International Journal of Modern Physics D

ANGULAR SIZE TEST ON THE EXPANSION OF THE UNIVERSE

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Received Day Month Year
Revised Day Month Year
Communicated by Managing Editor

Assuming the standard cosmological model as correct, the average linear size of galaxies with the same luminosity is six times smaller at $z = 3.2$ than at $z = 0$, and their average angular size for a given luminosity is approximately proportional to z^{-1}. Neither the hypothesis that galaxies which formed earlier have much higher densities nor their luminosity evolution, mergers ratio, or massive outflows due to a quasar feedback mechanism are enough to justify such a strong size evolution. Also, at high redshift, the intrinsic ultraviolet surface brightness would be prohibitively high with this evolution, and the velocity dispersion much higher than observed. We explore here another possibility to overcome this problem by considering different cosmological scenarios that might make the observed angular sizes compatible with a weaker evolution.

One of the models explored, a very simple phenomenological extrapolation of the linear Hubble law in a Euclidean static universe, fits the angular size vs. redshift dependence quite well, which is also approximately proportional to z^{-1} with this cosmological model. There are no free parameters derived ad hoc, although the error bars allow a slight size/luminosity evolution. The type Ia supernovae Hubble diagram can also be explained in terms of this model with no ad hoc fitted parameter.

WARNING: I do not argue here that the true Universe is static. My intention is just to discuss which theoretical models provide a better fit to the data of observational cosmology.

Keywords: Cosmology: miscellaneous; galaxies: statistics.

1. Introduction

The analysis of the dependence of the angular size of some sources with redshift was for many decades one of the most important geometric tests of cosmological models. Different cosmologies predict different dependences for a given linear size and this can be compared with the data from observations. The test, first conceived by Hoyle[1], is simple in principle but its application is not so simple because of the difficulty in finding a standard rod, a type of object with no evolution in linear size over the lifetime of the Universe.

It is well known that the application of the angular size (θ) vs. redshift (z) test gives a rough dependence of $\theta \propto z^{-1}$ for QSOs and radio galaxies at radio wavelengths[2,3,4,5,6,7], for first ranked cluster galaxies in the optical[8,9,10,11], and for the separation of brightest galaxies in clusters [12] or in QSO-galaxy pairs of the same redshift[13]. The deficit of large objects at high redshifts with respect to the predictions of an expanding Universe is believed to be an evolutionary effect by which galaxies were smaller in the past (e.g., [2]), or a selection effect (e.g., [14]). Thus, the $\theta \propto z^{-1}$ relationship, as predicted by a static Euclidean Universe, would be just a fortuitous coincidence of the superposition of the $\theta(z)$ dependence in the expanding Universe and evolutionary/selection effects.

Some other studies have tried to find better standard rods. Ultra-compact radio sources[15,16,17,18,19] were used to carry out an angular size test: a dependence different from $\theta \propto z^{-1}$ and closer to the predictions of expanding Universe models was found. The test was even used to ascertain not only whether or not the Universe is expanding but also to constrain the different cosmological parameters. However, these applications are not free from selection effects[18] and, as will be discussed in §5.2, interpretation of the results of these tests is not so straightforward.

Another proposal[20] used the rotation speed of high redshift galaxies as a standard size indicator since there is a correlation between size and rotational velocity of galactic disks. This method was indeed applied[21] over a sample of emission-line galaxies with $0.2 < z < 1$, with the result that Einstein–de Sitter cosmology is excluded to within 2-σ, and that small galaxies (with fixed rotation velocity lower than 100 km/s) should show a strong evolution in size (at $z = 1$ a factor two smaller than at $z = 0$) and no evolution in luminosity within the concordance cosmology, while large galaxies do not evolve significantly, either in size or in luminosity. Unfortunately, this method requires spectroscopy, so the sample has an upper limit of $z \sim 1$ even with very large telescopes, and the uncertainties are so large that they cannot be interpreted directly without certain assumptions concerning evolution models. In principle, looking at figures 4 and 5 of Marinoni *et al.*[21], one sees no reason to exclude a $\theta \propto z^{-1}$ dependence.

The aim of this paper is to repeat the angular size test for galaxies within a wide range of redshifts ($z = 0.2 - 3.2$) in optical–near infrared surveys (equivalent to the optical at rest). Data from high spatial resolution surveys available nowadays, such as those carried out with Hubble Telescope or FIRES, provide useful input for this old test of the angular size with new analyses and interpretations. Recent analysis of these data[22,23,24] has shown that the linear size of the galaxies with the same luminosity should be much lower than locally. However, this is true only if we considered the standard cosmological model as correct. In the present paper, I will do a reanalysis of these data and consider different cosmological scenarios, which will shed further light on the degeneracy between expansion + evolution and non-expansion.

2. Data

Angular effective radii, defined as circularized Sérsic half-light radii within which 50% of the light is present, are taken from [22,23] for galaxies with $0.2 < z < 1$ and from Trujillo *et al.*[24] for galaxies with $z > 1$. Both samples separate approximately early-type and late-type galaxies by means of the exponent (n_S) of their Sérsic profile: $n_S > 2.5$ for early-type galaxies and $n_S < 2.5$ for late type galaxies.

[22,23] provide data and angular size measurement of the GEMS survey with two Hubble Space Telescope colors (F606W and F850LP). In total, they have 929 galaxies. The data processing and photometry are discussed by Rix *et al.*[25].

Trujillo *et al.*[24] use near-infrared FIRES data of $z > 1$ galaxies in the HDF-S and MS 1054-03 fields to derive the angular size in the rest-frame V filter. In total, there are 248 galaxies with $1 < z < 3.2$. There are 14 more galaxies with $z > 3.2$, but they are very few, very luminous, and their photometric redshift determination was not very accurate; indeed, the available on-line data through the FIRES Website, http://www.strw.leidenuniv.nl/ fires, gives new recalculated redshifts and some of them are very different. The data processing and photometry are discussed in detail by Labbé *et al.*[26] for HDF-S and Förster-Schreiber *et al.*[27] for the MS 1054-03 field.

From these galaxies, I take only those with $3.4 \times 10^{10} < L_{V,rest}(L_{\odot,V}) < 2.5 \times 10^{11}$, a total of 393 galaxies (271 galaxies from GEMS with $z < 1$, and 122 galaxies from HDF-S/MS 1054-03 fields with $z > 1$). The lower limit is the same as that adopted by Trujillo *et al.*[24], avoiding the faintest ones in order to have a more homogeneous sample. The maximum limit is to avoid the galaxies away from the range of local galaxies for which the relationship between radius and luminosity was explored (explained in §3). This is an almost complete sample for redshifts up to ≈ 2.5 [24], and is incomplete for $2.5 < z < 3.2$. As we will see later, it is unimportant whether the sample is complete or not since the test is independent of the luminosity of the galaxies, but within this restriction I will concentrate on the analysis of the brightest galaxies. A higher limit at the lowest luminosity (the sample would be complete up to $z = 3.2$ for $L_{V,rest} > 6.7 \times 10^{10}$ $L_\odot$) would reduce the number of galaxies too severely and the statistics would be poorer. In any case, I will also comment on the results for these higher luminosity lower limits (see §3.2).

3. Angular size test

I am going to analyze the variation of the angular size, $\theta_{V,rest}$, of the galaxy rest-frame V with the redshift. The angular size in the $z < 1$ sample was measured at $\lambda_0 = 9450$ Å (filter F850LP), θ_{λ_0}, which, due to the color gradients, is slightly different from the angular size at V-rest. This difference is small[22,23] but I apply the following correction to it:

$$\theta_{V,rest} = \left\{ \begin{array}{l} \theta_{\lambda_0}\left[1 - 0.11\left(\frac{\lambda_V(1+z)}{\lambda_0} - 1\right)\right], \ \ n_S > 2.5 \\ \theta_{\lambda_0}\left[1 - 0.08\left(\frac{\lambda_V(1+z)}{\lambda_0} - 1\right)\right], \ \ n_S < 2.5 \end{array} \right\}, \tag{1}$$

with $\lambda_0 = 9450$ Å, and $\lambda_V = 5500$ Å. In any case, this correction is very small (less than 4%) and the results would not change significantly if it were not applied. In HDF-S/MS 1054-03 data, the size was already measured in the filter which gives V at rest[24], so no correction is necessary.

Since we have a wide range of luminosities and types of galaxies, there is a huge dispersion of sizes for a given redshift, with Malmquist bias, but this dispersion and bias can be reduced by defining θ_* as the equivalent angular size if the galaxy were early-type with a given V-rest luminosity (I take 10^{10} $L_{\odot,V}$; however, the variation of this number does not affect any result, just the calibration in size):

$$\theta_* \equiv \theta \frac{R(10^{10}\ \mathrm{L}_{\odot,\mathrm{V}}, n_S > 2.5)}{R(L_{V,rest}, n_S)}, \tag{2}$$

where $R(L_{V,rest}, n_S)$ is the average radius of a galaxy for a given luminosity and exponent (n_S) of the Sérsic profile. The number n_S is affected by an important uncertainty for galaxies with very small angular size ($\theta < 0.125$") [24], which produces some extra dispersion. In any case, the dispersion of R values is moderate[28], and, given that $\theta_*(z)$ does not contain the dispersion of luminosities, it will present a much lower dispersion than $\theta(z)$. Certainly, $\theta_*(z)$ contains the spread of θ and R so the dispersion due to random errors in these quantities is larger for θ_* than for θ; but the dispersion due to the spread of luminosities dominates, so, as said, θ_* will present a dispersion much lower than $\theta(z)$. This definition also avoids selection effects due to Malmquist bias, since θ_* for a given redshift should be nearly independent of the luminosity of the galaxy, at least on average.

Shen *et al.*[28](Eqs. 14-15) give the median radius of a galaxy of given r'_{SDSS}-luminosity (K-correction applied) and n_S for local SDSS galaxies. In total, the radius (measured for $z \approx 0.1$ in the r-band, which is more or less equivalent to V-band at rest) as a function of the absolute magnitude in r'_{SDSS}-rest $M_{r'-SDSS}$ is:

$$R(M_{r'-SDSS}, n_S) \tag{3}$$

$$= \left\{ \begin{array}{ll} 10^{-0.260 M_{r'-SDSS} - 5.06}, & n_S > 2.5 \\ 10^{-0.104 M_{r'-SDSS} - 1.71}[1 + 10^{-0.4(M_{r'-SDSS}+20.91)}]^{0.25}, & n_S < 2.5 \end{array} \right\} \mathrm{kpc}$$

We are not considering the evolution in this Eq. (3); all discussion of the effects of evolution will be considered in §4. To translate this relationship into a V-rest luminosity, we must make a color correction, as in McIntosh *et al.*[22] [however Trujillo *et al.*[24] interpolate between the rest-frame g-band and r-band]. Taking into account that $\langle(V - r'_{SDSS,AB})\rangle$=0.33 for early-type galaxies and = 0.30 for late-type galaxies[29] (this already includes the transformation of Vega to AB system; no evolution is considered here), and that $M_{V,\odot} = +4.79$ (Vega system) we will have (L_V in units of 10^{10} $L_{\odot,V}$)

$$R(L_V, n_S) = \begin{Bmatrix} A_0 L_V^{0.65}, & n_S > 2.5 \\ B_0 L_V^{0.26}(1 + B_1 L_V)^{0.25}, & n_S < 2.5 \end{Bmatrix} \text{ kpc}, \quad (4)$$

with $A_0 = 1.91$, $B_0 = 2.63$, $B_1 = 0.692$. These parameters are valid assuming the concordance cosmological model ($H_0 = 70$ km/s/Mpc, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$). For other cosmologies, we have to calibrate the relationship between luminosities and radii with the corresponding luminosity and angular distances for $z_{SDSS} \approx 0.1$. See Table 1 for the values of A_0, B_0 and B_1 with other cosmologies. This relationship of Shen *et al.*[28] is fitted with galaxies of $-19 > M_r > -24$ for early types and $-16 > M_r > -24$ for late types, which is equivalent to $2.5 \times 10^9 < L_V < 2.5 \times 10^{11}$ $L_{\odot,V}$ for early types and $1.6 \times 10^8 < L_V < 2.5 \times 10^{11}$ $L_{\odot,V}$ for late types. Our galaxies are within these limits.

In order to derive the luminosity, $L_{V,rest}$, for a given redshift (z) and the rest flux for the filter V, $F_{V,rest}$, we need the distance luminosity $d_L(z)$ from different cosmologies, such that (without considering neither evolution nor extinction)

$$d_L \equiv \sqrt{\frac{L_{V,rest}}{4\pi F_{V,rest}}}. \quad (5)$$

And the average equivalent angular size evolution with z should be compared with the prediction of the same cosmology, which, in principle, without taking into account the evolution, should be given by:

$$d_A(z) \equiv \frac{R_*}{\theta_{*,pred}(z)}, \quad (6)$$

where $d_A(z)$ is the angular distance, and R_* is the equivalent physical radius of the galaxy associated with the equivalent angular size θ_*; that is, if the galaxy were early-type of V-rest luminosity 10^{10} $L_{\odot,V}$.

