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Cosmological Implications of the Tetron Model of Elementary Particles

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Abstract

Based on a possible solution to the tetron spin problem, a modification of the standard Big Bang scenario is suggested, where the advent of a spacetime manifold is connected to the appearance of tetronic bound states. The metric tensor is constructed from tetron constituents and the reason for cosmic inflation is elucidated. Furthermore, there are natural dark matter candidates in the tetron model. The ratio of ordinary to dark matter in the universe is calculated to be 1:5.

1 Introduction

Particle physics phenomena can be described, for example, by the left-right symmetric Standard Model with gauge group $U(1)_{B-L} \times SU(3)_c \times SU(2)_L \times SU(2)_R$ [1] and 24 left-handed and 24 right-handed fermion fields which including antiparticles amounts to 96 degrees of freedom, i.e. this model has right handed neutrinos as well as righthanded weak interactions.

In recent papers [2, 3, 4] a new ordering scheme for the observed spectrum of quarks and leptons was presented, which relies on the structure of the group of permutations S_4 of four objects called tetrons, and a mechanism was proposed, how 'germs' of the Standard Model interactions might be buried in the representations A_1, A_2, E, T_1 and T_2 of this group. Furthermore, it was shown how to construct the Standard Model gauge fields with the help of tetrons.

In the present paper I will argue that this model is not just a strange observation in the realm of particle physics, but has a more fundamental meaning, so that also gravitational and astrophysical effects can also be understood on the tetron basis.

In modern cosmology there are 3 outstanding phenomena not completely understood: the underlying reason for inflation, the ratio of dark to ordinary matter and the appearance of dark energy:

i) Cosmic inflation [5] is the widely accepted hypothesis that the nascent universe passed through a phase of exponential expansion that was driven by a vacuum energy density of negative pressure. It resolves several problems in the Big Bang cosmology that were pointed out in the 1970s, like the horizon problem, the flatness problem and the magnetic monopole problem.

ii) Dark matter is defined to interact with ordinary matter essentially only via gravity. Gravitational effects in the rotation of galaxies as well as other observations (see e.g. [6]) suggest the existence of dark matter with an amount 4 or 5 times larger than ordinary matter which appears in stars, dust and gas.

iii) The present universe is appearantly undergoing a phase of accelerated expansion (see e.g. [7]). This can be explained either by a modification of the Einstein Lagrangian, the so called F(R) gravities, see [8] and references therein, or by the presence of dark energy, see e.g. [9], either in the form of a positive cosmological constant or of a scalar field, sometimes called 'quintessence' [10], that drives the accelaration and acts not unlike the 'inflaton' which is often introduced to drive inflation.

In the present paper I want to analyze these phenomena in the light of the tetron model. Tetron interactions will be assumed to describe the deepest level of matter, just above the Planck scale. I will show how

i) the tetron model may affect the inflationary scenario via the enormous energies set free when after the era of a tetron plasma tetron bound states are formed.

ii) some tetron bound states naturally contribute to the dark matter of the universe.

iii) tetron interactions may be related to the formation of spacetime and the appearance of gravitational forces and of dark energy (in the form of a quintessence field).

The outline of the article is as follows: in section 2 the main ingredients of the tetron model are reviewed. Sections 3, 4 and 6 contain improved arguments as to how the spin-$\frac{1}{2}$ properties of quarks and leptons can be obtained in this model. In section 5 the dark matter candidates of the tetron model are discussed. In section 7 a view on gravitational interactions and dark energy is taken from the standpoint of the tetron model. In section 8 I will discuss how shortly after the big bang a tetron plasma appears from which in a process of supercooling the ordinary quarks, leptons and gauge bosons arise. Finally, in the appendix I present an alternative description of the tetron idea by introducing both an inner symmetry lattice and a spatial lattice. This possibility is related to the fact that the permutation group S_4 is isomorphic to the symmetry group of a tetrahedral lattice. Although phenomenologically this approach leads to the same results as before, the microscopic interpretation is different because tetron bound states are now interpreted as lattice excitations of a yet unknown dynamics. Sections 5, 7 and 8 and the appendix contain completely new material which have not appeared elsewhere.

2 Short Review of the Tetron Idea

The starting point of refs. [2, 3, 4] was the observation that there is a natural one-to-one correspondence between the quarks and leptons and the elements of the permutation group S_4, as made explicit in table 1 and natural in the sense that the color, isospin and family structure correspond to the K, Z_2 and Z_3 subgroups of S_4, where Z_n is the cyclic group of n elements and K is the so-called Kleinsche Vierergruppe which consists of the 3 even permutations $\overline{2143}$, $\overline{3412}$, $\overline{4321}$, where 2 pairs of numbers are interchanged, plus the identity. Note that permutations $\sigma \in S_4$ will be denoted $\overline{abcd}, a, b, c, d \in \{1, 2, 3, 4\}$.

