APPARENT TIME-DEPENDENCE OF THE HUBBLE CONSTANT DEDUCED FROM THE OBSERVED HUBBLE VELOCITY-DISTANCE EQUATION

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Abstract

An apparent time dependence of the Hubble constant was deduced from the linear correlation between the recession velocity of galaxies and the traveled distance of their photons under the assumption of the space expansion being homologous. The time dependence of the space expansion velocity at early era implied that the currently used relativistic Doppler equation, invalid for accelerating/deaccelerating reference frames, would lead to inaccurate measurement of the cosmological recession velocity for highly redshifted galaxies/quasars.

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1. TWO OVERLOOKED ASSUMPTIONS FOR DEDUCING THE HUBBLE LAW FROM HUBBLE DIAGRAM

The Hubble equation (1) is experimentally derived from the Hubble diagram of galactic redshift (z[1]) and the photon-traveled distance (r'), which currently covers galaxies apparently billions light-years away or more precisely, galactic photon-emissions occurred billions years ago (see, for instance, Parker[1], Peebles[2]; Hetherington[3], Hartmann[4], Weinberg[5]).

$$v = H_0 r' \quad (H_0\text{: the Hubble constant at present epoch}) \qquad (1)$$
$$v = Hr \qquad (2)$$

To deduce from (1) the Hubble law (2), which correlates the radial recession velocity (v, converted from z by the Doppler effect) of these photon-emitters to their distance from the earth (r) *at the emission time*, one has to make the following two assumptions related to the time variable of space-time: (A) The positional/coordination displacement in space of the earth (or Milky Way) and these galaxies, since their early era photon-emission (that we are detecting at present), is negligible, so that the *ancient* earth-galaxy distance could be approximated by the distance these photons have traveled ($r \approx r'$); (B) The Hubble constant is essentially time-independent ($H \approx H_0$), so that the {v, r} data for galaxies at *different* epochs (from $\geq 10^9$ years in the past to present) could be fitted to a common, linear v – r correlation ($v \approx H_0 r$).

In contrast to the other two well-probed assumptions associated with (1) (*i. e*. the correlation of z to v by the Doppler effect and the application of the photon-emission mechanisms of local/present Cepheid variables, supernovae, and galaxies to remote/ancient photon-emitters), assumptions A and B have attracted little recognition or attention in the past seven decades. Obviously, assumption A contradicts the belief that new space has been continuously created between any two coordinates in our expanding universe, and assumption B contradicts the belief that *H*, being related to the scale

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[1] Abbreviations: H, Hubble constant; v, space expansion rate between the earth (Milky Way) and a galaxy, or apparent radial recession velocity of the galaxy as viewed from the earth; r, distance between the earth and a galaxy; r', apparent distance traveled by galactic photons; t, time since the Big Bang; z, redshift; Ω, ratio of the density of universe to the critical density; Λ, Cosmological constant, *c*, light speed in vacuum. Subscript 0, present epoch; subscript 1, a past epoch.

factor (R), is in general a function of time [1-5]. Then how the time variable, or the time dependence of H and r', would affect the deduction of (2) from (1) or the Hubble diagram?

2. DIFFERENTIATION OF THE DISTANCE BETWEEN TWO GALAXIES AT A GIVEN TIME POINT FROM THE DISTANCE TRAVELED BY PHOTON BETWEEN THE TWO GALAXIES DURING A TIME INTERVAL

Let's consider the detection of photons emitted from a source S at the past epoch t_1 by an observer O at present epoch t_0 (Fig. 1). Before the photons reach O, there could be four potential scenarios for the movement (world-line) of S and O in space-time: (i) both S and O could be stationary with a constant distance r_1, (ii) O could be stationary while S could have moved (either toward to or away from O), (iii) S could be stationary while O could have moved (either toward to or away from S), (iv) both S and O could have moved (either toward to or away from each other). Let's focus on the scenario (iv) with moving-away S and O (Fig. 1), because it would be the only one consistent with our current cosmological model of a homologously expanding universe. How could one measure r_1, to which v_1 (the apparent recession velocity of S as observed on O) should be related, by the distance r_1' that the S photons would have to travel to reach O for their detection?