3.1. *Different cosmological scenarios*

(1) Concordance model with Hubble constant $H_0 = 70$ km/s/Mpc, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$:

$$d_A(z) = \frac{c}{H_0(1+z)} \int_0^z \frac{dx}{\sqrt{\Omega_m(1+x)^3 + \Omega_\Lambda}}, \quad (7)$$

$$d_L(z) = (1+z)^2 d_A(z). \quad (8)$$

(2) Einstein–de Sitter model [Eq. (7) with $\Omega_\Lambda = 0$, $\Omega_m = 1$]:

$$d_A(z) = \frac{2c}{H_0(1+z)} \left[1 - \frac{1}{\sqrt{1+z}}\right], \quad (9)$$

$$d_L(z) = (1+z)^2 d_A(z). \tag{10}$$

Although this is not the standard model nowadays, there are some researchers who still consider it more appropriate than the concordance model (e.g., [30,31,32]). About the compatibility with type Ia supernovae data, see discussion in §5.3.

Since the distances for objects at $z = 0.1$ are 0.943 times the distances of the concordance model, the corrected relationship between luminosity and radius is Eq. (4) with $A_0 = 1.90$, $B_0 = 2.61$, $B_1 = 0.699$.

(3) Friedmann model of negative curvature with $\Omega = 0.3$, $\Omega_\Lambda = 0$, which implies a term of curvature $\Omega_K = 0.7$ [[33], Eq. (26)].

$$d_A(z) = \frac{c}{H_0(1+z)\sqrt{\Omega_K}} \sinh\left(\sqrt{\Omega_K}\int_0^z \frac{dx}{\sqrt{\Omega_m(1+x)^3+\Omega_K(1+x)^2}}\right), \tag{11}$$

$$d_L(z) = (1+z)^2 d_A(z). \tag{12}$$

I will use this model of an open universe to check that a model different to a flat universe does not change the results significantly. Since the distances for objects at $z = 0.1$ are 0.970 times the distances of the concordance model, the relationship between luminosity and radius with corrected calibration is Eq. (4) with $A_0 = 1.93$, $B_0 = 2.59$, $B_1 = 0.736$.

(4) Quasi-Steady State Cosmology (QSSC), $\Omega_m = 1.27$, $\Omega_\Lambda = -0.09$, $\Omega_c = -0.18$ (C-field density) [34]

$$d_A(z) = \frac{c}{H_0(1+z)}\int_0^z \frac{dx}{\sqrt{\Omega_c(1+x)^4+\Omega_m(1+x)^3+\Omega_\Lambda}}, \tag{13}$$

$$d_L(z) = (1+z)^2 d_A(z). \tag{14}$$

This cosmology is not the standard model, but it can also fit many data on angular size tests[34] or Hubble diagrams for SNe Ia[35,36,37]. The expansion with an oscillatory term gives a dependence of the luminosity and angular distance similar to the standard model, adding the effect of matter creation (C-field) with slight changes depending on the parameters. The parameters of this cosmology are not as well constrained as those in the standard model. Here, I use the best fit for a flat ($K = 0$) cosmology given by Banerjee *et al.*[34]: $\Omega_m = 1.27$, $\Omega_\Lambda = -0.09$, $\Omega_c = -0.18$, which corresponds to $\eta = 0.887$ (amplitude of the oscillation relative to 1), $x_0 = 0.797$ (ratio between actual size of the Universe and the average size in the present oscillation) and maximum allowed redshift of a galaxy $z_{max} = 6.05$ (note however that the maximum observed redshift has risen above 8 nowadays according to some authors, [38]). Other preferred sets of parameters give results that are close. The values most used are $K = 0$, $\Omega_\Lambda = -0.36$, $\eta = 0.811$, $z_{max} = 5$ [35,36,37,39], which imply $\Omega_m = 1.63$, $\Omega_c = -0.27$, but I avoid

them because they do not allow galaxies to be fitted with $z > 5$. Parameters with a curvature different from zero ($K \neq 0$) also give results that are very close in the angular size test[34].
Since the distances for objects at $z = 0.1$ are 0.943 times the distances of the concordance model, the relationship between luminosity and radius with corrected calibration is Eq. (4) with $A_0 = 1.94$, $B_0 = 2.56$, $B_1 = 0.778$.

(5) Static euclidean model with linear Hubble law for all redshifts:

$$d_A(z) = \frac{c}{H_0} z, \tag{15}$$

$$d_L(z) = \sqrt{1+z}\, d_A(z). \tag{16}$$

These simple relations indicate that redshift is always proportional to angular distance, a Hubble law. We assume in this scenario that the Universe is static; the factor $\sqrt{1+z}$ in the luminosity distance stems from the loss of energy due to redshift without expansion. There is no time dilation, and precisely because of that the factor is not $(1+z)$. The caveat is to explain the mechanism different from the expansion/Doppler effect, which gives rise to the redshift. This cosmological model is not a solution which has been explored theoretically/mathematically. However, from a phenomenological point of view, we can consider this relationship between distance and redshift as an ad hoc extrapolation from the observed dependence on the low redshift Universe. Our goal here is to see how well it fits the data, and forget for the moment the theoretical derivation of this law.
Since the luminosity distance for objects at $z = 0.1$ is 0.966 times the luminosity distance of the concordance model and the angular distance is 1.115 times the angular distance of the concordance model, we must adopt approximately Eq. (4) with $A_0 = 2.23$, $B_0 = 2.98$, $B_1 = 0.741$.

(6) Tired-light/simple static euclidean model :

$$d_A(z) = \frac{c}{H_0} \ln(1+z), \tag{17}$$

$$d_L(z) = \sqrt{1+z}\, d_A(z). \tag{18}$$

This is again a possible ad hoc phenomenological representation which stems from considering that the photons lose energy along their paths due to some interaction, and the relative loss of energy is proportional to the length of that path (e.g. [12]), i.e.

$$\frac{dE}{dr} = -\frac{H_0}{c} E. \tag{19}$$

Of course, as in the previous case, this ansatz is very far from being considered as the correct one by most cosmologists, but it is interesting to analyze its compatibility with the angular size test too.

For the calibration of Eq. (4), I use the fact that the luminosity distance for objects at $z = 0.1$ is 0.921 times the luminosity distance of the concordance model, and the angular distance is 1.063 times the angular distance of the concordance model, so $A_0 = 2.26$, $B_0 = 2.92$, $B_1 = 0.816$.

(7) Tired-light/"Plasma redshift" static euclidean model:

$$d_A(z) = \frac{c}{H_0}\ln(1+z), \tag{20}$$

$$d_L(z) = (1+z)^{3/2} d_A(z). \tag{21}$$

The plasma redshift application[40](§5.8) used $d_L(z) = (1+z)^{3/2} d_A(z)$ instead of $d_L(z) = (1+z)^{1/2} d_A(z)$ to take into account an extra Compton scattering which is double that of the plasma redshift absorption. For the calibration of Eq. (4): $A_0 = 2.00$, $B_0 = 2.78$, $B_1 = 0.674$.

The volume element in a static uiverse is different from the volume element in the standard concordance model. Particularly for the standard concordance model the comoving volume element in a solid angle $d\omega$ and redshift interval dz is (for null curvature, which is the case of the concordance model)

$$dV_{\rm concordance} = \left(\frac{c}{H_0}\right)^3 \frac{\left[\int_0^z \frac{dx}{\sqrt{\Omega_m(1+x)^3+\Omega_\Lambda}}\right]^2}{\sqrt{\Omega_m(1+z)^3+\Omega_\Lambda}} d\omega dz \tag{22}$$

while for the two first static models it is

$$dV_{\rm static-lin.Hub.} = \left(\frac{c}{H_0}\right)^3 z^2 d\omega dz, \tag{23}$$

$$dV_{\rm static-tir.l.} = \left(\frac{c}{H_0}\right)^3 \frac{[\ln(1+z)]^2}{(1+z)} d\omega dz. \tag{24}$$

Hence, if we wanted to evaluate the evolution of some quantity per unit comoving volume for the static universes, we must multiply the result in the concordance model by the factor $\frac{dV_{\rm concordance}}{dV_{\rm static}}$.

3.2. *Results*

The results of the test are plotted in Fig. 1 for the concordance model, with the equivalent angular size of each individual galaxy, and Fig. 2 for the different models, with the representation of the average value of $\log_{10}\theta_*$ in bins of $\Delta\log_{10}(z) = 0.10$. I do a weighted linear fit in the log–log plot. Since I am calculating an average of the logarithm for the angular sizes, the possible error in individual galaxies should not significantly affect the value of the average. The error bars in each bin of the plot are the statistical errors. Note that there are values of θ_* lower than $0.03''$, but they do indeed correspond to measured values of $\theta \ge 0.03''$; θ_* is lower than θ at the highest redshifts because the luminosity in V-filter of those galaxies is higher than 10^{10} $L_{\odot,V}$.

As can be observed, the average equivalent angular size gives a good fit to a $\theta_*(z) = Kz^{-\alpha}$ law, with values of K, α given in Table 2. No fit is totally in agreement with the cosmological prediction without evolution and extinction. The static model with a linear Hubble law is not very far from being compatible with the data: I get $\theta_*(z) = 0.136z^{-0.97}$, near the expected dependence ($\theta_*(z) = 0.109z^{-1}$) although with a slightly larger size.

NOTE*: the normalization of the angular size in any model prediction (solid line in Fig. 2) stems from the Shen et al.*[28] *calibration with the corresponding parameters A_0, B_0 and B_1 in each cosmology. It does not stem from a fit.*

If we separate elliptical ($n_S > 2.5$) and disk galaxies ($n_S < 2.5$) for the concordance model, the plots in Fig. 3 are obtained, with a higher slope for elliptical galaxies ($\alpha = 1.18 \pm 0.04$) than for disk galaxies ($\alpha = 0.80 \pm 0.07$). This might be due to a different evolution of disk and elliptical galaxies, but they are likely to be due to different systematic errors for elliptical galaxies and disk galaxies due to systematic errors in the value of n_S. The galaxies with $2.0 < n_S < 2.5$ are suspected of being strongly contaminated by ellipticals since the obtained n_S tends to be lower than the real one ([24], Fig. 1). If we take disk galaxies only with $n_S < 2.0$, then $\alpha = 0.92 \pm 0.06$, closer to unity. On the other hand, elliptical galaxies with $\theta < 0.125$" are also strongly contaminated by disk galaxies ([24], Fig. 2) which are compacted by a wrong measure of n_S and consequently give a smaller radius than the real one. If we take elliptical ($n_S > 2.5$) galaxies only with $\theta > 0.125$": $\alpha = 1.03 \pm 0.05$. Therefore, from the present analysis and within the systematic errors, we cannot be sure that the angular size test gives different results for elliptical and disk galaxies. However, when all the elliptical and disk galaxies are put together, the excesses and deficits more or less compensate; there are approximately as many elliptical galaxies misclassified as disk galaxies as disk galaxies misclassified as elliptical galaxies. See further discussion on the systematic errors in §3.3.

In Fig. 4, I analyze the dependence of α on the luminosity of the galaxies, and we see that there is no significant dependence. $\alpha = 1$ is more or less compatible with all the subsamples of different luminosity.

3.3. *Selection effects, and errors in the angular size measurement and luminosity-radius relationship*

As said previously, the definition of θ_* avoids selection effects due to Malmquist bias, since θ_* for a given redshift should be nearly independent on how luminous the galaxy is, at least on average.

Nonetheless, although $R_* = \theta_* d_A$ is independent of the luminosity and type of the galaxy at $z = 0$, its possible evolution might depend on both parameters and this would give different average θ_* when the selection of galaxies is different. In any case, whatever the sample of galaxies is, if we are going to interpret the factor

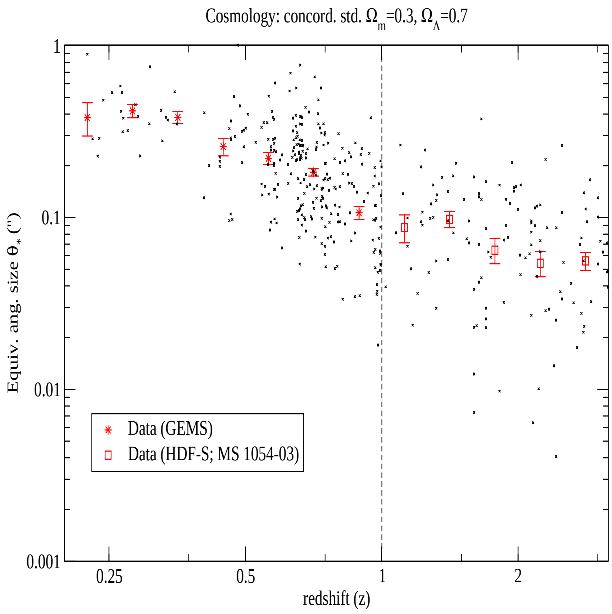

Fig. 1. Log–log plot of the equivalent angular size (θ_*) vs. redshift (z) in the concordance cosmology. Left: ($z < 1$) GEMS data. Right: ($z > 1$) HDF-S and MS 1054-03 data. The average of $\log_{10}\theta_*$ in bins of $\Delta\log_{10}(z) = 0.10$ is represented with asterisks and squares with statistical error bars.

Values of the parameters A_0, B_0 and B_1 in the relationship between radius and luminosity of a galaxy, Eq. (4), calibrated with different cosmological models.

