

NEW SOLUTIONS OF MAXWELL'S EQUATIONS- PROPOSING SPATIAL AMPLITUDE TO ELECTROMAGNETIC WAVE

V.A.Induchoodan Menon
Gujarat Univ. Campus,
Ahmedabad-380009, India

Abstract

We show that Maxwell's equations in vacuum have solutions which indicate a helical structure for the electromagnetic waves in space and which are at the same time circularly polarized. Our proposal goes against the universally accepted solutions which treat these waves as propagating along linear paths. It is shown that the fundamental state of the electromagnetic wave is circularly polarized and this finding is consistent with the quantum mechanical picture of photon which is theorized to exist only in circularly polarized states. In the process we show that the electromagnetic waves have a new property represented by its "spatial amplitude".

PACS numbers: 41.20.Jb, 42.25.Bs, 03.50.De

1 Introduction

In quantum mechanical description, electromagnetic field is an assembly of photons with specific spin states. The spin states of photon are classically understood in terms of the circular polarization of the electromagnetic waves. On the other hand, due to historical reasons, in classical electrodynamics it is assumed that any polarization state could be treated as a combination of two linearly polarized states [1]. The universally accepted solutions of Maxwell's equations are arrived at, based on such an assumption although it goes counter to quantum theory where a photon which is the most basic state of the electromagnetic field, is treated as a circularly polarized state [2]. In view of this it will be interesting to revisit Maxwell's equations based on the assumption that the circularly polarized states are the most fundamental states and explore the possible solutions.

We start with Maxwell's equations given by

$$\begin{aligned} (i) \quad \nabla \cdot \xi &= \rho/\epsilon_o, & (ii) \quad \nabla \times \xi &= -\partial \mathbf{B}/\partial t, \\ (iii) \quad \nabla \cdot \mathbf{B} &= 0. & (iv) \quad c^2 \nabla \times \mathbf{B} &= \mathbf{j}/\epsilon_o + \partial \xi/\partial t \end{aligned} \quad (1)$$

Since we propose to study the transmission of the electromagnetic waves in vacuum, we shall take the charge density, ρ and the electric current density \mathbf{j} as zero which gives

$$\begin{aligned} (i) \quad \nabla \cdot \xi &= 0, & (ii) \quad \nabla \times \xi &= -\partial \mathbf{B}/\partial t, \\ (iii) \quad \nabla \cdot \mathbf{B} &= 0. & (iv) \quad c^2 \nabla \times \mathbf{B} &= \partial \xi/\partial t. \end{aligned} \quad (2)$$

We shall now solve these equations to see how the concept of the electromagnetic wave emerges from them. Subsequently we shall modify the approach suitably to arrive at the new

solutions. We shall follow Feynman's insightful approach here in solving these equations in the conventional way [3]. We know that the first equation can be written as

$$\nabla \cdot \xi = \frac{\partial \xi_x}{\partial x} + \frac{\partial \xi_y}{\partial y} + \frac{\partial \xi_z}{\partial z} = 0. \quad (3)$$

Here we assume that there are no variations with x and y which means that the first two terms could be taken as zero. Hence, we have

$$\frac{\partial \xi_z}{\partial z} = 0. \quad (3A)$$

In other words ξ_z is a constant in the x-direction. If we study Maxwell's equation (2.iv), assuming that just as in the case of the electric field, there is no variation in x and y directions in the magnetic field also, it can be seen that E_z is also a constant in time. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore we may take $E_z = 0$. In other words, the electric field exists only in the x and y directions which are perpendicular to the direction of propagation. As a first step, for the sake of simplicity, we may assume that the electric field has a component only in the x-direction and obtain a solution on that basis. Later we may take up the case where the electric field has a component only in the y-direction. Then, the general solution could always be expressed as the superposition of the two cases.

