin{pmatrix} \sec\psi & -\tan\psi & 0 & 0 \\ -\tan\psi & \sec\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (49)$$

where, ψ takes on values in the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$ like $\phi\psi$, in the positive and negative universes.

The matrix L^* can be said to be the outward manifestation on flat four-dimensional spacetime of SR of the intrinsic matrix ϕL^* on flat two-dimensional intrinsic spacetime of ϕSR. It must be recalled however that while the intrinsic angle $\phi\psi$ in (48) measures actual rotation of intrinsic coordinates $\phi\tilde{x}'$ and $\phi c\phi\tilde{t}'$ of the primed frame relative to the intrinsic coordinates $\phi\tilde{x}$ and $\phi c\phi\tilde{t}$ of the unprimed frame, (as in Figs. 8a, 8b, 9a and 9b), in the context of ϕSR, the angle ψ in (49) represents intrinsic (i.e. non-observable or hypothetical) rotation of spacetime coordinates $\tilde{x}'$ and $c\tilde{t}'$ of the primed frame relative to $\tilde{x}$ and $c\tilde{t}$ of the unprimed frame.

The concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, 3\pi)$ wherein the intrinsic angle $\phi\psi$ and the angle ψ take on values in the positive and negative universes imply that the intrinsic matrix ϕL^* (the intrinsic Lorentz boost) and the Lorentz boost L^* in the two-world picture are unbounded. It must be recalled that the matrix L that generates the Lorentz boost in the Minkowski one-world picture given by Eq. (6) on page 4 is likewise unbounded because the parameter α in that matrix takes on values in the unbounded interval $(-\infty, \infty)$.

Also by letting $\phi\psi \to \frac{\pi}{2}$ and $\phi\psi \to -\frac{\pi}{2}$ or $\frac{3\pi}{2}$ in the intrinsic matrix ϕL^*, we have $\sec\phi\psi = \tan\phi\psi \to \infty$ and $\sec\phi\psi = \tan\phi\psi \to -\infty$ respectively, which shows that ϕL^* (or the intrinsic Lorentz boost) and hence the Lorentz boost L^* in the two-world picture are not closed. Whereas $\alpha \to \infty$, $\cosh\alpha \to \infty$, $\sinh\alpha \to \infty$, and $\alpha \to -\infty$, $\cosh\alpha \to \infty$, $\sinh\alpha \to -\infty$ in matrix L, which implies that the Lorentz boost in the Minkowski one-world picture is closed (since no entry of L is outside the range $-\infty < \alpha < \infty$ of the parameter α [6]). Thus L is not bounded but is closed, while ϕL^* and L^* are not bounded and not closed. The matrices L, L^* and the intrinsic matrix ϕL^* are therefore non-compact.

It is required that ϕL^* be both closed and bounded for it to be compact. Likewise the matrix L^*. It follows from this and the foregoing paragraphs that making the the intrinsic Lorentz boost (48) and consequently the Lorentz boost (49) in the two-world picture compact has not been achieved in this paper. As deduced in sub-section 1.1, making the Lorentz boost compact implies making SO(3,1) compact. Thus SO(3,1) has yet not been made compact in the two-world picture since the Lorentz boost has yet not been made compact.

There is good prospect for making SO(3,1) compact in the two-world picture however. This is so since the intrinsic matrix ϕL^* and consequently the matric L^* (the Lorentz boost in the two-world picture) will become compact by justifiably replacing the concurrent open intervals $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{3\pi}{2})$, in which the intrinsic angle $\phi\psi$ and the angle ψ take on values in ϕL^* and L^* respectively, by the concurrent closed intervals $[-(\frac{\pi}{2} - \epsilon), \frac{\pi}{2} - \epsilon]$ and $[\frac{\pi}{2} - \epsilon, \frac{3\pi}{2} - \epsilon]$, where ϵ is a small nonzero angle. This will make each of ϕL^* and L^* to be both closed and bounded and hence to be compact. It will certainly require further development of the two-world picture than in this initial paper to make SO(3,1) compact in two-world − if it will be possible.

This paper shall be ended at this point with a final remark that although the possibility of the existence of a two-world picture (or symmetry) in nature has been exposed, there is the need for further theoretical justification than contained in this initial paper and experimental confirmation ultimately, in order for any one to conclude the definite existence of the two-world picture. The next natural step will be to include the light-axis and the distinguished frame of reference of electromagnetic waves in the two-world picture that encompasses no light cones and to investigate the signs of mass and other physical parameters, as well as the possibility of invariance of natural laws in the negative universe.

References

1. Bonnor W. B. Negative mass in general relativity. *Gen. Relat. & Grav.*, 1989, v. 21, 1143–1157.

2. Segre E. Nuclei and particles. W. A. Benjamin Inc., Reading, 1977.

3. . Muirhead H. The Special Theory of Relativity. The McMillan Book Company, New York, 1973.

4. Einstein A. On the electrodynamics of moving bodies. In: *The Principle of Relativity*, a collection of memoirs on relativity, with notes by A. Sommerfeld, translated by W. Perrett and G. B. Jeffery, Methuen, 1923 (with reprints by Dover Publications).

5. Minkowski H. Space and time. *Ibid.*

6. Tung W. Group theory in physics. World Scientific Publishing Co. Pte Ltd., Singapore, 1985.

7. Loedel E. Geometric representation of Lorentz transformation. *Am. J. Phys.*, 1957, v. 25, 327.

8. Brehme R. W. A geometric representation of Galilean and Lorentz transformations. *Am. J. Phys.*, 1962, v. 30, 489.

9. Adler R., Bazin M., Schiffer M. Introduction to General Relativity. Second edition, McGraw-Hill Book Company, New York, 1975.

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