On the Precession of Mercury's Orbit

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Revised version v3, 11 August 2011, sent to Earth, Moon and Planets

Abstract.

The Sun's orbital motion around the Solar System barycentre contributes a small quadrupole moment to the gravitational energy of Mercury. The effect of this moment has until now gone unnoticed, but it actually generates some precession of Mercury's orbit. Therefore the residual 43arcsec/cy, currently allocated to general relativity, has to account for this new component as well as a *reduced* relativity component.

Keywords: precession, Mercury, planets.

1. Introduction

The orbit of planet Mercury has been calculated by several investigators; see Clemence (1947), Brouwer & Clemence (1961), an analysis in Roseveare (1982), and review in Pireaux & Rozelot (2003). In their calculations, the inverse square law has been applied to set up the differential equations of motion using the measured distances and velocities between Mercury, the Sun and planets. Then the observed precession of the perihelion of Mercury was explained as being due to general precession in longitude, the planets, solar oblateness, and relativity.

In this paper, an additional contribution to precession has been identified due to the actual motion of the Sun around the barycentre, which produces a very small quadrupole moment in the energy of Mercury. It is simply analogous to solar oblateness, as if the moving solar mass is extended equatorially on average. This causes additional precession of Mercury's orbit which has never before been recognised.

Consequently, previous investigators have calculated the orbit of Mercury and derived the residual 43arcsec/cy precession, which now has to account for general relativity *plus* this new component. So precession due to relativity is less than 43arcsec/cy.

2. Derivation of precession due to the moving Sun

The binding energy of Mercury, in the field of the Sun orbiting around the barycentre, may be calculated by using Newton's law. First, consider the *theoretical system* shown in Figure 1. Let Mercury (mass M_1) be regarded as stationary at distance (r_{1C} = 57.91x10^{6}km) from the origin C, while the Sun (mass M) travels rapidly around C at radius (r_{SC} = 7.43x10^{5}km). Then, for the Sun at distance r_1 from Mercury we can write:

$$r_1^2 = r_{1C}^2 + r_{SC}^2 - 2r_{1C}r_{SC}\cos\theta \qquad . \qquad (1a)$$

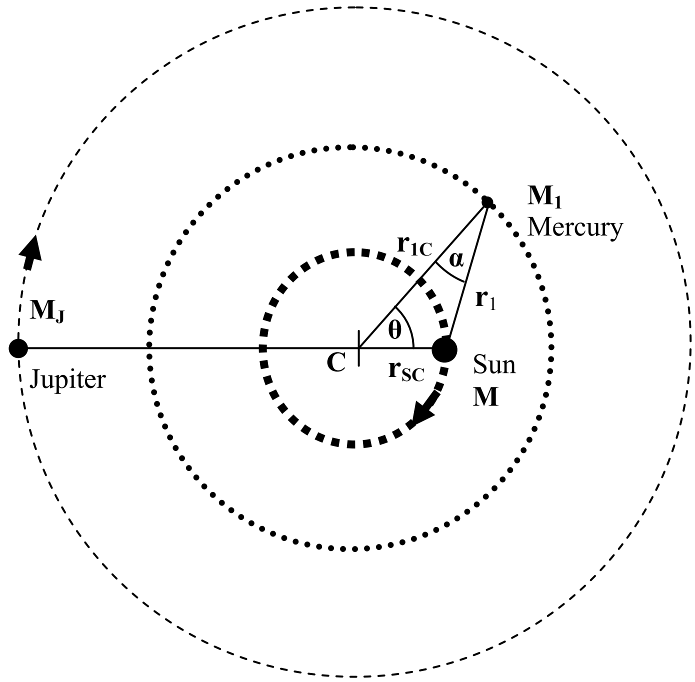

Figure 1. Schematic diagram showing Jupiter and the Sun moving around their centre of mass C. Theoretically, Mercury is considered to be stationary during one orbit of the Sun.