Let us take the Maxwell's equation (2.ii) and equate the projections along the three coordinate axes separately to obtain

$$(\nabla \times \xi)_x = \frac{\partial \xi_z}{\partial y} - \frac{\partial \xi_y}{\partial z}; \quad (\nabla \times \xi)_y = \frac{\partial \xi_x}{\partial z} - \frac{\partial \xi_z}{\partial x}; \quad (\nabla \times \xi)_z = \frac{\partial \xi_y}{\partial x} - \frac{\partial \xi_x}{\partial y}. \quad (4)$$

Here $(\nabla \times \xi)_z$ will be zero because the derivatives with regard to x and y are zero. Note that we could take ξ_x as a constant along the x and y directions while ξ_y is taken as zero. $(\nabla \times \xi)_x$ is zero because the first term which is a derivative of ξ_z is zero while the second term is zero as ξ_y is taken to be zero. The only component of $\text{curl} \xi$ which is not zero is $(\nabla \times \xi)_y$ which is equal to $\partial \xi_x / \partial z$. Setting the three components of $(\nabla \times \xi)$ equal to the corresponding components of $-\partial \mathbf{B} / \partial t$, we obtain

$$\frac{\partial B_x}{\partial t} = 0; \quad \frac{\partial B_y}{\partial t} = -\frac{\partial \xi_x}{\partial z}; \quad \frac{\partial B_z}{\partial t} = 0. \quad (5)$$

Since the z and x components of the magnetic field have zero time derivatives, they represent constant fields. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore, we may take $B_z = B_x = 0$. The second equation in (5) shows that the electric field has only the x-component while the magnetic field has only the y-component. This means ξ and \mathbf{B} are perpendicular to each other.

Let us now take the last Maxwell's equation whose components can be written as

$$\begin{aligned}
c^2[\nabla \times \mathbf{B}]_x &= c^2 \frac{\partial B_z}{\partial y} - c^2 \frac{\partial B_y}{\partial z} = \frac{\partial \xi_z}{\partial t} ; & c^2[\nabla \times \mathbf{B}]_y &= c^2 \frac{\partial B_x}{\partial z} - c^2 \frac{\partial B_z}{\partial x} = \frac{\partial \xi_y}{\partial t} ; \\
c^2[\nabla \times \mathbf{B}]_z &= c^2 \frac{\partial B_y}{\partial x} - c^2 \frac{\partial B_x}{\partial y} = \frac{\partial \xi_x}{\partial t} .
\end{aligned} \tag{6}$$

Out of these, only the term $\partial B_y / \partial z$ is not equal to zero. So these three equations simply to give

$$-c^2 \frac{\partial B_y}{\partial z} = \frac{\partial \xi_x}{\partial t} . \tag{7}$$

Now taking partial differentiation with regard to t and using the second equation in (5), we obtain the wave equations

$$\frac{\partial^2 \xi_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \xi_x}{\partial t^2} = 0 ; \quad \frac{\partial^2 B_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 B_y}{\partial t^2} = 0 . \tag{8}$$

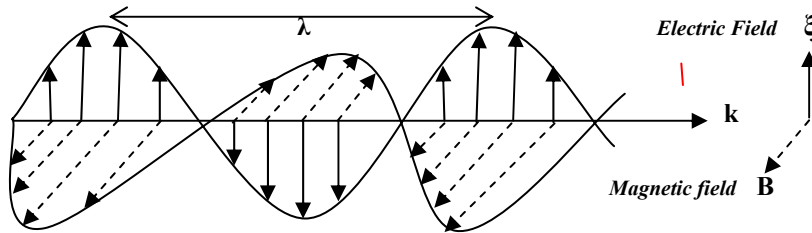
Note that the above equations represent waves having polarization in one plane. Similarly, we can obtain the equations for waves having polarization in a perpendicular plane involving only ξ_y and B_x as

$$\frac{\partial^2 \xi_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \xi_y}{\partial t^2} = 0 ; \quad \frac{\partial^2 B_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 B_x}{\partial t^2} = 0 . \tag{8A}$$

We know the solutions for the combined wave equations are given by

$$\begin{aligned}
\psi_x(\xi) &= \xi_x e^{-i(\omega t - k z)} ; & \psi_y(B) &= B_y e^{-i(\omega t - k z)} . \\
\psi_y(B) &= B_y e^{-i(\omega t - k z)} ; & \psi_x(\xi) &= \xi_x e^{-i(\omega t - k z)} .
\end{aligned} \tag{9}$$

where ω is the angular frequency and k is the wave vector. Combining both, the wave equation



The electric and the magnetic fields are perpendicular to each other and the direction of propagation. Note that the sinusoidal curve represents the field on the line of propagation.