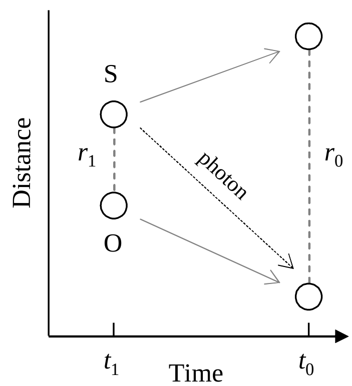

Fig. 1: Detection of photons emitted from S at t_1 (past) by O at t_0 (present) in a homologously expanding universe. S - O distance: r_1 at t_1, r_0 at t_0. Traveled distance by S photons: $r_1' = r_1 + (r_0 - r_1)/2$. Apparent S – O distance as measured from O at t_0: r_1'.

Driven by its homologous expansion, space stretches itself to create separation among galaxies. Immediately after leaving S, more space would have been created between the photons (as well as S) and O, making O to apparently move away from its position at t_1. Assuming that S and O are separated with a rate of v (equivalent to the apparent recession velocity of S as observed from O), which in general could be the function of both t and r, then the distant that the photons would have traveled within time interval t_0–t_1 to reach O is:

$$r_1' = r_1 + (r_0 - r_1)/2 = r_1 + (1/2)\int_{t_1}^{t_0} \mathrm{v}\,\mathrm{dt} = c(t_0 - t_1), \text{ (}c\text{: light speed)} \tag{3}$$

One can see that approximating r_1 with r_1' (measured/calibrated by luminosity, pulse period, brightness, line-broadening, flux, etc.) for the Hubble diagram could have the following two important consequences: First, r_1 could be severely overestimated for photon-emitters of very early era (large $t_0 - t_1$ interval) or with very high v. Secondly, photons emitted at different epochs (t variable) from sources at different distances (r variable) or under different expansion rate (v variable) could have the same r_1'. Both consequences could lead to a Hubble diagram in which more than one galaxies have the same v but different r', or the same r' but different v (*"n-to-1" correspondence*).

3. EVOLUTION OF INTERGALACTIC DISTANCE UNDER HOMOLOGOUS EXPANSION

Let's assume the space between S and O has been expanding homologously so that (2) stands for any given time [2]. Because v = dr/dt, (2) gives dr/r = Hdt, then:

$$\int_{r_1}^{r_0} \frac{\mathrm{dr}}{\mathrm{r}} = \int_{t_1}^{t_0} \mathrm{Hdt}\text{ , or } r_0 = r_1 e^{\chi},\ \ \chi = \int_{t_1}^{t_0} \mathrm{Hdt} \tag{4}$$

Thus the t-dependence of H would determine the positional displacement of O and S from t_1 to t_0. Substituting (4) and $v_1 = H_1 r_1$ at t_1 into (3), we find:

$$r' = r_1(1 + e^{\chi})/2 = (v_1/H_1)(1 + e^{\chi})/2 \tag{5}$$

4. DEDUCTION OF THE H FUNCTION FROM THE HUBBLE EQUATION

Using (1), (5) can be rewritten as $v_1/H_0 = (v_1/H_1)(1 + e^\chi)/2$, leading to:

$$\chi = \int_{t_1}^{t_0} \mathrm{H}dt = \ln(2\,H_1/H_0 - 1) \qquad (6)$$

To satisfy (6), H has to be the function of t as the following:

$$\mathrm{H} = (H_0/2)/\{1 - \exp[-(H_0/2)t]\} \qquad (7)$$

From (1), (2), (4), and (7), we find:

$$r = r_0H_0/(2\mathrm{H} - H_0) = r_0\{\exp(H_0t/2) - 1\} \qquad (8)$$

$$v = v_0\mathrm{H}/(2\mathrm{H} - H_0) = (v_0/2)\exp(H_0t/2) \qquad (9)$$

As for the scale factor R, because $\mathrm{H} = (dR/dt)/R$, then $dR/R = \mathrm{H}dt$, or $\int d(\ln R) = \int \mathrm{H}dt$. From (7) we have:

$$R = R_0/(2\mathrm{H}/H_0 - 1) = R_0\{\exp(H_0t/2) - 1\} \qquad (10)$$

5. APPARENT AGE OF THE UNIVERSE

When $t = t_0$, $\mathrm{H} = H_0$. Then (7) yields $t_0 = 2\ln 2/H_0 \approx 1.4/H_0$, which corresponds to the current age of the universe. This age is older than either $1/H_0$ or $(2/3)/H_0$ estimated by the current Big Band model assuming that Λ(cosmological constant) = 0 and Ω (ratio of the density of universe to the critical density) = 0 or 1, respectively [1-4]. Based on the current $60 \le H_0 \le 80$ km/s/mpc estimation [6], (7) would yield a kinematic age between 17 and 23 billion years for our universe.