S_4 is a semi-direct product $S_4 = K \diamond Z_3 \diamond Z_2$ where the Z_3 factor is the family symmetry and Z_2 and K can be considered to be the 'germs' of weak isospin and color symmetry (cf. [3]). At low energies this product cannot be distinguished from the direct product $K \times Z_3 \times Z_2$ but has the advantage of being a simple group and having a rich geometric and group theoretical interpretation as the rotational symmetry group of a regular tetrahedron and, up to a parity factor, the symmetry group of a 3-dimensional cubic lattice. Furthermore it does not only describe quarks and leptons (table 1) but also leads to a new ordering scheme for the Standard Model (plus some GUT-like) vector bosons, cf. table 2 and ref. [2]. In fact, 12 GUT-like heavy vector bosons can be constructed in the tetron model, which behave similar, though not identical, as the ones appearing in the standard SU(5) model.

Actually, the assignments in table 1 are only heuristic. Instead one has to take linear combinations of symmetry adapted wave functions, dictated by the 5 representations A_1, A_2, E, T_1 and T_2 of S_4 [3]. The content of table 1 may then be interpreted as the sum of representations $A_1{+}A_2{+}2E{+}3T_1{+}3T_2$.

Ordering the particle spectra according to representations of the permutation group S_4 one is naturally lead to the idea of a constituent picture where fermions (table 1) and gauge bosons (table 2) are built from 4 tetrons t^a with 'flavors' $a \in 1, 2, 3, 4$ and with the condition that in a bound state all 4 flavors must be different. The origin of this selection rule has been widely discussed in refs. [3, 2].

The most appealing solution is to allow only discrete values for the inner symmetry variable, i.e. t^1, t^2, t^3 and t^4 are assumed to be fixed vectors in the inner symmetry space which point to the corners of an inner tetrahedron, and then to assume that the interaction Hamiltonian is proportional to the volume of this tetrahedron. In that case non-permutation states like $t^1t^1t^1t^2$

	...1234... family 1	...1423... family 2	...1243... family 3
	τ, $b_{1,2,3}$	μ, $s_{1,2,3}$	e, $d_{1,2,3}$
ν	$\overline{1234}(id)$	$\overline{2314}$	$\overline{3124}$
u_1	$\overline{2143}(k_1)$	$\overline{3241}$	$\overline{1342}$
u_2	$\overline{3412}(k_2)$	$\overline{1423}$	$\overline{2431}$
u_3	$\overline{4321}(k_3)$	$\overline{4132}$	$\overline{4213}$
	ν_τ, $t_{1,2,3}$	ν_μ, $c_{1,2,3}$	ν_e, $u_{1,2,3}$
l	$\overline{3214}(1 \leftrightarrow 3)$	$\overline{1324}(2 \leftrightarrow 3)$	$\overline{2134}(1 \leftrightarrow 2)$
d_1	$\overline{2341}$	$\overline{3142}$	$\overline{1243}(3 \leftrightarrow 4)$
d_2	$\overline{1432}(2 \leftrightarrow 4)$	$\overline{2413}$	$\overline{3421}$
d_3	$\overline{4123}$	$\overline{4231}(1 \leftrightarrow 4)$	$\overline{4312}$

Table 1: List of elements of S_4 ordered in 3 fermion families. k_i denote the elements of K and $(a \leftrightarrow b)$ a simple permutation where a and b are interchanged. Permutations with a 4 at the last position form a S_3 subgroup of S_4 and may be thought of giving the set of lepton states. It should be noted that this is only a heuristic assignment. Actually one has to consider linear combinations of permutation states as discussed in section 2.

$B_\mu = \overline{1234}(id)$	$G_{3\mu} = \overline{2314}$	$G_{8\mu} = \overline{3124}$
$W_{3\mu} = \overline{2143}(k_1)$	$G_{1\mu} = \overline{3241}$	$G_{2\mu} = \overline{1342}$
$W_{1\mu} = \overline{3412}(k_2)$	$G_{4\mu} = \overline{1423}$	$G_{5\mu} = \overline{2431}$
$W_{2\mu} = \overline{4321}(k_3)$	$G_{6\mu} = \overline{4132}$	$G_{7\mu} = \overline{4213}$
$X_{1\mu} = \overline{3214}(1 \leftrightarrow 3)$	$X_{4\mu} = \overline{1324}(2 \leftrightarrow 3)$	$X_{5\mu} = \overline{2134}(1 \leftrightarrow 2)$
$Y_{1\mu} = \overline{2341}$	$Y_{2\mu} = \overline{3142}$	$Y_{3\mu} = \overline{1243}(3 \leftrightarrow 4)$
$X_{2\mu} = \overline{1432}(2 \leftrightarrow 4)$	$Y_{4\mu} = \overline{2413}$	$Y_{5\mu} = \overline{3421}$
$X_{3\mu} = \overline{4123}$	$X_{6\mu} = \overline{4231}(1 \leftrightarrow 4)$	$Y_{6\mu} = \overline{4312}$