Figure 1

in a general direction will be given by

$$\psi(\xi) = \xi e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} . \quad (9A)$$

Similarly, we may obtain the wave equation for the magnetic component also which may be written as

$$\psi(B) = \mathbf{B} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} . \quad (9B)$$

Note that the magnetic field will always be perpendicular to the electric field. Another important point to be kept in mind is that the solutions represent not a single wave, but a wave front which has its components of the electric field ξ_x and ξ_y (also the corresponding magnetic fields) constant along the x and y directions at any instant.

2 New Solutions of Maxwell's Equations

In the conventional approach, for the wave propagating is along z-axis, Maxwell's equation (1.i) is satisfied by taking $\partial \xi_x / \partial x = \partial \xi_y / \partial y = \partial \xi_z / \partial z = 0$ [3]. This allows us to take ξ_x and ξ_y as constants in the transverse directions which could conveniently be equated to zero except on the line of propagation, while ξ_z could be taken as zero everywhere. We observe that to satisfy the condition $\nabla \cdot \xi = 0$, we could as well have taken

$$\frac{\partial \xi_x}{\partial x} = - \frac{\partial \xi_y}{\partial y} \quad (10)$$

provided we assume $\partial \xi_z / \partial z = 0$. The simplest way to interpret (3) is to assume that ξ_x and ξ_y are the field components at a point which undergo simple harmonic motion along x and y directions simultaneously. We may attribute the field at the point to be directly proportional to its transverse displacement from z-axis. If we represent the displacement along the x-direction by η_x and that along the y-direction by η_y , then the simple relation between ξ and $\boldsymbol{\eta}$ that satisfies (3) is given by

$$\xi_x = \alpha_1 \eta_x \quad \text{and} \quad \xi_y = -\alpha_1 \eta_y , \quad (11)$$

$$\eta_x = R \sin[\omega t] \quad \text{and} \quad \eta_y = R \cos[\omega t] , \quad (12)$$

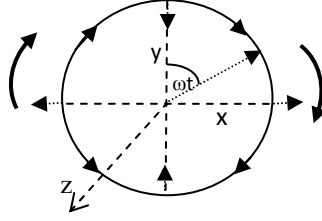
where α_1 is a constant. Now taking cue from the existing solutions of Maxwell's equation, we may assume that η_x and η_y depend on z and express (12) as

$$\eta_x = R \sin[\omega t - kz] \quad \text{and} \quad \eta_y = R \cos[\omega t - kz] . \quad (13)$$

These equations represent propagating waves. Note that ultimately the expressions for η_x and η_y assumed here have to satisfy Maxwell's equations in a consistent manner.

To understand the picture emerging from the above equations, we shall take the projection of the progressive wave on to the x-y plane at the origin. The projection will be a

circle of radius R with center at the origin. Note that (13) represents a helical wave progressing along z -axis in the clockwise direction (figure 1). From (11) and (13) we observe that the direction of the resultant electric vector which pertains to a point on the circle remains anti-parallel to the radial vector along the y -axis and parallel to it along the x -axis. Recall that the electric field is located at the tip of the radius vector.



The figure gives the transverse sectional view of the helical wave propagating along z -axis.

Figure. 1

It is obvious that ξ and η defined by (11) and (13) fully satisfy (3). This can be proved by substituting (13) into (11) keeping in mind that $x = R \sin \omega t$ and $y = R \cos \omega t$. Expanding the right hand side of (11) and replacing for $R \sin \omega t$ and $R \cos \omega t$ with x and y , we obtain

$$\begin{aligned}\xi_x &= \alpha_1 [x \cos kz - y \sin kz] ; \\ \xi_y &= -\alpha_1 [y \cos kz + x \sin kz] ;\end{aligned}\quad (14)$$

and

$$\partial \xi_x / \partial x = \alpha_1 \cos kz = -\partial \xi_y / \partial y .$$

Let us now take Maxwell's equation (1.ii). We know that based on (14), $(\nabla \times \xi)_z$ will be zero. Since ξ_z is taken as zero everywhere, $\partial \xi_z / \partial y$ will also be zero. But $\partial \xi_y / \partial z$ will not be zero as η_y is dependent on z . Hence the x -component of the curl will be given by

$$(\nabla \times \xi)_x = -\frac{\partial \xi_y}{\partial z} ; = -\frac{\partial B_x}{\partial t} \quad (15)$$

Using (14) and (15), we obtain

$$\frac{\partial B_x}{\partial t} ; = \frac{\partial \xi_y}{\partial z} = -k \alpha_1 n (\omega t - kz) . \quad (16)$$