The *instantaneous* gravitational force exerted by the Sun on Mercury is given by the inverse square law, ($F_1 = -GMM_1 / r_1^2$), and the force directed towards C is ($F = F_1\cos\alpha$), where α is the angle between the Sun and centre C given by:

$$r_{SC}^2 = r_{1C}^2 + r_1^2 - 2r_{1C}r_1\cos\alpha \ . \qquad (1b)$$

Upon eliminating $\cos\alpha$, the force is:

$$F = F_1\cos\alpha = -\left(\frac{GMM_1}{r_1^2}\right)\left(\frac{r_{1C} - r_{SC}\cos\theta}{r_1}\right) . \qquad (2a)$$

Now eliminate variable r_1 and get all the $\cos\theta$ terms in the numerator:

$$F \approx -\left(\frac{GMM_1}{r_{1C}^2}\right)\left(1 - \frac{3}{2}\frac{r_{SC}^2}{r_{1C}^2} + \frac{2r_{SC}}{r_{1C}}\cos\theta + \frac{9}{2}\frac{r_{SC}^2}{r_{1C}^2}\cos^2\theta\right) \qquad (2b)$$

After averaging θ over a complete orbit of the Sun, the average force towards C becomes:

$$\widetilde{F} \approx -\left(\frac{GMM_1}{r_{1C}{}^2}\right)\left[1+\frac{3}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right] . \qquad (2c)$$

This force is slightly stronger than an inverse square law for a stationary Sun at C. By integrating this force from r_{1C} to infinity ($r_{1C} < r < \infty$), the potential energy of Mercury in this theoretical system would be:

$$PE_{av} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1+\frac{1}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right] . \qquad (3)$$

If Mercury is now allowed to orbit this rapidly moving Sun, its angular momentum and kinetic energy would be slightly greater than that around a Sun stationary at C; but we are far more interested in the way that this small quadrupole moment causes precession.

In reality, the Sun orbits the barycentre at a radius of around $7.43\text{x}10^5$km over 11.86 years due to Jupiter which, being longer than the period of Mercury's orbit, produces a smaller quadrupole moment. Therefore, Eq.(2c) is correct for a rapidly moving Sun, but a slow Sun allows Mercury time to track the Sun's wobble, thereby replacing r_{SC} in these equations by the smaller compensated value ($r^{/}_{SC} << r_{SC}$). The conserved angular momentum of Mercury resists perturbation by the Sun's movement, but this cannot eliminate completely the quadrupole moment because Mercury is chasing an elusive accelerating Sun. A modified Eq.(2c) gives the average acceleration of Mercury towards C:

$$\widetilde{a} \approx -\frac{GM}{r_{1C}{}^2}\left[1+\frac{3}{4}\left(\frac{r^{/}_{SC}}{r_{1C}}\right)^2\right] . \qquad (4)$$

After substituting ($u = 1/r_{1C}$), plus Mercury's specific angular momentum [$h = (GMr_{1C})^{1/2}$], then orbit theory yields a differential equation for the trajectory:

$$\frac{d^2u}{d\phi^2}+u=\frac{-\widetilde{a}}{h^2u^2}=\frac{GM}{h^2}+\left(\frac{GM}{h^2}\frac{3}{4}(r^{/}_{SC})^2\right)u^2 . \qquad (5)$$

General relativity theory gives a similar expression for the trajectory of Mercury, (see Rindler, 2001):

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \left(\frac{3GM}{c^2}\right)u^2 \quad , \tag{6}$$

wherein the final term is responsible for the well-known 43arcsec/cy precession of Mercury's orbit. Hence, by comparison, we can calculate the precession to expect from the quadrupole moment in Eq.(5):

$$\delta\omega = \left(\frac{c}{2h} r'_{SC}\right)^2 \times 43\text{arc}\sec/\text{cy} \quad . \tag{7}$$

The effective value of r'_{SC} is found by considering Figure 2 wherein Mercury is allowed to orbit around and track the slow moving Sun with period τ_1. Initially, the orbit of Mercury is focussed on a moving point P, distance r_x from C towards the Sun, where:

$$r_x = r_{SC}\left(\frac{\tau_{SC}}{\tau_{SC} + \tau_1}\right) \quad , \tag{8a}$$

and τ_{SC} is the Sun's period around C. Then the distance from P to the Sun is:

$$x = (r_{SC} - r_x) = r_{SC}\left(\frac{\tau_1}{\tau_{SC} + \tau_1}\right) \quad . \tag{8b}$$