Cosmology	A_0	B_0	B_1
Concord. $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$	1.91	2.63	0.692
Einstein-de Sitter	1.94	2.56	0.778
Friedmann $\Omega_m = 0.3$	1.93	2.59	0.736
QSSC $\Omega_m = 1.27$, $\Omega_\Lambda = -0.09$, $\Omega_c = 0.18$	1.94	2.56	0.778
Static linear Hubble law	2.23	2.98	0.741
Static, tired light/simple	2.26	2.92	0.816
Static, tired light/plasma	2.00	2.78	0.674
St. lin. Hub. law, ext. $a_V = 1.6 \times 10^{-4}$ Mpc^{-1}	2.21	3.03	0.696
St. tired light, ext. $a_V = 3.4 \times 10^{-4}$ Mpc^{-1}	2.22	3.01	0.719

between the cosmological prediction and the observed average θ_* as a product of size evolution, this factor would reflect the average evolution of R_* in that sample. In our case, we are observing the factors for the average population with $3.4 \times 10^{10} < L_{V,rest}(L_\odot) < 2.5 \times 10^{11}$. For the concordance model the value of R_* at $z = 3.2$ is on

Weighted fit of the average equivalent angular size for the data in Fig. 2 to a law $\theta_*(z) = Kz^{-\alpha}$. Error bars give only statistical errors and do not include systematic errors as explained in §3.3.

Cosmology	K	α
Concord. $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$	0.1251 ± 0.0041	0.957 ± 0.045
Einstein-de Sitter	0.1721 ± 0.0054	0.834 ± 0.044
Friedmann $\Omega_m = 0.3$	0.1412 ± 0.0046	0.956 ± 0.045
QSSC $\Omega_m = 1.27$, $\Omega_\Lambda = -0.09$, $\Omega_c = 0.18$	0.1810 ± 0.0057	0.817 ± 0.044
Static linear Hubble law	0.1363 ± 0.0044	0.969 ± 0.045
Static, tired light/simple	0.2132 ± 0.0066	0.717 ± 0.043
Static, tired light/plasma	0.0915 ± 0.0031	1.177 ± 0.047
Concord., early type ($n_S > 2.5$)	0.1059 ± 0.0044	1.181 ± 0.043
Concord., late type ($n_S \le 2.5$)	0.1441 ± 0.0070	0.805 ± 0.067
Concord., early type ($n_S > 2.5$), $\theta > 0.125$”	0.1226 ± 0.0060	1.031 ± 0.051
Concord., late type ($n_S \le 2.0$)	0.1600 ± 0.0073	0.915 ± 0.057
Concord., $L_V > 6.8 \times 10^{10}$ $L_{\odot,V}$	0.1315 ± 0.0083	0.995 ± 0.098
Concord., $L_V > 1.02 \times 10^{11}$ $L_{\odot,V}$	0.1106 ± 0.0117	1.313 ± 0.186
St. lin. Hub. law, ext. $a_V = 1.6 \times 10^{-4}$ Mpc^{-1}	0.1055 ± 0.0035	1.113 ± 0.046
St. tired light, ext. $a_V = 3.4 \times 10^{-4}$ Mpc^{-1}	0.1565 ± 0.0050	0.801 ± 0.044

average ≈ 6 lower than R_* at $z = 0$. At low redshift we know that the relationship of Eq. (4) is correct (hence, the average size R_* should not depend on the luminosity of these galaxies). Therefore, the ratio of sizes between high/low redshift objects will depend only on the variation of Eq. (4) in galaxies selected at high redshift. We can say that the average size of the selected galaxies at $z = 3.2$ is ≈ 6 times lower than the galaxies at low redshift with the equivalent luminosity and Sérsic profile. In other words, if there is a (very strong) evolution in the size of the galaxies for a fixed luminosity (or a variation in luminosity for a fixed radius), the factor by which the galaxies are smaller will depend on which galaxies are selected, but in any case this factor will represent an average galaxy shrinking factor. An average factor of 6 in size reduction will mean that there are galaxies with size reduction by a factor larger than 6, and other galaxies with size reduction by a factor smaller than 6.

Trujillo *et al.*[24](§4.3) discussed the robustness of the “average” angular size measurement, whether it is affected by some other biases or selection effects. Their tests showed that the results, as presented here, are more or less robust and the difference in the results if we apply some minor corrections due to uncertainties or incompleteness in the radius distribution of the sources are $\Delta(\langle \log_{10} R \rangle) \sim 0.1$, thus $\Delta\theta_*/\theta_* \sim 20\%$.

There is a minimum angular size that Hubble telescope or FIRES can observe below which the uncertainties of θ and n_S are very high, with important systematic

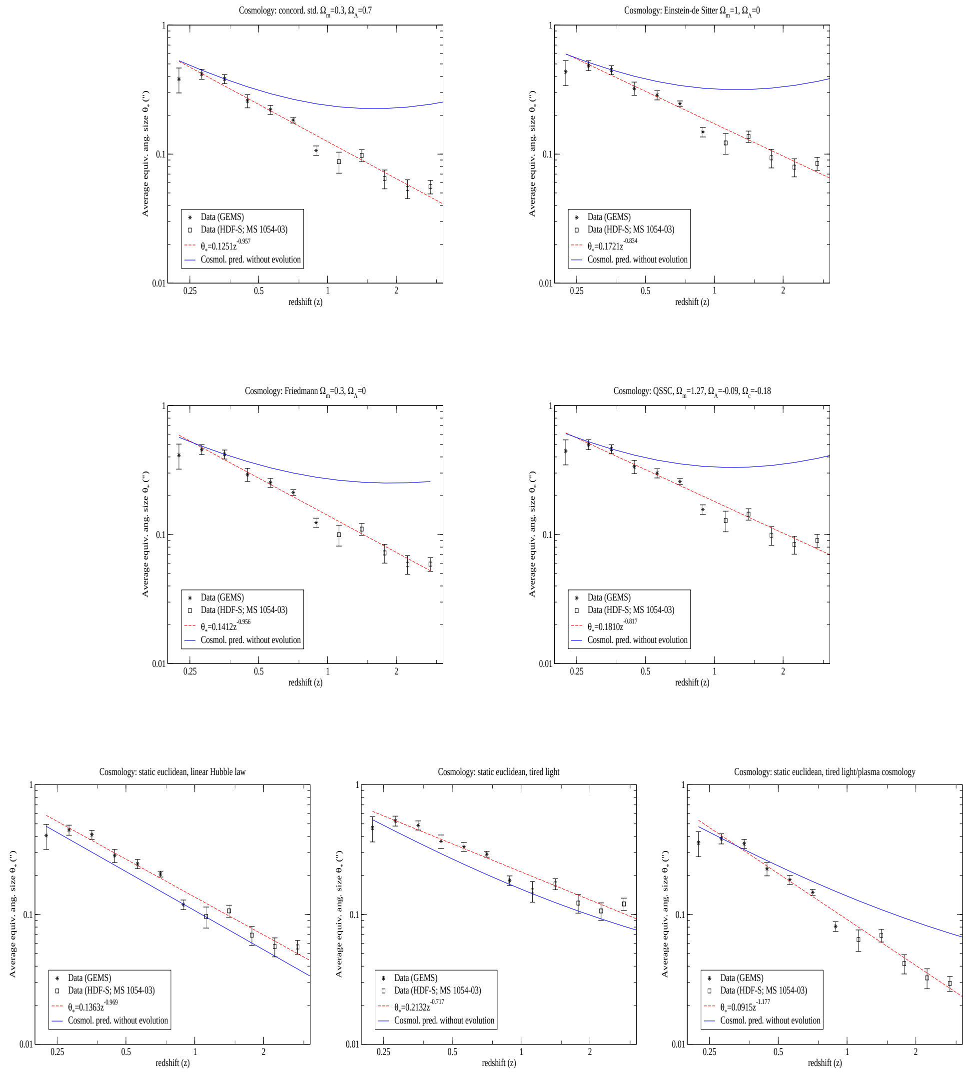

Fig. 2.

Log–log plot of the average of $\log_{10}\theta_*$, where θ_* is the equivalent angular size, vs. redshift (z). Bins of $\Delta\log_{10}(z) = 0.10$. Error bars only represent statistical errors; for the systematic errors, see text in §3.3. The seven plots are for the seven different cosmologies described in §3.1. Solid lines are the model predictions (the normalization stems from the Shen *et al.*[28] calibration with the corresponding parameters A_0, B_0 and B_1 in each cosmology). Dashed lines are the best weighted linear fits.

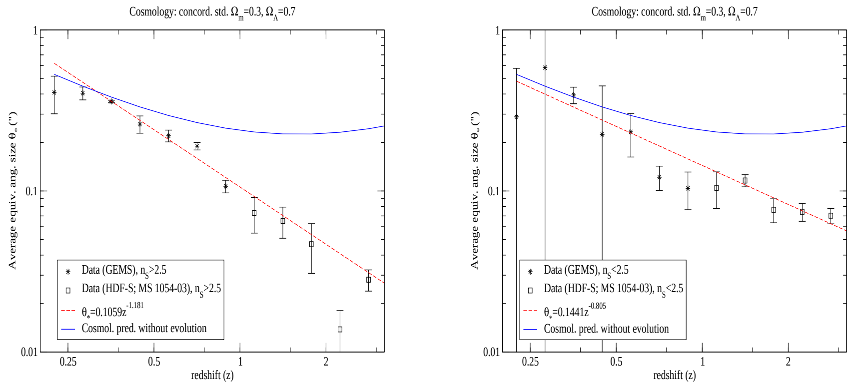

Fig. 3. Log–log plot of the average of $\log_{10}\theta_*$, where θ_* is the equivalent angular size, vs. redshift (z). Bins of $\Delta\log_{10}(z) = 0.10$. The two plots are for the concordance cosmology separating elliptical galaxies ($n_S > 2.5$) from disk galaxies ($n_S \leq 2.5$).

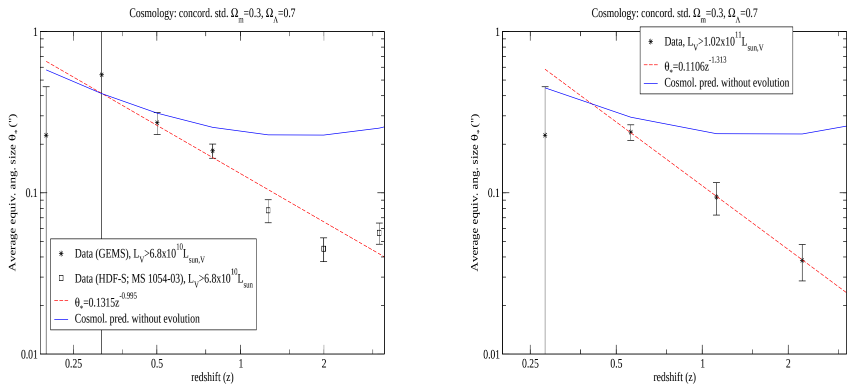

Fig. 4. Log–log plot of the average of $\log_{10}\theta_*$, where θ_* is the equivalent angular size, vs. redshift (z). Bins of $\Delta\log_{10}(z) = 0.20$ and $\Delta\log_{10}(z) = 0.30$ respectively. The two plots are for the concordance cosmology only for galaxies with $L_V > 6.8 \times 10^{10}$ $L_{\odot,V}$ (122 galaxies) and $L_V > 1.02 \times 10^{11}$ $L_{\odot,V}$ (48 galaxies) respectively.

deviations. Normally, the angular sizes are overestimated for these low angular sizes, so the observed average θ_* increases with z as more and more of the smaller angular size galaxies are included in the average. Moreover, the very compact galaxies are not included in the sample because they might be classified as stars. This affects mainly the $z > 1$ data, where the limit for low systematic errors is around $\theta = 0.125$" [24]. Suppression of these galaxies would change the average of θ_* at $z > 1$ by 10–30%. Systematic errors for $\theta < 0.125''$ are up to 50% or lower ([24], Fig. 2), which produces a systematic error on the average θ_* lower than $\approx 15\%$.