Keeping in mind $\omega = kc$, it can be easily seen that

$$B_x = \frac{\alpha_1 k R \cos(\omega t - kz)}{\omega} = (\alpha_1 / c) \eta_y \quad (17)$$

would satisfy equation (1.iv). Here we have taken the constant of integration as zero as we are interested only in variable fields. Similarly, it can be seen that

$$B_y = (\alpha_1 / c) \eta_x . \quad (18)$$

It can be easily seen that the divergence of the magnetic vector \mathbf{B} is equal to zero as required by (1.iii). Since the electric vector ξ and the magnetic vector \mathbf{B} are given by

$$\xi = \mathbf{i}\xi_x + \mathbf{j}\xi_y = R [\mathbf{i}\sin(\omega t - kz) - \mathbf{j}\cos(\omega t - kz)] \quad (19)$$

$$\mathbf{B} = \mathbf{i}B_x + \mathbf{j}B_y = \frac{\alpha_1}{c} R [\mathbf{i}\cos(\omega t - kz) + \mathbf{j}\sin(\omega t - kz)] \quad (20)$$

it is obvious that $\xi \cdot \mathbf{B} = 0$. Therefore, the electric and the magnetic vectors are orthogonal to each other and to z-axis. Note the difference between (11) and (17).

From (11), (17) and (18) one may get the impression that there is nothing to limit the magnitude of the electric and the magnetic fields. However, the fact that the frequency of the oscillations in the transverse directions has to be equal to that of the wave propagation introduces a limit to the value of R and makes it equal to $\lambda/2\pi$. We shall discuss this issue in detail in section 3.

Let us now express equation (1.ii) as

$$\frac{\partial \xi_y}{\partial x} - \frac{\partial \xi_x}{\partial y} = -\frac{\partial B_z}{\partial t}, \quad \frac{\partial \xi_z}{\partial y} - \frac{\partial \xi_y}{\partial z} = -\frac{\partial B_x}{\partial t}, \quad (21)$$

$$\frac{\partial \xi_x}{\partial z} - \frac{\partial \xi_z}{\partial x} = -\frac{\partial B_y}{\partial t} \quad (22)$$

On the basis of (14), we observe that $\partial \xi_y / \partial x = \partial \xi_x / \partial y$. Further, since ξ_z vanishes everywhere, its derivative will also be zero. This would leave us with

$$\frac{\partial B_z}{\partial t} = 0; \quad \frac{\partial B_x}{\partial t} = \frac{\partial \xi_y}{\partial z}, \quad \frac{\partial B_y}{\partial t} = -\frac{\partial \xi_x}{\partial z} \quad (23)$$

Let us now take the last Maxwell's equation whose components could be written as

$$\begin{aligned} (i) \quad c^2 [\nabla \times \mathbf{B}]_z &= c^2 \frac{\partial B_y}{\partial x} - c^2 \frac{\partial B_x}{\partial y} = \frac{\partial \xi_z}{\partial t}, \\ (ii) \quad c^2 [\nabla \times \mathbf{B}]_x &= c^2 \frac{\partial B_z}{\partial y} - c^2 \frac{\partial B_y}{\partial z} = \frac{\partial \xi_x}{\partial t}, \\ (iii) \quad c^2 [\nabla \times \mathbf{B}]_y &= c^2 \frac{\partial B_x}{\partial z} - c^2 \frac{\partial B_z}{\partial x} = \frac{\partial \xi_y}{\partial t}. \end{aligned} \quad (24)$$

From (17) and (18) we have $\partial B_y / \partial x = \alpha_1 / c = \partial B_x / \partial y$. This will make $[\nabla \times \mathbf{B}]_z = 0$. Of the remaining derivatives given in (14), only $\partial B_y / \partial z$ and $\partial B_x / \partial z$ will not be zero. Therefore (24) may be simplified to give

$$-c^2 \frac{\partial B_y}{\partial z} = \frac{\partial \xi_x}{\partial t}; \quad c^2 \frac{\partial B_x}{\partial z} = \frac{\partial \xi_y}{\partial t}. \quad (25)$$

Now taking partial differentiation with regard to t and z and using (23) we obtain the wave equations

$$\frac{\partial^2 \xi_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \xi_x}{\partial t^2} = 0; \quad \frac{\partial^2 B_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 B_y}{\partial t^2} = 0. \quad (26)$$

$$\frac{\partial^2 \xi_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \xi_y}{\partial t^2} = 0; \quad \frac{\partial^2 B_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 B_x}{\partial t^2} = 0. \quad (27)$$

