QUANTIZING TORSION EFFECTS IN A DE SITTER UNIVERSE

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Abstract. We derive quantization relations in the case when torsion effects are added in a De Sitter spacetime metric with or without a black hole at the Planck mass and Planck length limit. To this end we use Zeldovich's definition of the cosmological constant.

Key words: gravitation – quantization – torsion – spin – black holes.

1. INTRODUCTION

A natural way to talk about spin effects in gravitation is through torsion. Its introduction becomes significant for the understanding of the last stage in black hole evaporation. It could be the case of an evaporating black hole of mass M_{H} that disappears via an explosion burst, which can last for a time $t \approx 10^{-43}$ s, when it reaches a mass of the order of Planck mass

$$m_{\mathrm{P}} = (\hbar c / G)^{1/2} = 10^{-5}\,\mathrm{g}\ . \tag{1}$$

If this happens, there might be three distinct possibilities for the fate of the evaporating black hole (De Sabbata et al. 1990):

– The black hole may evaporate completely leaving no residue, in which case it would give rise to a serious problem of quantum consistency.

– If the final state of evaporation leaves a naked singularity behind, then it might violate cosmic censorship at the quantum level.

– If a stable remnant of residue of approximately the Planck mass remains, the emission process might stop.

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2. THEORY

If somebody tries to quantize the gravitational field, he must be aware of the fact that quantization has to be directed with the unique structure of the spacetime itself. Quantization will also imply that somebody might try to discretize space and probably time. Progress in this direction will also be related with the introduction of spin in the theory of general relativity.

When it comes to elementary particles, spin and mass are some of their most important original properties. In general relativity, matter can be represented by the energy-momentum tensor, which provides us with a description of the mass energy distribution in spacetime. This mass energy idea can now describe the properties of the macroscopic bodies.

On the other hand, and at subatomic level, particles are characterized by their mass and spin, and therefore the energy-momentum tensor is not enough to describe matter, unless the spin density tensor is taken into account. This is the easiest way to include spin in Einstein's general relativity.

Torsion thus can be described by the antisymmetric part of the Christoffel symbols of the second kind, and therefore the torsion tensor reads (Johri 1996):

$$Q^{\mu}_{\nu\lambda} = \frac{1}{2}[\Gamma^{\mu}_{\nu\lambda} - \Gamma^{\mu}_{\nu\lambda}] \equiv \Gamma^{\mu}_{[\nu\lambda]}. \qquad (2)$$

In the presence of torsion, the spacetime is called a Riemann-Cartan manifold, and is denoted by U_4. When torsion is taken into consideration, one can define distances in the following way and, supposing that we consider a small close circuit, we can write (Johri 1996):

$$\ell^{\mu} = \oint Q^{\mu}_{\nu\lambda} \mathrm{d}x^{\nu} \mathrm{d}x^{\lambda}, \qquad (3)$$

where $\mathrm{d}x^{\nu}\mathrm{d}x^{\lambda}$ is the area element enclosed by the loop, and ℓ^{μ} represents the so-called closure failure. In other words, the geometrical meaning of torsion can be represented by the failure of the loop closure, and ℓ^{μ} has now the dimension of length, where torsion tensor itself has the dimension of L^{-1}. From the above relation, we can see that the intrinsic spin $\hbar$ and, as the spin is quantized, we can further deduce that any defect in spacetime topology should occur in multiple values of the Planck length (l_{P}), and therefore we have (Johri 1996):

$$\oint Q^{\mu}_{\nu\lambda} \mathrm{d}x^{\nu} \mathrm{d}x^{\lambda} = n\left(\frac{\hbar G}{c^3}\right)^{1/2} n^{\mu}, \qquad (4)$$