Another effect to investigate is the error in the relationship of Eq. (4). This represents the average radius for a given luminosity and there is some dispersion of values with respect to it (given by Shen *et al.*[28]). I am not interested here in these statistical errors of R because this would just affect a dispersion of values of θ_* without changing its average value. Our concern is about the systematic errors in Eq. (4). I will analyze five sources of systematic errors:

(1) The color correction of $\langle(V-r'_{SDSS,AB})\rangle$=0.33 for early-type galaxies and $=0.30$ for late-type galaxies [29] might change if there is a preferential galactic type within the group of early- or late-type galaxies is given for some redshift. Indeed, Fukugita *et al.*[29] give a value of $\langle(V-r'_{SDSS,AB})\rangle$ of 0.36 for E type, 0.31 for S0, 0.32 for S_{ab}, 0.29 for S_{bc}, 0.28 for S_{cd}. That is, there are variations up to ≈ 0.03 with respect to the average in early types, and 0.02 for late types. In the worst cases, these systematic errors in colors would produce a systematic error of $\approx 2\%$ in θ_*, which is negligible.
(2) A misclassification of an elliptical galaxy as disk galaxy or vice versa produces an error up to a factor 2 in the value of R derived from Eq. (4), which leads to an error of factor < 2 in θ_*. The effect of the uncertainty in n_S for small angular sizes has already been checked by Trujillo *et al.*[24](§4.3) and the error of the average of θ_* is lower than $\sim 20\%$ for the sample with all of the galaxies. However, when I divide the galaxies into elliptical and disk types (see 3) the effect might be larger, as was noted previously concerning Fig. 3.
(3) Systematic errors in the luminosity at V-rest due to errors in the photometric redshift amount less than 4% [41], which implies errors in θ_* less than 2.5%. However, the systematic error of the luminosity measurement in faint elliptical galaxies amounts around 15% [24], which translates into 6% of error in θ_*.
(4) The calibration of eq. (4) (calibration of the parameters A_0, B_0 and B_1) is done for cosmologies different to concordance model assuming that the SDSS galaxies have redshift $z_{SDSS} = 0.1$. There is indeed a dispersion of redshift values in SDSS galaxies around this average. If we set $z_{SDSS} = 0.15$ we would obtain, for instance for the static linear Hubble law ansatz, $A_0 = 2.32$, $B_0 = 3.15$, $B_1 = 0.721$; the variations in the average measured θ_* are negligible ($<< 1\%$), but the predictions of the cosmological model for θ_* (proportional to A_0) increase by $\approx 6\%$. That is, the ratio between measured and predicted θ_* decreases by 6%. An error of 0.05 in the average redshift is among the worst cases, so we can say that the systematic error in the calibration should be less than $\approx 5\%$.
(5) Another systematic error comes from the V_{max} correction of the selection effects in the SDSS data made by Shen *et al.*[28](§3.1). Shen *et al.* [28] make a correction of selection effects by assigning a weight to each galaxy inversely proportional to the maximum comoving volume within which galaxies identical to one under consideration can be observed[42]. This correction mainly affects faint elliptical galaxies, as can be observed in fig. 2 of Shen *et al.*[28] and amounts to $\frac{\Delta R}{R} <\sim$

$0.7 \times exp(-0.7 L_V)$ in the average radius of the galaxies for elliptical galaxies of luminosity L_V (in units of 10^{10} $L_{\odot,V}$).
The weights depend on the cosmological model, since the comoving volumes at a given redshift change in the different cosmological models. Moreover, there is an intrinsic systematic error in using the standard model: Shen *et al.*[28] used the integration of physical volumes instead of comoving volumes [see Eq. (8) of Shen*et al.*[28]], which leads to systematic errors of V^*_{max} up to $\sim 50\%$ [there is a factor $(1+z)^3$ between comoving and physical volumes]. In order to calculate the effect of a change of cosmology, apart from the recalibration of A_0, B_0 and B_1, I would need to have all their SDSS data at hand and repeat the full analysis of their fit with the new conditional maximum volume V^*_{max} values for each cosmology [see Eq. (9) of Shen *et al.*[28]] using the correct comoving volume V_{max} instead of the integration of physical volumes, which is not possible for us. Assuming a total systematic error in the volume of $\Delta V^*_{max} \sim 0.5 V^*_{max}$,

$$\Delta(\langle \log_{10} R \rangle) <\sim 0.15 \times exp(-0.7 \langle L_V \rangle). \tag{25}$$

Since $L_V > 3.4$ in our selected sample, this may justify systematic errors in $R(L)$ up to 3% .

Summing up, apart from the statistical errors plotted in Fig. 2, there are systematic errors in the average θ_* that may amount up to 30–40% for the general case with all the galaxies

3.4. *Relationship with the surface brightness test*

The average surface brightness of a galaxy with total flux $F_{\lambda,rest}$ and half-light circularized angular size θ is:

$$SB = \frac{F_{\lambda,rest}/2}{\pi\theta^2}. \tag{26}$$

Using eqs. (5), (6); and $d_L = (1+z)^{i/2} d_A$, with $i = 1$ if static, and $i = 4$ if expanding, it is

$$SB = \frac{L_{\lambda,rest}}{8\pi^2 R^2 (1+z)^i}. \tag{27}$$

The intrinsic surface brightness [without the $(1+z)^i$ dimming factor] is

$$SB_0 = \frac{L_{\lambda,rest}}{8\pi^2 R^2}. \tag{28}$$

Given the relationship between radius and luminosities of eq. (4), we find that the average intrinsic surface brightness in V-rest (assuming no size evolution) should follow:

$$SB_{0,V-rest}(L_V,n_S)=\left\{\begin{array}{ll}\frac{L_V^{-0.30}}{8\pi^2A_0^2}, & n_S>2.5\\ \frac{L_V^{0.48}}{8\pi^2B_0^2(1+B_1L_V)^{0.50}}, & n_S<2.5\end{array}\right\}. \quad (29)$$

The surface brightness for a given luminosity should be independent of the redshift if there is no evolution in size. I can also define an equivalent surface brightness to avoid the dependence on the luminosity:

$$SB_*\equiv\frac{F_{V,rest}}{2\pi\theta^2}\frac{SB_0(10^{10}\ \mathrm{L}_{\odot,\mathrm{V}},>2.5)}{SB_0(L_{V,rest},n_S)}=\frac{10^{10}\ \mathrm{L}_{\odot,\mathrm{V}}}{8\pi^2R_*^2(1+z)^i}. \quad (30)$$

This is indeed the surface brightness test, or Tolman test[43]. In Fig. 2 for the static models, it is observed that the data of $\theta_*(z)$ without size evolution are fitted more or less (within the statistical+systematic error). Thus, R_* is nearly constant in a static model for all redshifts and we obtain $\langle SB_*\rangle\propto(1+z)^{-1}$, while for the expanding model we would need a strong evolution in $R_*(z)$ and the average surface brightness would be $\langle SB_*\rangle\propto R(z)^{-2}(1+z)^{-4}$. In order to give the same $\langle SB_*\rangle(z)$ as in the static Universe, assuming that $d_L(z)$ is approximately the same (which is nearly true for the concordance and the linear Hubble law luminosity distances; see §5.3), $R(z)\approx R(z=0)(1+z)^{-3/2}$.

Lubin & Sandage[44] obtained $\langle SB\rangle\propto(1+z)^i$ with $i=1.6-3.2$ for a sample with $z<0.9$ depending on the filter (R or I), and the initial hypothesis (expanding or static). Lerner[45] obtained $i=1.03\pm0.15$ for $z<\sim 5$, with data in wavelengths from ultraviolet to visible from Hubble Space Telescope (see also Lerner[46]), while Nabokov & Baryshev[33] obtained $i=3-4$ with the same type of data but without including K-corrections. Andrews[32] obtained $i=0.99\pm0.38$, $i=1.15\pm0.34$, with two different samples of cD galaxies. Lubin & Sandage argue that their data are compatible with an expanding Universe if we take evolution into account. Lerner criticizes Lubin & Sandage for not using the same range of wavelengths at rest for the high and low redshift galaxies but instead using K-corrections with many free parameters, which is less direct and more susceptible to errors. Moreover, Lerner argue against the evolution that the intrinsic ultraviolet surface brightness of high redshift galaxies would be extremely large, with impossible values (see §5.4.1).

4. Evolution of galaxies in expansion models

From the plot in Fig. 2(concord. model), we see that, in order to make the concordance model compatible with the data on angular size, we must assume an evolution such that the galaxies at high redshift are much smaller than at low redshift ($z<\sim 0.2$). For instance, at redshift $z=2.5$, the galaxies should be on average ≈ 4.6 times smaller. Trujillo *et al.*[24] obtained a lower average factor (~ 3) because they used only galaxies with the constraint $\theta>0.125''$; however in the error bars of Trujillo et al. result, this bias effect of avoiding $\theta\leq 0.125''$ galaxies is included, and within this error bar it is compatible with our result. The most massive galaxies are thought to be contracted by a factor of ≈ 4 up to $z = 1.5$ [47] and of 5.5 up to $z = 2.3$[48], but Mancini *et al.*[49] think that the extra compactness of these galaxies can be understood in terms of fluctuations due to noise preventing the recovery of the extended low surface brightness halos in the light profile, so those factors might be not real. For galaxies with redshift $z = 3.2$ in our Fig. 2(concord. model), the average ratio of sizes would increase to a factor 6.1 (comparing the linear fit with the theoretical prediction). Separating by types, the ratio would be 4.5–6 for disk galaxies and 6–10 for elliptical galaxies, taking into account the systematic uncertainties commented in §3.2. Other authors[50,51,52,53,54,56,33] find similar results at high redshift.

If we have galaxies that are on average 6 times smaller than the equivalent galaxies (of same type and luminosity) at low redshift, this means that the V-rest luminosity density (inversely proportional to the cube of the size) of these galaxies is ≈ 200 times higher than at low redshift. The surface brightness in V-rest is increased by a factor ~ 40. Is this situation possible?

4.1. *Effect of the expansion of the Universe*

In the standard picture of the expanding models, the expansion does not affect the galaxies because there is no local effect on particle dynamics from the global expansion of the universe: the tendency to separate is a kinematic initial condition, and once this is removed, all memory of the expansion is lost[57]. Note, however, that there are other views on the topic of whether there is expansion of galaxies due to the expansion of the Universe and the nature of the expansion itself [58,59]. Dark energy or cosmological vacuum might have some influence on the size of the galaxies [60,61], but this effect would be small. Lee[62] suggested instead that the size of the galaxies increases as the scale factor of the universe assuming dark matter models based on a Bose–Einstein condensate or scalar field of ultra-light scalar particle, which is another heterodox idea to explore. Nonetheless, apart from this kind of proposals, within standard scenarios, the galaxies do not expand with the Universe.

An indirect way in which the expansion has an effect is in the formation of galaxies. In the theoretical ΛCDM hierarchical scenarios, galaxies formed at higher redshift should be denser[63] since, to decouple from expansion, structures must have a given density ratio with the surrounding density, which is larger at higher redshift. Some authors (e.g., [24,50]) have used this argument to explain the apparent size evolution. However, the observed redshift in galaxies is not its formation redshift, so the application of this idea is not straightforward. As a matter of fact, the stellar populations of most local massive elliptical galaxies are very old[64], and formed before the age corresponding to $z \sim 3$, so we cannot say that galaxies now are larger because of this effect since most of them were formed > 10 Gyr ago. The difference of formation epoch of the galaxies observed now and at $z \sim 3$ is not large. Moreover, the hierarchical scenario in which massive galaxies form first do not represent the observed Universe appropriately, as I argue in §4.3.

Furthermore, the theoretical claim by Mo *et al.*[63] derived from the models that galaxies which formed earlier are denser is not in general observed. If it were true, we should observe that at low redshift the youngest galaxies (formed later) should be much larger for a given mass than the oldest galaxies of age 12–13 Gyr. There is already evidence that this is not the case: the densest galaxies are young instead of old[65]. And we can check with our own sample within $0.2 < z < 3.2$ that the color of elliptical galaxies is not correlated with size: Fig. 5. Redder elliptical galaxies are older and, for a given redshift, indicate earlier formation, which should be equivalent to smaller size, at least statistically. This correlation is not observed at all: linear fits in the four redshift ranges of Fig. 5 all give slopes compatible with zero within 1σ except for the range $-0.4 < \log_{10}(z) < -0.1$, which gives $\frac{d(log_{10}(R_*/A_0))}{d(B-V)_{\rm rest}} = +1.0 \pm 0.5$, a 2σ correlation but in a direction opposite to prediction that galaxies are larger when redder=older (formed earlier).

Therefore, it is not a question of comparing the formation of galaxies at different redshifts but the evolution of galaxies already formed, either isolated or in interaction/merging with other systems.

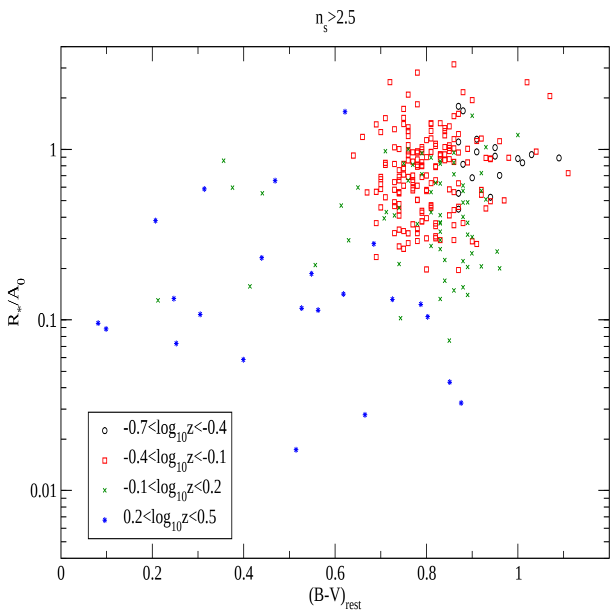

Fig. 5.

Equivalent linear size (normalized with Shen *et al.*'s[28] calibration) vs. $(B-V)$ color at rest for elliptical galaxies in the concordance cosmology.

4.2. *Younger population of high redshift galaxies*

The main argument in favour of the evolution in size for a fixed luminosity is that the younger a galaxy is the brighter it is, and we expect to see younger galaxies at high redshift. Therefore, galaxies with radius smaller than R_* in the past will produce the same luminosity as galaxies with that radius at present. How much brighter?