Note that

$$\xi_x = \alpha_1 R \sin[\omega t - kz] \quad ; \quad \xi_y = -\alpha_1 R \cos[\omega t - kz]. \quad (28)$$

satisfy the Maxwell's equations for the electric field. This shows the form of $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ in (11) and (12) gives us a consistent solution of the Maxwell's equations. At a glance these equations appear to have the same form as the ones obtained in the conventional approach. However, there is one major difference. Here $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ cannot exist independent of each other. They are coupled. The same holds good for \mathbf{B} also.

Note that the amplitude of ξ_x and ξ_y are the same and is equal to $\alpha_1 R$. Combining the two equations given above we may express them in a more general complex form as

$$\boldsymbol{\xi} = -(\xi_{y_0} - i\xi_{x_0}) e^{-i(\omega t - kz)}, \quad (29)$$

where $\xi_{x_0} = i\alpha_1 R$ and $\xi_{y_0} = j\alpha_1 R$. Similarly, we have

$$\mathbf{B} = (\mathbf{B}_{x_0} + i\mathbf{B}_{y_0}) e^{-i(\omega t - kz)}. \quad (30)$$

Here we should keep in mind that the solutions given by (29) and (30) are not complete as we have not taken into consideration the spatial displacement of the wave represented by η_x and η_y . We may express the wave equations representing the spatial oscillations also by way of separate equations. For this, we may substitute for ξ_x and ξ_y from (11) into (26) and (27) to obtain

$$\frac{\partial^2 \eta_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \eta_x}{\partial t^2} = 0; \quad \frac{\partial^2 \eta_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \eta_y}{\partial t^2} = 0. \quad (31)$$

It is obvious that η_x and η_y given in (5) are solutions to the above equations and could be expressed as

$$\boldsymbol{\eta} = (\boldsymbol{\eta}_{y_0} + i\boldsymbol{\eta}_{x_0}) e^{-i(\omega t - kz)}, \quad (32)$$

where $\boldsymbol{\eta}_{x_0} = \mathbf{i} R$ and $\boldsymbol{\eta}_{y_0} = \mathbf{j} R$. Note that the solutions given by (29), (30) and (31) together represent a circularly polarized wave moving in a helical path with its axis along z -axis.

Remember that $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ which are solutions to Maxwell's equations are defined by (11) and (12) and they represent the case where the radius vector is parallel to the electric vector along x -axis and anti-parallel along y -axis. In fact, it can be seen that the only requirement for

zero divergence of the electric field is that as the radius vector rotates in one direction, the electric vector should undergo rotation in the opposite direction with the same frequency. The solutions we had examined till now pertain to the special case where the electric and the radius vectors are in phase along the x-axis.

We may now consider another state where the helical wave rotates in the counterclockwise direction given by

$$\eta_x = -R \sin[\omega t - kz] ; \eta_y = R \cos[\omega t - kz] \quad (33)$$

In this case also we can have similar relation between ξ and η . This shows that essentially there are only two states for the electromagnetic wave, one having counter-clockwise rotation for the helical wave in the direction of progression (spin up state) and the other having clockwise rotation (spin down state). An important point to be kept in mind is that in this new approach the electromagnetic wave has two component oscillations, one in the physical space and the other in the electromagnetic field. But the phases of these oscillations are coupled. Using (11) and (33) the general solutions in the complex form is given by

$$\xi = (\xi_{y_0} + i\xi_{x_0}) e^{-i(\omega t - kz)} \quad (34)$$

$$\mathbf{B} = -(\mathbf{B}_{x_0} - i\mathbf{B}_{y_0}) e^{-i(\omega t - kz)} \quad (35)$$

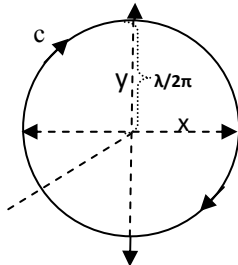
$$\eta = (\eta_{y_0} - i\eta_{x_0}) e^{-i(\omega t - kz)} \quad (36)$$

It is easy to see that these solutions would coincide with the conventional approach if the radius of the helical path is taken to be zero. In that sense, the helical wave solutions are more general than the conventionally accepted ones.