This distance has not included the effect of compensation, due to Mercury conserving angular momentum while chasing the Sun. Total compensation (x = 0) could only exist for rigid coupling between Mercury and the Sun, and would be the same as for a stationary Sun. Therefore an average compensation value will be taken, which reduces *x* by half. From the viewpoint of an observer on Mercury, after 49 orbits the Sun has moved in a circle of radius r'_{SC}, given by:

$$r'_{SC} = \frac{x}{2} = \left(\frac{1}{2}\right)r_{SC}\left(\frac{\tau_1}{\tau_{SC} + \tau_1}\right) \quad . \tag{9}$$

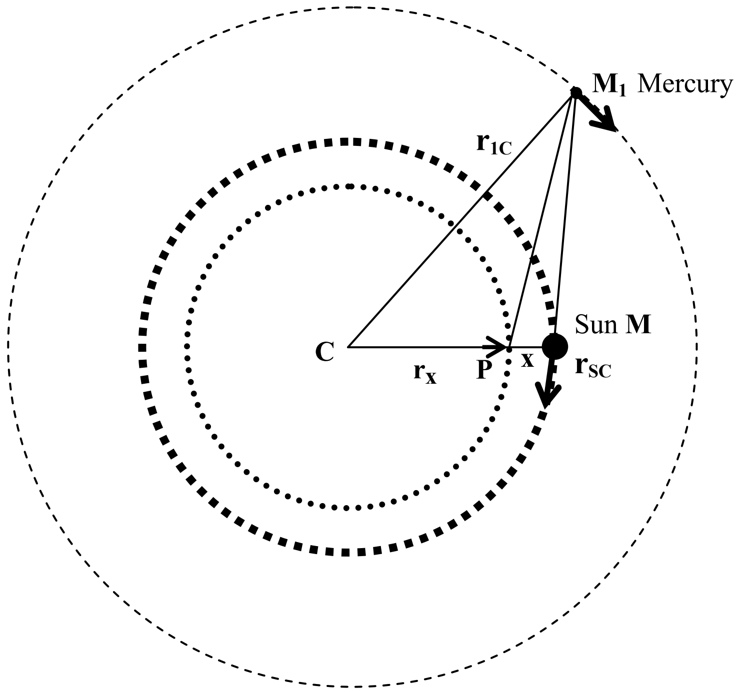

Figure 2. Schematic diagram showing the Sun moving around the centre of mass C. Mercury is considered to be orbiting focal point P between C and the Sun.

For (τ_1 = 88days) and (τ_{SC} = 4333days), this evaluates to (r'_{SC} = 7544km), which can be substituted in Eq.(7) to yield the actual precession due to the quadrupole moment:

$$\delta\omega = 0.166 \times 43 = 7.15 \text{arc sec/cy} \quad . \qquad (10)$$

Precession due other planets increasing the Sun's wobble is variable because together they cause great fluctuation in r_{SC} , with a long-term average at around 8×10^5km (Landscheidt, 2007).

Precessions currently attributed to general relativity in the orbits of Venus, Earth and Icarus, will also be affected by the Sun's quadrupole moment, (Shapiro et al (1968), Lieske & Null (1969), Sitarski (1992)).

3. Conclusion

Motion of the Sun, around the Solar System barycentre, adds a small quadrupole moment to the gravitational binding energy of Mercury. This term has been overlooked previously, but it is responsible for significant precession in the orbit of Mercury. Therefore, the well-known 43arcsec/cy residual of the observed precession is due to this new term *plus* general relativity. Fortunately, Einstein's theory can accommodate this change, see Wayte (1983).

Acknowledgements

I would like to thank Imperial College Libraries, R. Glovnea for computing assistance, and A. Rutledge for typing. I also thank the reviewers for suggesting some beneficial improvements.

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