Using Vazdekis *et al.*'s[66] synthesis model, with revised Kroupa IMF, we can derive the mass–luminosity ratio in a passively evolving elliptical galaxy as a function of its intrinsic (B-V) color and its luminosity. The metallicity degeneracy is broken with an iterative method which uses the relationship between stellar mass and metallicity[67]. With this method, the average mass–luminosity ratio of elliptical galaxies (not affected by extinction) at the last bin of Fig. 2 ($z = 2.5 - 3.2$) is $\langle M_*/L_V\rangle = 0.5$ [$N = 6$, $\langle L_V\rangle = 10^{11}$ $L_\odot$, $\langle(B - V)\rangle = 0$]. Rudnick *et al.*'s[41](§4.4) analysis of the same galaxies also obtain a quite similar mass–luminosity ratio. For the elliptical galaxies with $\langle L_V\rangle > 5\times10^{10}$ $L_\odot$ of the two first bins ($z = 0.20-0.32$): $\langle M_*/L_V\rangle = 2.2$ [$N = 7$, $\langle L_V\rangle = 7 \times 10^{10}$ $L_\odot$, $\langle(B - V)\rangle = 0.94$], that is, a mass–luminosity ratio 4.4 times larger. Assuming a variation of the luminosity linearly dependent on the time, the variation of this ratio would be by a factor 5.8 between z=0.1 (the average redshift of SDSS galaxies, which are the reference of the size calibration in Shen *et al.*[28]) and $z = 3.2$. This is an acceptable estimation for elliptical galaxies. Kauffmann *et al.*[68](Fig. 13) showed that SDSS late-type galaxies have a mass–luminosity ratio around 2 times lower than early-type ones, and Rudnick *et al.*[41](Fig. 9) showed that blue galaxies at $z \approx 3$ also have mass–luminosity ratios around 0.2-0.3, so we also keep this number of ≈ 6 for disk galaxies. This factor of 6 might be affected by important errors (see §5.4), and could be much lower, though not much larger.

Given that we are comparing galaxies with the same luminosity, the galaxies at $z = 3.2$ would have 6 times lower stellar mass than at $z = 0.1$. Hence, from Eqs. (17)-(18) of Shen *et al.*[28], we find that galaxies at $z = 3.2$ should be 2.7 times smaller than at $z = 0.1$ if they are elliptical, or 2.0 if they are disk galaxies. These factors are much lower than the measured values of 6–10 and 4.5–6 respectively. A factor in size 2–4 for elliptical or 2–3 for disk galaxies, including this luminosity evolution correction remains (in rough agreement with the results by Trujillo *et al.*[24]). Therefore, the argument of "younger population in higher redshift" does not serve to justify the present data. In order to explain the observed size increase in terms only of luminosity evolution of the stellar population, we would need to set M_*/L_V with respect to the SDSS galaxies a factor 25–60 for elliptical galaxies, or 50–100 for disk galaxies, too much!

4.3. *Mergers*

If the luminosity density is not due to an increase of the luminosity of each star, it must be due to a redistribution of the mass density to make it more compact. Why?

Explanations in terms of mergers proliferate in the literature (see discussion by Refs. [24,47] and references therein). Refs. [24,47] support the merger scenario calculating the size vs. mass ratio and observing how it also decreases with redshift[a], but this calculation also depends on the cosmological model used and can give different results for different cosmologies. Furthermore, Khochfar *et al.*[69] point out that the presence of higher amounts of cold gas at high redshift mergers of ellipticals also produces size evolution.

Trujillo *et al.*[47] cites the paper of Boylan-Kolchin *et al.*[70] as a possible powerful mechanism to increase the radius in massive elliptical galaxies. Each merger would follow a law $R \propto M_*^{-\alpha}$ [70] with $\alpha = 1.0$ for a pericenter of 15 kpc and lower for higher pericenter distances or higher for lower pericenter distance. So when two galaxies of the same luminosity approach each other reaching a minimum distance of 15 kpc and merge, the total radius will be ≈ 2.00 times the radius of each individual galaxy, while Shen *et al.*[28] for local galaxies give $\alpha = 0.56$: a factor ≈ 1.47 in radius on average when we double the mass. This means that each major merger (fusion of galaxies of the same luminosity) gives an extra factor of 1.36 for ellipticals in angular size with respect to eq. (4) relationship. We would need an average of 2.3–3.6 ($= \frac{ln(2-3)}{ln(1.36)}$, i.e. $1.36^{2.3-3.6} = 2-3$) major mergers along the life of each elliptical galaxy to justify a factor 2–3 in radius.

The observed rate of mergers is indeed much lower than the necessary rate to justify these numbers. Lin *et al.*[71] with statistics of close galaxy pairs ($r < 20h^{-1}$kpc) up to z=1.2 show that only $\sim$ 9% of the luminous galaxies ($-21 < M_B < -19$) would have a major merger during their lives since $z = 1.2$. De Propis *et al.*[72] get similar merger ratios for local galaxies ($z < 0.12$) than Lin *et al.*[71] for $0.5 < z < 1$. The number of cumulative mergers increases up to 22% [73] if we allow larger pair separation ($r < 30h^{-1}$kpc) and up to 54% [73] if we allow a broader definition of major merger including pairs of galaxies with mass ratio up to four. Ryan *et al.*[74] calculated a number of 42% galaxies undergoing some merger up to $z = 1$ for more luminous galaxies ($M_B > -20.5$) [and 62% for all z], also for mass ratios up to four, and $r < 20h^{-1}$ kpc. In our case, comparison with major mergers of equal mass galaxies and with average pericenter around 15 kpc should be made, so a number of 10–20% (up to $z = 3.2$ is $\sim$ 50% larger than up to $z = 1$) of mergers would be the amount to compare with the 2–4 mergers (200–400%) we need for the size evolution. Or we may account for mergers with mass ratios up to 4 and pericenter radii larger than 15 kpc on average, although these would not produce an increase of an extra factor of 1.36 for ellipticals in angular size with respect to the Eq. (4) relationship but lower. Therefore, we would have to compare the number

[a]In Refs. [24,41], the masses were estimated from the colors of the galaxies assuming solar metallicity for the determination of the mass–light ratio and a Salpeter IMF model. Trujillo *et al.*[47] multiply the masses by a factor 0.5 to correct for the difference of Kroupa and Salpeter IMF, which is only a very rough approximation. The masses of Trujillo*et al.*[47] are more accurately calculated, with an uncertainty of a factor two. All these assumptions introduce significant errors into the calculation of the mass–light ratio. Therefore, we must take these results with care.

of $\sim 50\%$ with the large number of mergers of this kind to produce the observed evolution in radius (~ 10 mergers per galaxy; probability of merging $\sim 1000\%$).

Moreover, I suspect that the number of mergers is overestimated. First because not all galaxy pairs become mergers. Second, because of the contamination of interlopers. There is an uncertainty of radial distance due to proper motion of the galaxies apart from the Hubble flow, and many of the identified pairs of galaxies in the projected sky are not real pairs in 3D space. There may be a chance superposition in the line of sight of galaxies. The situation is worst for Ryan *et al.*[74], who use spectrophotometric redshifts; and interlopers do not introduce a statistical error, as they say, but a systematic one. However, the order of magnitude of the pairs with or without interlopers should be more or less the same, the interlopers being at $z < 3$ lower than $\sim 30\%$ [75]. Also, De Ravel *et al.*[76], for instance, using spectroscopically confirmed pairs, get similar merger ratios. Third, because timescales of mergers increase slightly with redshift and are longer than assumed in most observational studies[77].

About the CAS method of estimation of merger ratios based on the identification of major mergers with highly asymmetric galaxies[78,79] two things may be commented: 1) The authors consider that all asymmetries in principle associated with starbursts are due to major mergers, but the mechanisms which trigger important amounts of star formation might be different from major (ratio 1:1) mergers; in particular, minor mergers may also trigger asymmetric star formation. Strongly disturbed systems, indicative of recent strong interactions and mergers, account for only a small fraction of the total star formation rate density[80]. 2) Interlopers, galaxies and stars with very different redshifts projected as background or foreground objects in the line of sight produce an important amount of apparent distortion in the galaxies, especially at high redshift, where Conselice *et al.*[79] claim that a high fraction of galaxies are major mergers. We must bear in mind that Hubble images may detect very faint galaxies and there are more than 5 million galaxies per square degree with $m_z < 30$ ([81]; extrapolated from the counts up to magnitude 28), one galaxy in each square of $1.6'' \times 1.6''$ on average, which, mixed with the main galaxy ($m_z < 27$ in Conselice *et al.*[79]), produce apparently distorted galaxies. Also, De Propis *et al.*[72] checked that the contamination is happening in low redshift galaxies with foreground stars. Moreover, we see the galaxies once they are "presumably" merged but we do not see enough galaxy pairs at $1 < z < 2$ when both galaxies of the merger are separated (e.g., according to the already overestimated ratio by Ryan *et al.*[74]). This indicates at least that the asymmetries are not produced by major mergers but by minor mergers (ratio of masses larger than 4) or other effects.

Stellar population analyses do not even agree with these merger rates. Mergers or captures of smaller galaxies can occasionally occur, but hierarchical-scheme subunits fusing together and made of gas and stars is not the dominant one by which massive elliptical galaxies are made, at least for $z < 2$ [82]. Most massive elliptical galaxies have a passive evolution since their creation according to stellar population analyses[82]. Mergers are beautiful, spectacular events, but not the dominant mechanism by which elliptical galaxies are assembled. Most early-type galaxies with a velocity dispersion exceeding 200 km/s formed more than 90% of their current stellar mass at redshift $z > 2.5$ [64]. Elliptical galaxies formed in a process similar to monolithic collapse, even though their structural and dynamical properties are compatible with a small number of dry mergers[83], far from the number of mergers necessary in our case. Dry mergers do not decrease the galaxy stellar-mass surface density enough to explain the observed size evolution[84], and the high density of the high-z elliptical galaxies does not allow them to evolve into present-day elliptical galaxies[84].

Mergers are searched for in the Local Group galaxies too. In the case of the Milky Way, some authors try to find evidence of major fusion events of big galaxies, but up to now we do not see evidence in favour but against such scenarios[85]. There are minor mergers, of course, and absorption of small clouds of the intergalactic medium, but the presence of intermediate mass galaxies at short distances from the center, at present or in the past, have yet to be identified. There are certain attempts to find something, for instance the recent discovery of a galaxy with a relatively large diameter at only 13 kpc from the Galactic center called Canis Major, but that discovery resulted in a fiasco[86]. In any case, two to four major mergers on average per galaxy is too much.

Another argument against the merger scenario of the hierarchical CDM cosmology is that galaxy formation is controlled by a single parameter[87]. One would expect in the merger scenario that the properties of individual galaxies be determined by a number of factors related to the star formation history, merger history (masses, spins and gas content of the individual merging galaxies), etc., but that is not so; all the different parameters of the galaxies are correlated[87] and there is only one single independent parameter based on their mass.

On the other hand, major mergers of disk galaxies of comparable mass should give place to elliptical galaxies, so it is not easy to understand in this scenario how the radius of disk galaxies grows. Conselice *et al.*[88] show in fact that there is little to no evolution for disk galaxies at $z < 1.2$, for the K-band, in the stellar-mass Tully–Fisher relation, and in the ratio of stellar/total mass. Ferguson *et al.*[89] also find that accretion flows play only a minor role in determining the evolution of the disk scalelength. In models in which the main infall phase precedes the onset of star formation and viscous evolution, they find the exponential scalelength to be rather invariant with time. On the other hand, models in which star formation/viscous evolution and infall occur concurrently result in a smoothly increasing scalelength with time, reflecting the mean angular momentum of material which has fallen in at any given epoch.

Furthermore, selection effects go apparently in the opposite direction of observing pre-merger galaxies at high redshift. At very high redshift, we are observing only galaxies with stellar masses over 10^{11} $M_\odot$ and some of them over 10^{12} $M_\odot$ [24].

And at low redshifts there are galaxies of all masses but the average stellar mass is much lower than that. Thinking that very massive galaxies at high z are building blocks of even more massive low z galaxies is counterintuitive. After 2–4 mergers of equal mass galaxies in average, the galaxies should be 4–16 times more massive than the original building blocks at high redshift. We should be observing some galaxies at low redshift with stellar masses of $\sim 10^{13}$ M$_\odot$. With a mass-luminosity ratio of $M_*/L_V = 6$ (for a very old population [66]; if it were younger than 12 Gyr it would be lower so the luminosity would be even higher), this would mean galaxies with $L_V = 2 \times 10^{12}$ L$_{\odot,V}$, or absolute magnitude $M_V = -26$. Even cD galaxies in the centers of the clusters are not as bright as that. Where are these galaxies, then?

Thinking that low- to intermediate-mass galaxies are the final stage of major merger processes is a reasonable possibility, since we cannot see their building blocks at high redshift (they are very faint). However, for only high luminous galaxies I get more or less the same shrinking factors at high-z with respect to low-z (see Fig. 4). It is not a question of some merging which affects low luminosity galaxies more. I could even concentrate our analysis on galaxies with $L_V > 1.02 \times 10^{11}$ L$_{\odot,V}$ and, in spite of the poorer statistic, a very strong size evolution between galaxies at $z = 0.5 - 1$ and $z > 2$ can be appreciated. Refs. [47,55] even get higher evolution for higher stellar masses, but this has been criticized[49].

Another element that is not consistent with these hierarchical merging scenarios is that superdense massive galaxies should be common in the early universe ($z > 1.5$), and a non-negligible fraction (1-10%) of them should have survived since that epoch without any merging process retaining their compactness and presenting old stellar populations in the present universe. However, Trujillo *et al.*[65] find only a tiny fraction of galaxies ($\sim 0.03\%$) of these superdense massive galaxies in the local Universe ($z < 0.2$) and they are relatively young (~ 2 Gyr) and metal-rich ($[Z/H] \sim 0.2$). Clearly a case of how some authors (Trujillo *et al.*[65]) try to find proofs in favour of the hierarchical merging scenario, and when they find that the observations point out exactly the opposite thing of what is expected, instead of claiming that the hierarchical merging scenario is wrong, they try to deviate attention by giving less importance to the observed facts for the validation of the standard theory.