The reason why the possibility of the “new solutions” was never explored earlier may be traced to the historical origin of the concept of the electromagnetic waves whose fundamental state was considered to be plane polarized one. The circularly polarized wave was treated as a composite state formed by two plane polarized states. It can be easily seen that such a bias confines us to the conventional solutions of Maxwell’s equations.

3 The Spatial Amplitude of the Electromagnetic Wave

We saw that the circularly polarized electromagnetic wave represents the locus of a point possessing electric field that undergoes two mutually orthogonal oscillations in physical space in the transverse directions with a phase difference $\frac{1}{2}\pi$, with the magnitude of electric fields directly proportional to its displacement from the line of propagation. The path taken by such a point in the transverse plane will be a circle (figure 2). The simplest case of the circular motion could be obtained if we assume that it is executed at the velocity of light. Any other velocity





The figure is the transverse view of a wave. The vertical and the horizontal lines stand for two oscillations having a phase difference of $\frac{1}{2}\pi$. The resultant circular velocity is taken as c .

Figure.2

would involve the introduction of a new attribute to the electromagnetic wave. Since the wave is also progressing with the velocity c , it is obvious that by the time the wave travels one wave length, it would have executed one full circle in the transverse directions. Therefore, we may conclude that the radius, R of the circle would be given by

$$R = \lambda/2\pi = 1/k \quad . \quad (37)$$

Once the value of R is fixed, then from (11) we obtain

$$\xi = \alpha_1 R = \alpha_1 / k = \lambda \alpha_1 / 2\pi \quad . \quad (38)$$

The above equations connecting ξ with R highlight their direct dependence on the wave length or momentum of the electromagnetic wave.

4 Discussion

It is quite surprising that the solutions of the Maxwell's equations in terms of the helical waves have been overlooked for so long. We know that photon, as it possesses unit spin, has to be treated as in a circularly polarized state. The new solutions obtained above are in conformity with this treatment. Here we are forced to redefine the term helicity because now we have two types of rotations which have to be reckoned. But it is obvious that the concept of spin of the electromagnetic wave can be understood classically on the helical structure of the wave. Since the point on the electromagnetic wave executes rotational motion in the transverse direction, we may take the angular momentum for this motion to be

$$S = \mathbf{r} \times \mathbf{p} = (\lambda/2\pi)(h/\lambda) = \hbar \quad (39)$$

This means that the helicity of the wave is determined by the rotation of the radius vector. It is interesting to note that the new solutions would approach the conventional ones if the radius of the helical wave is taken to be infinitesimally small, approaching zero.

The question that comes to one's mind while studying the new solutions introduced above would be why such a structure did not show up in any of the experiments till now. Actually the problem we face when we try to experimentally verify the helical nature of the electromagnetic wave is that this property gets camouflaged by certain other properties exhibited by it. For example, if we pass micro waves having wave length λ through a wave guide having circular cross section with diameter d , then, there is a sharp cut off of the output

at $\lambda = \frac{1}{2}d$, . This is because standing waves are set up in the transverse mode in the wave guide blocking propagation of energy through it. Therefore, the question of observing a sharp reduction in the output at $d = \lambda/\pi$, the cut off proposed in our theory does not arise. This may be the reason why the helical nature of the electromagnetic wave has not been observed in any experiment as yet. It should be possible to set up an experiment in which this property of the electromagnetic wave gets manifested without the interference by any other phenomenon.

An interesting aspect which emerges from the helical structure proposed here is that the electromagnetic wave is constituted by two types of oscillations, one spatial and the other electromagnetic. If we now assume that the helical wave which emerges from the spatial oscillations as the most basic entity, then it would seem that the electromagnetic wave is generated when the disturbance in the electromagnetic field gets coupled to this basic wave. In other words, here we may be dealing with two fields, and not one. In all probability the helical wave may be representing oscillations in the most basic field which may be identified with the Higgs field.

References:

- [1] J. D. Jackson, Classical Electrodynamics – Second edition- Wiley eastern Limited, (1989), p.269
- [2] J.J.Sakurai, advanced quantum mechanics.(Second Indian Reprint) Addison –Wesley Publishing Co., (1999), P.30-31
- [3] Richard Feynman, The Feynman Lectures on Physics (The Definitive Edition), Vol.2, Dorling Kindersley (India) Pvt. Ltd.,(2009). p.20.4-20.6.