6. EVOLUTIONS OF H, R, V, AND R IN RECENT PAST

Rewriting $\exp(-H_0t/2)$ as $\exp\{(H_0/2)(t_0 - t)\}/2$, then when $\Delta t = (t_0 - t) \ll 2/H_0$, $\exp(\pm H_0/2) \approx 1$, (7) to (10) yield $\mathrm{H}/H_0 \approx 1$, $r/r_0 \approx 1$, $v/v_0 \approx 1$, and $R/R_0 \approx 1$. If $(H_0/2)\Delta t \le 0.1$ is considered a good approximation ($|\exp(\pm H_0\Delta t/2) - 1|/\exp(\pm H_0\Delta t/2) \le 11\%$), then based on the current $H_0 \approx 70$ km/s/mpc value, this would cover the past 3 billion years from present. During this period, the universe seems to be "static" in terms of these parameters. Thus for those galaxies whose photons are no more than 10^9 light-years old, the observed r_1' would approximate r_0 ($\approx r_1$) well, so that (1) could be regarded as $v_0 = H_0r_0$, which is the Hubble law (2) of present epoch.

Since v ≈ v_0, the apparent radial recession velocity of a given galaxy (at a given r from the earth) could be regarded as constant during this time interval (≤ 3 billion years). Thus the special theory of relativity, which deals with inertial or constant velocity reference frames, can be applied to calculate the time-dilation experienced by O, leading to the Doppler equation (11) of deducing the v for high z galaxies/quasars [1, 3, 4]:

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{11}$$

7. EVOLUTIONS OF H, R, V, AND R AT EARLY ERA

When t « $2/H_0$, $\exp(\pm H_0 t/2) \approx 1 \pm H_0 t/2$, then (7) to (10) yield H ≈ 1/t, $r/r_0 \approx H_0 t/2$, $v/v_0 \approx 2$, and R ≈ $H_0 t/2$. If $H_0 t/2 \leq 0.1$ is considered a good approximation ($|\exp(\pm H_0 t/2) - 1|/\exp(\pm H_0 t/2) \leq 11\%$), then based on a $H_0 \approx 70$ km/s/mpc, such early era would cover the first 3 billion years since the Big Bang.

Between the first and recently past 3 billion years, the t dependence of v (9) would be significant. When v becomes comparable to c, O in Fig. 1 would experience a time dilation to find a "more recent" emission time t_1', or a shorter travel (from t_1' to t_0), for the S-photons. To take this relativistic effect into account, however, (11) should be replaced by another formula suited for noninertial or accelerating reference frames.

8. OVERALL REMARKS

The Hubble law is regarded as one of the three pillars of the current, prevailing cosmology doctrine, the Big Bang theory. Strictly speaking, what the *observed* Hubble equation (1) claims is the proportionality of the galactic radial recession velocity to the distance/time the galactic photons travel to reach Earth, rather than to the *actual* galactic distance at the emission time (as traditionally interpreted [1-4]). The distance covered by photons we detect today contains the positional displacement of Earth (or Milky Way) caused by the space expansion during the travel of the photons. As discussed above for (3), approximating the actual galactic distance (at the emission) with the distance traveled by photons can result in an "n-to-1 correspondence" in the high z region of the Hubble diagram. Such effect might contribute to the significant scattering of the z and apparent magnitude (related to distance) observed for high z quasars [1, 2].

In a homologously expanding universe, a simple H function (7) is deduced from the observed (1). It is not clear at present how (7) could be theoretically established or reconciled to the well-known H function derived from the general theory of relativity, which contains two undetermined parameters, Ω and Λ [1-3, 5, 7].

The relativistic effect becomes significant with high v. The analysis of (7) and (9) validates the use of (11) for galactic photons emitted ≤ 3 billion years ago (or $z = v/c = H\Delta t \leq 3H_0 \approx 0.2$), when the relativistic effect involves inertial or constant velocity reference frames. For older emissions, such as those used to probe the large-scale structure of universe [1, 2], the relativistic effect should be considered in a different way, since the significant t-dependence of v requires noninertial or accelerating reference frames. Thus, the v estimation of quasars of high z (≥ 0.2) by (11) might be erroneous and contradict other cosmological measurements. For instance, quasars of z = 5 have been reported as of 13 billion-year old [8]. According to the Big Bang model, the dimension of the universe of 13 billion years ago should be a few (~ 3 to 4) billion light-years [3], indicating that a photon would need only a few (~ 3 to 4) billion light-years to across the universe then. Thus the photons of these quasars, if their ages are right, should have reached the earth a few billion years ago!

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