4.4. *Quasar feedback*

Fan *et al.*[90] realized that the evolution+merger luminosity solution is not enough to explain the strong size evolution and they claimed, "no convincing mechanism able to account for such size evolution has been proposed so far". Nonetheless, they have proposed a new mechanism to explain this extraordinary size evolution. Fan *et al.*[90] propose that in elliptical galaxies it is directly related to a quasar feedback: part of the energy released by the QSO would be spent to produce outflows of huge amounts of cold gas expelled from the central regions, a rapid (few tens of Myr) mass loss which induces an expansion of the stellar distribution.

Although this mechanism might explain some small part of the expansion of elliptical galaxies, there are some aspects in this hypothesis which do not fit with the observed facts well, at least while we do not clarify some points. First, we do not see such supermassive outflows which are necessary to maintain the Fan *et al.*[90] idea, although since their life is very short only a few high redshift QSOs would show it. Second, elliptical galaxies expend their gas to produce stellar formation which gives rise to their stellar mass. If a QSO swept the gas away, no young stellar populations would be observed, but there is now compelling evidence for a significant post-starburst population in many luminous AGN[91]. There is also detection of large amounts of warm, extended molecular gas indicating that QSOs have vigorous star formation[92], and that the gas is not being expelled. The blue color of host galaxies, $(B-V)_{rest} \sim 0$ [93], indicates a young population too. Third, if QSOs produced such massive outflows ejecting the gas of the galaxies, this would also apply to disk galaxies. Around 40% of host galaxies in QSOs are disk galaxies[94], and it is clear that disk galaxies still have gas and active star formation in their disks. Fourth, a continuous increase in the average size of ellipticals, as shown in Fig. 3/left, would require the continuous expulsion of gas, but most massive elliptical galaxies have had a passive evolution since their creation at $z > 2$ according to stellar population analyses[82], which indicate that the gas was already drained in them at $z > 2$. Fifth, for an average increase of a factor 3 in size in the elliptical galaxies, we would need to assume that most elliptical galaxies have hosted very luminous QSOs during their lives. With a minimum QSO lifetime of around 40 Myr, as required for the massive outflow mechanism of Fan *et al.*[90], the number of very luminous QSOs should be around 1/300 of the number of elliptical galaxies. This number is too large, given that the density of QSOs with $L_{bol} >\sim 10^{48}$ erg/s (the necessary luminosity, 5% of which is spent to produce outflows larger than 1000 $M_\odot/yr$, as posited by Fan *et al.*[90]; $L \sim \frac{M\dot{M}}{0.05R}$ with $M > 2\times 10^{10}$ $M_\odot$ and $R = \frac{1}{3}R_{SDSS}$) is $\sim 10^{-8}$ Mpc^{-3} [95].

Nonetheless, I would not dare to say that Fan *et al.*'s[90] hypothesis is incorrect. I think it is an elegant and interesting solution to the problem, and it must be considered as a serious possibility, provided it is able to solve their caveats, and give some support to the scenario with some observations directly interpretable as massive outflows.

4.5. *Rotation or dispersion velocity analysis*

If this hypothesis of lower radius at high redshift for a given mass were true, due either to mergers or quasar feedback, we would expect a significant increase in the rotation speed or dispersion velocity of those galaxies at high redshift with respect to the local ones. For a constant mass within a radius R, one would roughly expect rotation speed in a galaxy $v_{rot} \propto R^{-1/2}$, and something similar for the dispersion velocities in ellipticals. Let us analyze whether this change of velocity is taking place.

An analysis of Marinoni *et al.*'s [21] data with galaxies with $z < 1.2$ does not show (see Fig. 6) a significant change in the rotation velocity–size relationship or the rotation velocity–absolute magnitude relationship. According to the Saintonge *et al.*[96] analysis for low redshift galaxies, the rotation velocity is proportional to $R^{0.98\pm0.01}$, and is also related to the absolute magnitude in the I-band by their eq. (3), so we could derive an average expected velocity from the luminosity of the galaxy, $v(M_i)$. In Fig. 6, I see that these relationships remain more or less constant: the variations in $v_{rot}/R^{0.98}$ and $v_{rot}/v(M_i)$ are compatible within 1-σ to be null. Particularly, the best linear fits give:

$$\frac{v_{rot}}{R^{0.98}} = (26.4 \pm 5.9) + (8.9 \pm 8.8)z, \tag{31}$$

$$\frac{v_{rot}}{v(M_i)} = (1.04 \pm 0.20) - (0.09 \pm 0.16)z. \tag{32}$$

Within the error bars, I cannot exclude an increase in these ratios with redshift compatible with the hypothesis of radius decrease. An interesting test would be to measure the rotational velocity in some of the very compact galaxies with redshift 3. Although getting a spectrum of these faint galaxies is technically difficult, this would give a proof of whether either they are really so compact or the cosmological parameters are wrong.

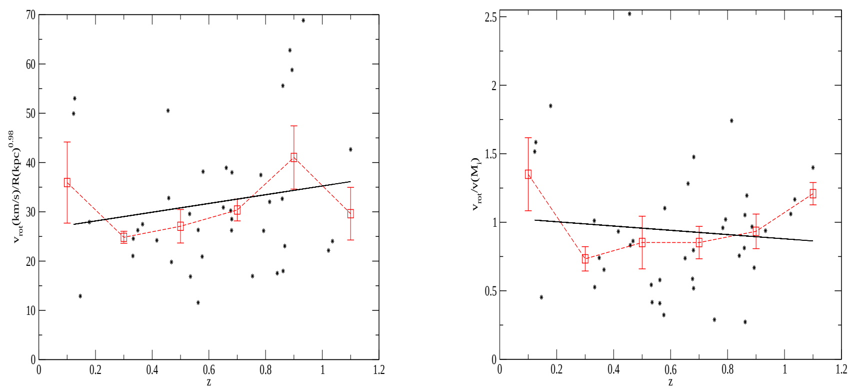

Fig. 6.

Plot of $v_{rot}/R^{0.98}$ and $v_{rot}/v(M_i)$ for the 39 galaxies of the sample from Marinoni *et al.*[21]. Squares with error bars are the average of the data (asterisks) with steps of $\delta z = 0.2$. The solid line is the best linear fit, given by eqs. (31) and (32).

The critical assumption of a variable effective radius is also counter-argued by the proofs in favour of a constant radius[97] showing that high redshift first-rank elliptical galaxies, with similar absolute magnitudes, have the same velocity dispersions as low redshift first-rank elliptical galaxies. Van Dokkum *et al.* [97] point out, however, that these galaxies are predicted by galaxy formation models to be those whose formation finished at very high redshift and so it would not be surprising that these galaxies had the same radius as at low redshift, specifically because they are in clusters where mergers may not be likely.

A surprising new result also points in this direction[98]: the velocity dispersion of giant elliptical galaxies with average redshift $z \approx 1.7$ from Cimatti *et al.*'s [54] sample is similar or very slightly larger than the dispersion for the same kind of galaxies with the same stellar mass in the local Universe at 240 km/s, while at $z = 0$ it is around 180 km/s. Since Cimatti *et al.*[54] galaxies at $z \approx 1.7$ are more compact than the average size with that luminosity at that redshift, it is normal to have a slightly higher velocity dispersion. It should be much higher (over 400 Km/s) at high redshift since a much lower radius is attributed to them [54], but it is not. This result points directly to the conclusion that the galaxies have not strongly changed their radii. Other alternative ad hoc ideas in terms of a conspiracy of effects in which the dark matter ratio has increased the amount necessary to compensate for the radius increase[98] sound like a queer coincidence, and have no clear basis in terms of galaxy formation scenarios.

4.6. *Discussion on expansion+evolution models*

All these considerations may make us think that the concordance model cannot explain the present data. The other expanding models present similar problems: a factor in average size evolution up to $z = 3.2$ equal to 5.8 for Einstein–de Sitter, 5.6 for Friedmann with $\Omega_m = 0.3$, and 5.9 for the QSSC model. Phenomenologically, it is possible to fit the data to any of the expanding models with appropriate evolution of galaxies. In practice, this evolution for a constant luminosity galaxy should be so strong (up to a factor 200 in average in the luminosity density up to $z \approx 3.2$; systematic errors may change up to a factor 2 this number, but this does not change the situation) that the explanations for it seem unrealistic.

The situation for the expanding models becomes even more dramatic if we go to higher redshift. At redshift 6, the linear size of the galaxies assuming a concordance model is even lower, approximately a factor two lower than at $z = 3.2$ ([51] Fig. 4). If we were going to consider a reduction in size by a factor 12, all the arguments given in this section would become even stronger.

5. Analysis of the static universe cases

In the first two static Universe cases, there is an excess of size ($\sim 20 - 30\%$) for most redshifts. A possible interpretation of this discrepancy is that there may be a systematic error in the calculation of the ratio between measured and predicted average θ_*. In §3.3, I have discussed the possible systematic errors and I concluded that they should be lower than $\sim 30 - 40\%$. These systematic errors might be enough to justify the departures of the data with respect to the prediction in Fig. 2 for these static models.

The third case of plasma redshift is much more discrepant and can only fit the data with a significant evolution of galaxies, although less strong than in the expanding models: a factor 3 instead of a factor 6 in the concordance model for a given luminosity from $z = 0$ to $z = 3.2$.

5.1. *Including extinction*

Apart from the systematic error considerations, another solution to make the two first static cosmological models compatible with our data would be related to extinction rather than the evolution. Extinction would make the galaxies look fainter, which means that, through Eqs. (2) and (4), when the corrections of extinctions are made, the luminosity is larger and θ_* is smaller than their values without corrections. Let us check this hypothesis with a rough calculation. Instead of Eq. (5), the inclusion of the IGM extinction with absorption coefficient κ (area per unit mass) will give the following relationship

$$L_{V,rest} = 4\pi F_{V,rest} d_L^2 e^{\rho_{dust} \int_0^{d_A} dr\ \kappa[\lambda_V \frac{1+z(d_A)}{1+z(r)}]}. \quad (33)$$

I have assumed a constant dust density ρ_{dust} along the line of sight, which is an appropriate approximation for a homogeneous Universe without expansion and moderate amounts of dust ejection by the galaxy. If we considered an expanding Universe with a strong dust emission rate by the galaxies, we should include $\rho_{dust}(r)$ within the integral, but it is not the case here. The absorption coefficient can approximately be described with a wavelength dependence:

$$\kappa(\lambda) = \kappa(\lambda_V) \left(\frac{\lambda}{\lambda_V}\right)^{-\alpha}. \quad (34)$$

Hence, and using Eqs. (15) and (17),

$$L_{V,rest} = 4\pi F_{V,rest} d_L^2 e^{\frac{c\ a_V}{H_0(\alpha+m)}[(1+z)^m - (1+z)^{-\alpha}]}, \quad (35)$$

with $m = 1$ for the cosmology with linear Hubble law, and $m = 0$ for the tired light case. $a_V \equiv \kappa(\lambda_V)\rho_{dust}$ is the absorption in V per unit length, which means there are $1.086 a_V$ magnitudes in V of extinction per unit length.

The value of the exponent α is not totally independent of λ but I take it approximately as constant. I adopt $\alpha = 2$, as observed in near-infrared bands in our Galaxy[99]. For lower wavelengths (optical, ultraviolet) α would be lower. Since most of the sources have the wavelength equivalent to V-rest in the near-infrared, and the extinction curve of dust in the intervening QSO absorbers resembles the SMC extinction curve[100], this approximation is reasonable.

The absolute value of a_V is not well known and neither do we know whether it is significant or null. There are only some constraints for the maximum value (e.g., [101], although based on standard cosmology). The values of a_V which give the best fit to our data are: $a_V = 1.6 \times 10^{-4}$ Mpc^{-1} for the linear Hubble law case, and $a_V = 3.4 \times 10^{-4}$ Mpc^{-1} for the simple tired light case. Assuming $\kappa(\lambda_V) \sim 10^5$ cm^2/gr [102], the value for the dust density necessary to produce such an extinction would be $\rho_{dust} \sim 6 \times 10^{-34}$ g/cm^3, and $\rho_{dust} \sim 1.2 \times 10^{-33}$ g/cm^3 respectively, which is within the range of possible values. Vishwakarma[37] gives values of $\rho_{dust} = 3 - 5 \times 10^{-34}$ g/cm^3, but for the QSSC model. Inoue & Kamaya[101] allow values as high as $\rho_{dust} \sim 10^{-33}$ g/cm^3 for the high z IGM within the standard concordance cosmology. For comparison, the average baryonic density of the Universe is (taking $\Omega_b = 0.042$; [103]) $\rho_b = 3.9 \times 10^{-31}$ g/cm^3, so this would mean that IGM dust constitutes 0.15 or 0.30% of the total baryonic matter, reasonable amounts.

Whether the extinction used in the models with extinction would be grey or would introduce some small reddening is not totally clear, but this is not a question for the present analysis. I just note that reddening in the optical would depend on the value of α in the visible at intermediate to high redshift and the variability of $\kappa(\lambda)$ with respect to λ in the UV. The features of dust extinction in the UV are not easy to model in an IGM with unknown composition.