where n is an integer, and n^{μ} is a unit point vector. This is a analogous relation to the Bohr-Sommerfeld relation in quantum mechanics. The torsion tensor $Q^{\mu}_{\nu\lambda}$ plays the role of a field strength, which is analogous to that of the electromagnetic field tensor $F_{\mu\nu}$. Finally, the geodesic equations in the case of a nonzero spin turn to

$$\frac{\mathrm{d}^2 x^{\mu}}{\mathrm{d}p^2} + \Gamma^{\mu}_{\nu\lambda}\frac{\mathrm{d}x^{\nu}}{\mathrm{d}p}\frac{\mathrm{d}x^{\lambda}}{\mathrm{d}p} = -2Q^{\mu}_{\nu\lambda}\frac{\mathrm{d}x^{\nu}}{\mathrm{d}p}\frac{\mathrm{d}x^{\lambda}}{\mathrm{d}p}, \tag{5}$$

where p is an affine parameter.

3. INTRODUCTION OF LINE ELEMENTS

To understand spin effects in gravitation, we can use torsion. Consequently, let us first write the two modified De Sitter metrics including torsion effects (De Sabbata and Zhang 1992), as well as a De Sitter metric with torsion, also containing a black hole (Gibbons et al. 1985)

$$\begin{aligned} \mathrm{d}s^2 = c^2\left[1-\frac{\Lambda}{3}r^2 \pm \frac{3G^2s^2}{2r^4c^6}\right]\mathrm{d}t^2 - \\ -\left[1-\frac{\Lambda}{3}r^2 \pm \frac{3G^2s^2}{2r^4c^6}\right]^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathrm{d}s^2 = c^2\left[1-\frac{\Lambda r^2}{3}-\frac{2GM}{rc^2} \pm \frac{3G^2s^2}{2r^4c^6}\right]\mathrm{d}t^2 - \\ -\left[1-\frac{\Lambda r^2}{3}-\frac{2GM}{rc^2} \pm \frac{3G^2s^2}{2r^4c^6}\right]^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \end{aligned} \tag{7}$$

where Λ is the cosmological constant, s is the torsion, which could also be written as $s = \sigma r^3$ (with σ the spin density), M is the mass of the black hole, whereas the element of the solid angle is $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2$. The surface gravity of a black hole in general is given by (Birrel and Davies 1982);

$$\vec{k} = \frac{1}{2}\left(\frac{\partial F}{\partial r}\right)_{r=R_{\mathrm{H}}}, \tag{8}$$

where $F(r) = g_{ii}$ are the diagonal metric elements.

4. CALCULATION

Let us now proceed by using the De Sitter metric in which

$$g_{44} = 1 - \frac{\Lambda}{3} r^2 \pm \frac{3G^2 s^2}{2r^4 c^4} , \tag{9}$$

$$g_{44} = 1 - \frac{2GM}{rc^2} - \frac{\Lambda}{3} r^2 \pm \frac{3G^2 s^2}{2r^4 c^6} , \tag{10}$$

these being the corresponding metric elements without and with a black hole. The above relationships would imply that a spin $s \approx \hbar$ is parallel or antiparallel to gravitation. For such a black hole the surface gravity (and hence the temperature) vanishes. Therefore, in this case, the torsion effects entering with the same or opposite sign to gravitation cancel those of gravity.

Now, let us consider the metric element of the De Sitter universe containing a black hole. Going through the same analysis, we obtain the two expressions below. Taking into account expression (9) for torsion effects antiparallel to gravity, and expression (10) for torsion effects parallel to gravity, for antiparallel spin we have

$$\vec{k} = \frac{GM}{r^2 c^2} - \frac{\Lambda}{3} r - \frac{3G^2 s^2}{c^4 r^5} = 0 , \tag{11}$$

where for parallel spin we obtain

$$\vec{k} = \frac{GM}{r^2 c^2} - \frac{\Lambda}{3} r + \frac{3G^2 s^2}{c^4 r^5} = 0 . \tag{12}$$