With this simple correction for extinction, the results are significantly improved, as shown in Fig. 7. The tired light case gives a better fit.

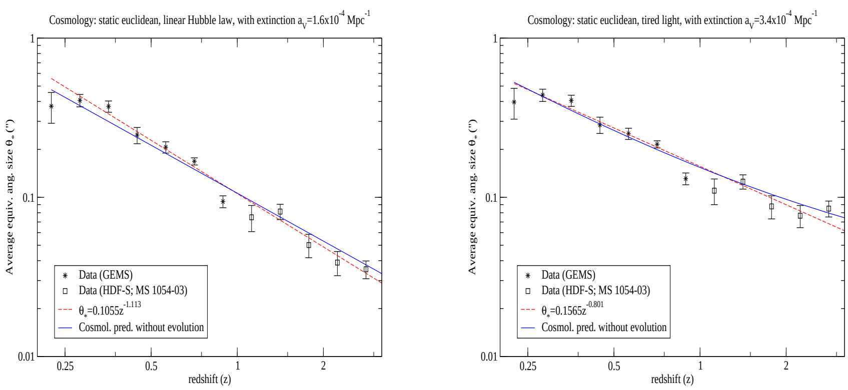

Fig. 7. Log–log plot of the average of $\log_{10}\theta_*$, where θ_* is the equivalent angular size, vs. redshift (z). Bins of $\Delta\log_{10}(z) = 0.10$. The two plots are for the first two static cosmologies with the inclusion of a constant IGM extinction which gives the best fit.

5.2. *Comparison with angular size test for ultra-compact radio sources*

Compact radio sources have been used by several authors to carry out the angular size test because these sources were thought to be free of evolutionary effects. However, the different results obtained with these sources has raised the suspicion that they may not be such good standard rods. Apparently, these rods are somewhat flexible. For example, Kellermann[15] claimed that the angular size test for these sources fitted Einstein–de Sitter expectations very well, when Einstein–de Sitter was the fashionable model. Jackson & Dodgson[16] claimed the opposite: that it was not compatible with Einstein–de Sitter, and that, given that $\Omega_m = 0.2$, the best fit for the cosmological constant term was $\Omega_\Lambda = -3.0$; flat cosmological models were excluded with $> 70\%$ C.L. Jackson[18], in the era of the concordance model as the fashionable cosmology, again carried out the analysis of the same data used by Jackson & Dodgson[16], doing some new corrections due to selection effects and bias, and they get the best fit for $\Omega_m = 0.29$, $\Omega_\Lambda = 0.37$, compatible within 1-σ with the concordance model. With further data, Jackson & Jannetta[19] get the best fit for $\Omega_m = 0.25^{+0.04}_{-0.03}$, $\Omega_\Lambda = 0.97^{+0.09}_{-0.13}$ (68% C.L.). It seems that the general trend is to obtain the result expected from fashionable cosmologies on the date in which the test is carried out, and when incompatibilities appear, some selections effects, biases, small evolution effects are sought to try to make the results compatible. In my opinion, this is not a very objective way to do science, but let us leave the discussion of the methodology of cosmology aside.

One important selection effect is derived from the fact that linear sizes depend on radio luminosities. Jackson[18] tries to take this effect into account and suggests a method of correcting it: binning the data into groups of 42 points in the redshift distribution and taking as representative of each bin the mean of points between 11 to 17 within each one, counting from the smallest objects. This is supposed to be done to compensate for the dependence $R_{rad} \propto L_{rad}^{-1/3}$ for a given redshift and for the fact that the lowest luminosity points cannot be observed at high redshift. In my opinion, this is not the right way to correct the selection effect. Jackson[18] is just doing a median which gives more weight to smaller objects, but this median is shifted at high redshift by the lack of low luminosity objects.

Nevertheless, my concern is not about Jackson's method of correcting for the Malmquist bias, but about the relationship between radius and luminosity. The relationship in Fig. 1 of Jackson[18] is applicable only to the concordance model and it will be far different for static Universes. Particularly in Figs. 1 and 2 of Jackson[18] we see that the linear size is almost independent of redshift between $z = 0.5$ and $z = 4$: around 10 pc, with some scattering due in part to the range of luminosities for each redshift. However, if I use eq. (15) of the static Universe with linear Hubble law instead of eq. (7) of the concordance model to calculate the angular size distances, I see that the linear sizes at $z \approx 3.5$ should be a factor of 10 larger (~ 100 pc instead of ~ 10 pc). Therefore, the linear sizes would not be independent of redshift but highly dependent on it. In such a case, Figs. 1 and 2 of Jackson[18] would be transformed into a plot with a continuous increase in linear size with radio luminosity for all ranges of luminosity for all redshifts: roughly $R_{rad} \propto L_{rad}^{1/2}$. Since at high redshift we cannot observe objects with low L_{rad}, this means that we would be losing objects of low R_{rad} at high z, so the median of θ would be highly overestimated with respect to low to intermediate redshift objects. This would explain why the angular size test for ultra-compact radio sources does not give a z^{-1} dependence. Therefore, a static Universe is not excluded by this test unless we demonstrate with data at low redshift that the radii of the ultra-compact sources does not depend on their luminosities.

Ultra-compact objects could be used to carry out a right angular size test, but we cannot directly compare objects of low luminosity at low redshift with objects of high luminosity at high redshift because we have no guarantee that the linear size is the same in both cases (and it is not valid to assume a cosmological model a priori to prove that it is good a standard rod, because the method should be independent of any cosmological assumption if we want to derive from it which is the best cosmological model). We should either i) compare objects of the same luminosity (different for each cosmology), or ii) define a θ_* as in the present paper in which we need to calibrate the size–luminosity relationship in the low-z Universe. This second option has the caveat that we do not have very high luminosity compact radio sources at low redshifts so we need to extrapolate the local radius–luminosity relationship for high luminosities.

5.3. *Hubble diagram for the different cosmologies*

In Fig. 8, I show the different distance moduli for the different cosmologies without extinction, together with some real data of SNe Ia compiled by Kowalski *et al.*[104]:

$$m_{V,rest} - M_{V,rest} = 5\log_{10}[d_L(z)(\mathrm{Mpc})] + 25. \tag{36}$$

In the first two static models, if we wanted to include the extinction, we should sum $A_{V,rest}(z) = \frac{1.086c\ a_V}{H_0(\alpha+m)}[(1+z)^m - (1+z)^{-\alpha}]$ to the distance modulus.

One aspect is remarkable: the value of the distance modulus for the concordance model is very similar to its value for the static model with a linear Hubble law, and it can be seen in Fig. 8 how the data of SNe Ia are approximately compatible with this scenario. The fit for the concordance model over the data gives a reduced χ^2: $\chi_r^2 = 3.34$; while for the static model with linear Hubble law $\chi_r^2 = 4.20$, slightly worst but not by much: the concordance model reduces the χ^2 by only 20%. Lerner[46] also show this by comparing the residuals of the Hubble diagram in the concordance model and in the static universe ansatz and realizing that they are both similar. Is it a coincidence[b]? With a slight extinction of $a_V \sim 1{-}2\times10^{-4}$ Mpc^{-3} (the range of the best fit obtained in §5.1), the agreement is also quite conspicuous at least for $z < 1$.

[b]The degree of coincidence depends on the maximum redshift, z_{max}, we use. For instance, the value of Ω_Λ in a flat Universe which gives a best minimum square fit to the Hubble diagram for a static model with linear Hubble law and without extinction is $\Omega_\Lambda \approx 0.39 + 0.108 z_{max} - 0.0085 z_{max}^2$. For $z_{max} = 6$, as in our plot of Fig. 8, it gives $\Omega_\Lambda = 0.74$, but it falls to 0.55 if we only consider it up to redshift 1.7, as usually for SNe Ia data. If we set $\Omega_m = 0.3$ and we searched for the best value of Ω_Λ with any value of Ω_{total}, we would get the best fit for $\Omega_\Lambda \approx 0.76 + 0.171 z_{max} - 0.053 z_{max}^2 + 0.0038 z_{max}^3$. For $z_{max} = 6$, it gives the best fit for $\Omega_\Lambda = 0.70$, and it increases to 0.91 if we only consider it up to redshift 1.7.

For the simple tired light model, only with extinction of the order $a_V \sim 5 \times 10^{-4}$ Mpc^{-3} (not very far from the best fit obtained in §5.1) would get the coincidence for $z < 1$. For the plasma redshift tired light model without extinction, the agreement with SNe Ia for $z < 1$ is also acceptable, as noted by Brynjolfsson[40]. This means that, with the static models, we can fit nearly the same Hubble diagrams as the concordance model with its cosmological constant, particularly for supernovae fits. It is not the place here to extend the discussion on the analysis of the compatibility of the static model with a linear Hubble law and the supernovae Ia data; this would require a discussion of the systematic errors, the selection effects, etc. At present, I just want to emphasize that there are no major problems to make compatible the static model of linear Hubble law with SNe Ia data.

For other cosmologies, and without extinction, the difference from the concordance model is larger, especially for the highest redshifts. This does not mean that they are discarded, because the objects used as standard candles (particularly supernovae) might have an absolute magnitude which is not constant with redshift or some extinction along their lines of sight. Several authors[105,106,107,108,109,110,111,32,112] also think that SNe Ia data used to derive $\Omega_\Lambda = 0.7$ are affected by several systematic uncertainties that make the Λ-CDM cosmology uncertain.

5.4. *Evolution, and other tests*

Another topic to discuss is the apparent evolution of galaxies at different redshift. I have not included any evolution correction for the static models in the plots of Fig. 7 although some slight evolution might be compatible with them, in the sense that brighter populations are present at higher redshifts. Some evolution might be present because, as discussed in §3.2, elliptical galaxies get lower angular sizes at high redshifts than disk galaxies, possibly due to the different mean ages of their populations or merger rate; although some of these differences might be due to systematic errors too, as said in §3.3. Also, the most massive galaxies present a higher ratio of angular size with respect to local galaxies[47,48,55], which may be interpreted here as a real higher compactness with respect to the least massive galaxies. Note, however, that, as said above, this extra-compactness can be understood in terms of fluctuations due to noise preventing the recovery of the extended low surface brightness halos in the light profile[49].

There is also a slight color at rest evolution[67] and a mass–luminosity ratio evolution, as said in §4.2; but these mass calculations are subject to important errors depending on the synthesis model, IMF assumption, etc.; hence, there is a wide range of possible values of mass–luminosity ratio evolution. The bluer (at rest) color of high redshift galaxies might be due to bias, because bluer galaxies with younger populations are brighter. We must also bear in mind that photometric errors are larger at higher redshift (because of the fainter fluxes), the photometric redshifts have higher uncertainties and consequently might affect the determination of the luminosities

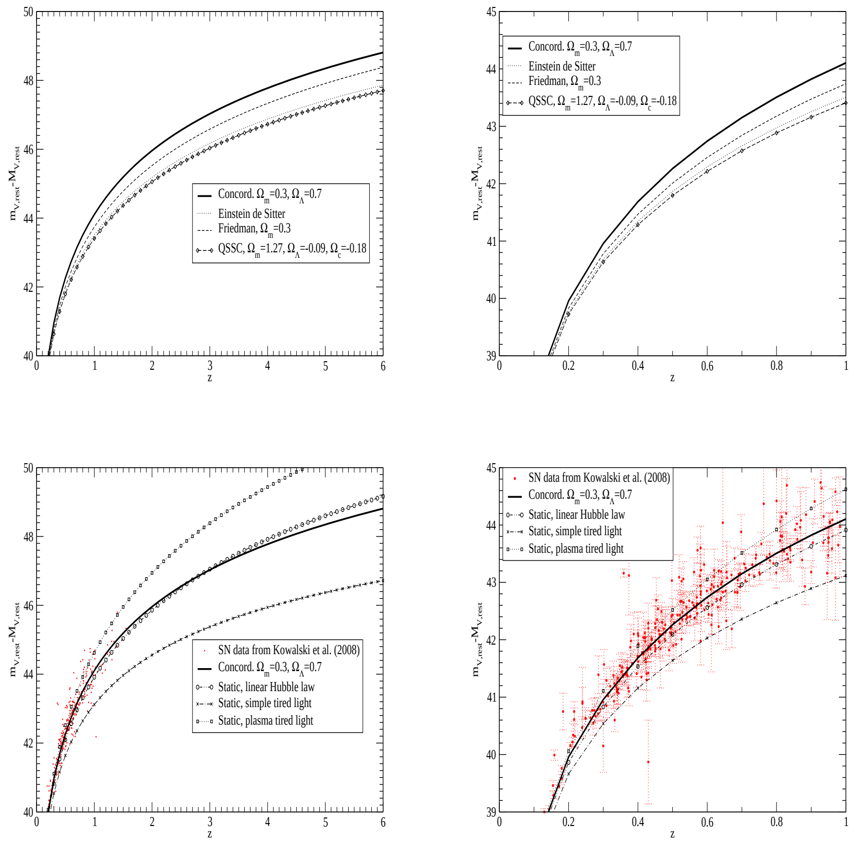

Fig. 8.