From here we can find that s is given by

$$s^2 = \frac{c^4 M}{3G} r^3 - \frac{\Lambda c^6}{9G^2} r^6 , \tag{13}$$

$$s^2 = -\frac{c^4 M}{3G} r^3 + \frac{c^6 \Lambda}{9G^2} r^6 . \tag{14}$$

Substituting for $r = r_{\mathrm{P}}\left(\frac{\hbar G}{c}\right)^{1/2}$ and $M = m_{\mathrm{P}}\left(\frac{\hbar c}{G}\right)^{1/2}$ as before, we obtain for antiparallel spin

$$s^2 = \frac{c^4}{3G}\left(\frac{\hbar c}{G}\right)^{1/2}\left(\frac{\hbar G}{c^3}\right)^{3/2} - \frac{c^6}{9G^2}\left(\frac{\hbar^3 G^3}{c^9}\frac{G^2}{\hbar^4}\frac{\hbar^3 c^3}{G^3}\right), \tag{15}$$

and for parallel spin respectively we have

$$s^2 = -\frac{c^4}{3G}\left(\frac{\hbar c}{G}\right)^{1/2}\left(\frac{\hbar G}{c^3}\right)^{3/2} + \frac{c^6}{9G^2}\left(\frac{\hbar^3 G^3}{c^9}\frac{G^2}{\hbar^4}\frac{\hbar^3 c^3}{G^3}\right). \tag{16}$$

For a mass $m = m_{\mathrm{P}}$ and a radius $r = r_{\mathrm{P}}$, the zero-surface gravity would correspond to an expression given by (6) and (7). Therefore, for the De Sitter metric without the black hole we obtain from the torsion antiparallel to gravitation:

$$\vec{k} = \frac{\Lambda}{3}r - \frac{3G^2 s^2}{c^6 r^5} = 0\,, \tag{17}$$

whereas for the parallel one we obtain

$$\vec{k} = -\frac{\Lambda}{3}r + \frac{3G^2 s^2}{c^6 r^5} = 0\,. \tag{18}$$

Therefore the corresponding spin expressions for torsion effects in the same and also opposite to gravity directions are

$$s^2 = \pm\frac{\Lambda c^6}{9G^2}r^6\,, \tag{19}$$

where the negative sign is for antiparallel spin and the positive for parallel, respectively.

5. MAXIMUM CURVATURE RELATIONS

Next, let us write Zeldovich's (1968) definition of the cosmological constant. This arises in the theory of quantum vacuum, and so we have

$$\Lambda_{\mathrm{Z}} = \frac{G^2 m^6}{\hbar^4}\,. \tag{20}$$

If we substitute (20) into (17) and (18), along with the fact that $m = m_{\mathrm{P}}$ and $r = r_{\mathrm{P}}$, we obtain:

$$s^2 = \frac{c^2}{9G^2}\left(\frac{G^2\hbar^3 c^3 \hbar^3 G^3}{\hbar^4 G^3 c^9}\right), \tag{21}$$

which finally simplifies to the following equation

$$s^2 = \pm\frac{1}{9}\hbar^2 . \tag{22}$$

Similarly working out the details in (13) and (14), we have the following values for s and for parallel and antiparallel spin, respectively:

$$s^2 = -\frac{2}{9}\hbar^2 \quad \text{and} \quad s^2 = \frac{4}{9}\hbar^2 \tag{23}$$

and

$$g_{44} = 1 - \frac{\Lambda}{3}r^2 - \frac{2GM}{rc^2} \pm \frac{3G^2\sigma^2}{2c^6}r^2 . \tag{24}$$

Next, calculating an expression for the surface gravity from (24), we have

$$\vec{k} = \frac{1}{2}\left(\frac{\partial\, g_{44}}{\partial\, r}\right) = -\frac{\Lambda}{3}r + \frac{GM}{r^2c^2} \mp \frac{3G^2\sigma^2}{2c^2}r . \tag{25}$$

Solving for σ, we further obtain

$$\sigma^2 = \pm\frac{2c^6}{9G^2}\Lambda \mp \frac{2c^4}{3Gr^3}M . \tag{26}$$

Substituting for $\Lambda = \Lambda_{\mathrm{Z}}$ and $m = m_{\mathrm{P}}$ as before, we obtain the following expression

$$\sigma^2 = \pm\frac{2c^6}{9G^2}\left(\frac{c^3}{G\hbar}\right) \mp \frac{2c^6}{3G^2}\left(\frac{c^3}{G\hbar}\right). \tag{27}$$

If quantum effects are present in the geometry of the spacetime, this would also imply a maximum curvature that is given by the relation $\Lambda_{\max} = \dfrac{c^3}{G\hbar} = 10^{66}\,\mathrm{cm}^{-2}$ (Sivaran and De Sabbata 1991). Taking into account the combination of the signs for opposite and same-to-gravity effects, we obtain all separate spin density cases expressed in terms of $\Lambda_{\max}$:

$$\sigma^2 = \pm\frac{8c^6}{9G^2}\Lambda_{\max} , \tag{28}$$

and

$$\sigma^2 = \pm\frac{4c^2}{9G^2}\Lambda_{\max} . \tag{29}$$

Looking at the above expressions, we can see how the spin density of a quantum black hole, is related to the maximum curvature of the spacetime, or in an another way that the maximum curvature of the spacetime, where quantum effects are important, can be expressed in terms of the spin density of the quantum black holes that are present. This would imply

$$\Lambda_{max} \pm\frac{9G^2}{8c^6}\sigma^2 , \tag{30}$$

or

$$s^2 = -\frac{4}{9}\hbar^2 \text{ or } s^2 = \frac{2}{9}\hbar^2 . \tag{31}$$

From the equations above we can see that s is quantized in units of $\hbar$ corresponding to torsion effects that are parallel and antiparallel to those of gravitation.

6. CONCLUSION

Using Zeldovich's definition of the cosmological constant in a De Sitter universe where torsion effects are present in the spacetime metric, we have obtained expressions for the spacetime torsion. In the case where a black hole is present in the metric, we got similar torsion expressions, and we have showed that in both cases the torsion is quantized in units of $\hbar$.

In the case of a De Sitter universe without a black hole, torsion effects are quantized in a smaller fraction of $\hbar$. The addition of black-hole torsion effects increases the quantization to a higher fraction of $\hbar$.

For the De Sitter metric without a black hole, spin effects are quantized according to the relation $s^2 = \pm\hbar^2/9$. In the case where a black hole is added in the metric, the values change to $s^2 = \pm 2\hbar^2/9$, $s^2 = \pm 4\hbar^2/9$. Furthermore, for the Planck-mass-type black holes, a connection was established, which relates the spin density effects to the maximum curvature of the universe at very early times, i.e., $\sigma \propto \sqrt{\Lambda_{\max}}$.

REFERENCES

Birrel, N. D., Davies, P. C. W.: 1982, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, England.

De Sabbata, V., Zhang, Z. (eds): 1992, *Black Hole Physics*, NATO ASI Series C, Kluwer Academic Publishers, Dordrecht, The Netherlands.

De Sabbata, V, Wang, D., Sivaram, C.: 1990, *Ann. Phys.*, **47**, 508.

Gibbons, G. W., Hawking, S. W., Siklos, S. T. C.: 1985, *The Very Early Universe*, Cambridge University Press, Cambridge, England.

Johri, V. B.: 1996, *The Early Universe*, Hadronic Press, Istituto per La Ricerca di Base, Molise. Italy.

Sivaran, C., De Sabbata, V.: 1991, *Astrophys. Space Sci.*, **176**, 145.

Zeldovich, Ya. B.: 1968, *Soviet Phys. Uspekhi*, **11**, 381.

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