Distance modulus as a function of redshift for different cosmological models. Data of supernovae Ia compiled by Kowalski *et al.*[104] were added. Right plots are zooms of the left plots.

at rest, etc. At present, with the data used from Refs. [24,41] at $z \geq 1$, the correlation between z and $(B-V)_{rest}$ is: $\langle z(B-V)_{rest}\rangle - \langle z\rangle\langle(B-V)_{rest}\rangle = -0.048 \pm 0.021$ for elliptical galaxies and -0.033 ± 0.015 for disk galaxies. The variation in color with the variation in redshift is much lower than the dispersion of colors and possible systematic effects. Therefore, we cannot exclude the possibility that the luminosity evolution is small enough to be within the error bars of our data with the static models.

Note however that the evolution of some quantities per unit comoving volume [for instance, the star formation ratio, expressed in mass per unit time per unit comoving volume, is claimed to be significantly higher in the past[113]] must be corrected with respect to the concordance cosmology with Eqs. (22), (24), which reduces by a factor of 10 at $z = 2$ the ratio in the static model with a linear Hubble law with respect to the concordance one.

In any case, there may be a real evolution which should be explained, either in expanding or static models. Note that "static" does not mean "no evolution"; "static" means "no expansion"; there is not necessarily a contradiction in observing evolution in a static Universe, provided that the creation of galaxies is not a continuous process.

Explaining the cosmic microwave background radiation (CMBR) and its anisotropies is not the purpose of the present paper. Note, however, that there exist alternative scenarios to explain them apart from the model of standard hot Big Bang (e.g., [114,39]; see the discussion in another paper of mine[115], §1). Concerning other arguments/tests in favor of the expansion (e.g., [44,116,117]), we must bear in mind that they are usually a matter of discussion. Tests such as the time dilation in SNe Ia, which were claimed to be a definitive proof of the expansion of the Universe, find counterarguments and criticisms from opponents[118,119,120], who claim that a static Universe is compatible with the data. The same thing happens with any other test, including the present one of angular size. Apart from the present angular size test, there are other tests that also present results in favor of a static Universe and against an expanding Universe (e.g., [12,121,122,123,124,125,32,45,46]).

Perhaps the most immediate problem with the static Universe is understanding the cause of the redshift of galaxies, but there are several proposals for alternative mechanism to produce redshifts without expansion or Doppler effect, so the hypothesis of a static Universe is not an impossible one. Other facts, such as the formation of the large scale structure, the creation of the light elements, etc., also provide alternative explanations different from the standard model. Further discussion of all the questions raised in this paragraph are given in my review[126].

5.4.1. *UV surface brightness test*

The main discrepancy in the different tests such as angular size, surface brightness, Hubble diagram, etc., is the evolution of galaxy luminosities, which is very large for the defenders of the expansion and not so large for the defenders of the static models. Lerner[45] proposes a test of the evolution hypothesis that is also useful in the present case. There is a limit on the UV surface brightness of a galaxy, because when the surface density of hot bright stars and thus supernovae increases large amounts of dust are produced to absorb all the UV except that from a thin layer. Further increase in surface density of hot bright stars beyond a given point just produces more dust, and a thinner surface layer, not an increase in UV surface brightness. Based on this principle, there should be a maximum surface brightness in UV-rest wavelengths independent of redshift. Scarpa *et al.*[127] measured in low redshift galaxies a maximum FUV (1550 Å at rest) emission of 18.5 $\mathrm{mag}_{AB}/\mathrm{arcsec}^2$ (the average is $24-25$ $\mathrm{mag}_{AB}/\mathrm{arcsec}^2$); no galaxy should be brighter per unit angular area than that.

Using eqs. (27), (28) with the flux at wavelength 1550 Å at rest[c] from the subsample MS 1054-03 in Trujillo *et al.*[24] galaxies, and the angular sizes $\theta_{1550\mathring{A}} = 1.14\theta_V$ [22] for $n_S > 2.5$ and $\theta_{1550\mathring{A}} = 1.10\theta_V$ [23] for $n_S < 2.5$, I get the values of SB_0 for all the galaxies in this subsample for the expanding or static cases (Fig. 9). For the expanding universe, many galaxies have average intrinsic surface brightness (SB_0) lower(brighter) than 18.5 $\mathrm{mag}_{AB}/\mathrm{arcsec}^2$, the galaxy MS 1054-03/1356 being the brightest one per unit angular area: 14.8 $\mathrm{mag}_{AB}/\mathrm{arcsec}^2$ (30 times brighter than the limit). The angular size of this galaxy MS 1054-03/1356 ($0.027''$ circularized in V; $0.031''$ in FUV) might be affected by some error since it is below $0.125''$ in V, but even an error of 100% in angular size would produce an error of 1.5 magnitudes in surface brightness, not 3.7 magnitudes as we observe here. The dispersion (r.m.s.) of $\frac{\theta_{1550\mathring{A}}}{\theta_V}$ might be around 20% ([23], Fig. 2), which means an uncertainty of around 0.4 $\mathrm{mag}/\mathrm{arcsec}^2$ (1-σ) in SB_0, much lower than the differences between SB_0 and its limit of 18.5. Moreover, even avoiding galaxies with angular size less than $0.125''$, there are some galaxies with surface brightness over the limit, up to 6 times brighter than the limit. Too high intrinsic FUV surface brightness. Lerner[45] also argues why other alternative explanations (lower production of dust at high redshift, winds or others) are not consistent. However, for the static models, all galaxies have average intrinsic surface brightness (SB_0) within θ higher(fainter) than the limit 18.5 $\mathrm{mag}_{AB}/\mathrm{arcsec}^2$, a result which may be interpreted again in favor of the static scenario.

5.5. *Is a static model theoretically impossible?*

Apart from the discussion on the observations, which are inconclusive, a static model is usually rejected by most cosmologists on the grounds of a belief/prejudice that a static model is impossible. However, from a purely theoretical point of view, without taking into account the astronomical observations, the representation of the Cosmos as Euclidean and static is not excluded. Both expanding and static space are possible for the description of the Universe, even with evolution.

Before Einstein and the rise of Riemannian and other non-Euclidean geometries to the stage of physics, attempts to describe the known Universe with a Euclidean Universe were given, but with the problem of justifying a stable equilibrium. Within a relativistic context, Einstein[128] proposed a static model including a cosmological constant, his biggest blunder according to himself, to avoid a collapse.

[c]An interpolation with the publicly available fluxes in filters U, B, V, I_{814} and J was used to get it. These fluxes are given in flux per unit frequency so the dimming factor must be multiplied by a factor $(1+z)$ with respect to the flux in the whole filter. That is, $SB_0 = SB(1+z)^3$ for the expanding case, and $SB_0 = SB$ for the static case.

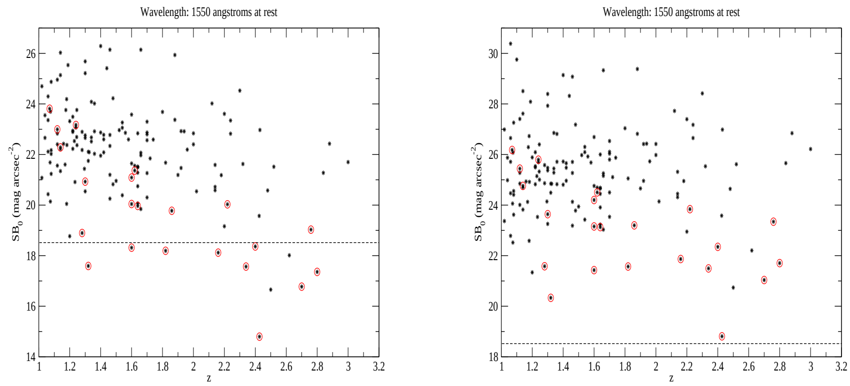

Fig. 9. Intrinsic FUV-rest surface brightness of the galaxies in the subsample MS 1054-03 in an expanding (left) or static (right) universe. The dashed line stands for the minimum value of 18.5 mag/arcsec^2 over which all galaxies should be located. Points with circles stand for data with $\theta < 0.125$".

This model still has problems to guarantee the stability, but it might be solved somehow. Narlikar & Arp[129] solve it within some variation of the Hoyle–Narlikar conformal theory of gravity, in which small perturbations of the flat Minkowski spacetime would lead to small oscillations about the line element rather than to a collapse. Boehmer *et al.*[130] analyze the stability of the Einstein static universe by considering homogeneous scalar perturbations in the context of $f(R)$ modified theories of gravity and it is found that a stable Einstein cosmos with a positive cosmological constant is possible. Other authors solve it with the variation of fundamental constants [133,134]. Another idea by Van Flandern[135] is that hypothetical gravitons responsible for the gravitational interaction have a finite cross-sectional area, so that they can only travel a finite distance, however great, before colliding with another graviton. So the range of the force of gravity would necessarily be limited in this way and collapse is avoided.

As said in §4.1, the very concept of space expansion has its own problems[58,59]. The curved geometry (general relativity and its modifications) has no conservation of the energy-momentum of the gravity field (the well-known problem of the pseudo-tensor character of the energy-momentum of the gravity field in general relativity). However, Minkowski space follows the conservation of energy-momentum of the gravitational field. One approach with a material tensor field in Minkowski space is given in Feynman's gravitation[131], where the space is static but matter and fields can be expanding in a static space. It is also worth mentioning a model related to modern relativistic and quantum field theories of basic fundamental interactions (strong, weak, electro-magnetic): the relativistic field gravity theory and fractal matter distribution in static Minkowski space[132].

Olber's paradox for an infinite Universe also needs subtle solutions, but extinction, absorption and reemission of light, fractal distribution of density and the mechanism which itself produces the redshift of the galaxies might have something to do with its solution. These are old questions discussed in many classical books on cosmology (e.g., [136], ch. 3) and do not warrant further discussion here.

6. Conclusions

Summing up, the main conclusions of this paper are the following:

- The average angular size of the galaxies for a given luminosity with redshifts between $z = 0.2$ and 3.2 is approximately proportional to $z^{-\alpha}$, with α between 0.7 and 1.2, depending on the assumed cosmology.
- Any model of an expanding Universe without evolution is totally unable to fit the angular size vs. z dependence. The hypothesis that galaxies which formed earlier have much higher densities does not work because it is not observed here that the smaller galaxies are precisely those which formed earlier; in any case, the galaxies observed today were formed mostly at redshifts not very different from the galaxies observed at higher redshifts. A very strong evolution in size would be able to get an agreement with the data but there appear caveats to justify it in terms of age variation of the population and/or mergers and/or ejection of massive outflows in the quasar feedback. An average of the necessary two to four major mergers per galaxy during its lifetime is excessive, and neither is it understood how massive elliptical galaxies may present passive evolution in this scenario or how spiral galaxies can become larger during their lifetimes. The depletion of gas in ellipticals by a QSO feedback mechanism does not appear to be in agreement with the observed star formation and other facts. Moreover, no evolution is observed in the rotation/dispersion velocities and, the FUV surface brightness turns out to be prohibitively high in some galaxies at high redshift.
- Static Euclidean models with a linear Hubble law or simple tired light fit the shape of the angular size vs. z dependence very well: there is a difference in amplitude of 20–30%, which is within the possible systematic errors. An extra small intergalactic extinction may also explain this difference of 20–30% . Some weak evolution of very high redshift sources is allowed, although non-evolution is a possible solution too. For the plasma redshift tired light static model, a strong (albeit weaker than in expanding models) evolution in galaxy size is necessary to fit the data. The SNe Ia Hubble diagram can also be explained in terms of these models.
- It is also remarkable that the explanation of test results with an expanding Universe requires four coincidences:

  (1) The combination of expansion and (very strong) size evolution gives nearly the same result as a static Euclidean universe with a linear Hubble law alone: $\theta \propto z^{-1}$.

(2) This hypothetical evolution in size for galaxies is the same in normal galaxies as in QSOs, as in radio galaxies, as in first ranked cluster galaxies, as the separation among bright galaxies in clusters. Everything evolves in the same way to produce approximately a dependence $\theta \propto z^{-1}$.
(3) The concordance model gives approximately the same (differences of less than 0.2 mag within $z < 4.5$) distance modulus in a Hubble diagram as the static Euclidean universe with a linear Hubble law.
(4) The combination of expansion, (very strong) size evolution, and dark matter ratio variation gives the same result for the velocity dispersion in elliptical galaxies (the result is that it is nearly constant with z) as for a simple static model with no evolution in size and no dark matter ratio variation.

These four coincidences might make us think that possibly we should apply Occam's razor *"Entia non sunt multiplicanda praeter necessitatem"*, we should use the simplest models that can reproduce the same things as a complex model with many more free parameters does.

It would be an irony of fate that, after all the complex solutions pursued by cosmologists for the last century, we had to come back to simple scenarios such as a Euclidean static Universe without expansion. None the less, we cannot at present defend any of these simple models apart from the standard one because this would require other analyses. The conclusion of this paper is just that the data on angular size vs. redshift present some conflict with the standard model, and that they are in accordance with a very simple phenomenological extrapolation of the Hubble relation that might ultimately be linked to a static model of the universe.

Acknowledgments

Thanks are given to: the anonymous referee for very helpful suggestion to improve this paper; Ignacio Trujillo and Eric J. Lerner for useful discussions on many topics related to this paper and comments on a draft of it; Riccardo Scarpa, Carlos M. Gutiérrez, Juan Betancort-Rijo also for their comments on a draft of this paper; Daniel MacIntosh for providing me the GEMS data from the papers [22,23]; Terry J. Mahoney for proof-reading this paper. MLC was supported by the *Ramón y Cajal* Programme of the Spanish Ministry of Science.

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