Table 2: List of elements of S_4 ordered as 1+3+8+6+6=24 vector bosons, half of which are the Standard Model vector bosons, while the rest can be identified with GUT-like X- and Y-bosons. In ref. [2] they where shown to lead to the correct standard gauge interaction terms. The decomposition follows the class structure of the group S_4, which consists of 5 classes usually called I, C_2, C_3, C_4 and C_2' with 1, 3, 8, 6 and 6 elements, respectively. In principle one has 2 separate S_4 tables, i.e. 2 separate S_4 multiplets, one for 'left' and one for 'right' vector bosons V_L and V_R which can formally be united in one large table by using the octahedral group $O_h \cong S_4 \times P_i$, where P_i is an inner parity operation defined to transform $V_L \leftrightarrow V_R$. Note that the question of (spatial) parity violation and vector like interactions has been discussed in ref. [3].

etc are automatically suppressed and one ends up with 24 possible bound states transforming under representations of S_4. It may be noted that models with a discrete inner symmetry space have been extensively studied in the framework of lattice physics [18, 19]. I will come to a lattice interpretation of this point in the appendix.

Another important question is how the spin-$\frac{1}{2}$ behavior of quarks and leptons arise from the spin of the 4 constituents. This is the so called 'tetron spin problem' and will be discussed next.

3 A possible Solution to the Tetron Spin Problem

One could have an easy living if one would assume quarks and leptons to be composed e.g. of four scalar tetrons and a neutral nucleus with spin-$\frac{1}{2}$. In the present paper, a different approach will be followed.

For simplicity, only spatial transformations will be considered. Extension to Minkowski space, i.e. going from rotational SO(3) to SO(3,1), essentially amounts to introduce antitetrons.

Let me start with a few well-known facts about half-integer spin: in a physical experiment one cannot distinguish between states which differ by a complex phase. Therefore, in addition to ordinary representations one may include projective, half-integer spin representations of the rotation group SO(3), and also of its $T_d \cong S_4$ subgroup[1]. These are true representations of the corre-

[1] T_d is the rotation symmetry group of a regular tetrahedron. It is a subgroup of $O(3)$ and isomorphic to S_4. T_d is also isomorphic to the octahedral group O, i.e. the group of proper rotations of a cube which is a subgroup of SO(3).

sponding covering groups SU(2) and $\tilde{S}_4$, respectively.

To solve the tetron spin problem I suggest to give up the requirement of continuous rotation symmetry and assume that tetrons live and interact in microscopical environments, in which only permutation symmetry survives. The latter is much less restrictive than rotational SO(3), because the idea of rotation assumes concepts of angle and length, which may be obstacled by quantum fluctuations when approaching the Planck scale. In contrast, the idea of permutation merely presupposes the more fundamental principle of identity. This is why permutation groups may enter theoretical physics at finer levels of resolution and higher energies than the Lorentz group. Tetrons may be more basic than spinors.

I call this assumption the 'spatial permutation hypothesis'. It amounts to introducing a second permutation index called i, j, k or l and taking values 1, 2, 3 and 4 (in addition to the tetron 'flavor' index a, b, c and d) and being responsible for the spatial ('spin') transformation behavior of tetrons and its compound states.

It is true that the phenomenological observation of 24 quarks and leptons and their interactions imply a permutation principle only on the level of inner symmetries. However, the assumption of 4 different tetron 'spins' within a fermion bound state comes closest to the original intuition of a spatial tetrahedral structure as discussed in ref. [3] where a generic ansatz for the composite wave function $t_i^a t_j^b t_k^c t_l^d$ with $a, b, c, d, i, j, k, l \in \{1, 2, 3, 4\}$ has been proposed.

As a consequence of the spatial permutation hypothesis a new type of particle statistics will arise (called *tetron statistics*) which differs from Fermi and Bose statistics and will play a role in the interpretation of the Big Bang and cosmic inflation presented below.

4 The Details

According to the spatial permutation hypothesis, the spin part of a 4-particle fermionic compound state should transform according to a (projective) representation of S_4. Besides the ordinary representations A_1, A_2, E, T_1 and T_2 there are 3 irreducible projective representations (representations of the covering group $\tilde{S}_4$), namely G_1, G_2 and H of dimensions 2, 2 and 4, respectively [14]. The sum 4+4+16 of the dimensions squared accounts for the 24 additional elements due to the Z_2 covering of S_4. Among them, G_1 uniquely corresponds to spin-$\frac{1}{2}$, i.e. is obtained as the restriction of the fundamental SU(2) representation to $\tilde{S}_4$. Similarly, H can be obtained from the spin-$\frac{3}{2}$ representation of SU(2), whereas G_2 is obtained from G_1 by reversing the sign for odd permutations. The combination $G_2 + H$ corresponds to a restriction of the spin-$\frac{5}{2}$ representation of SU(2) to $\tilde{S}_4$.

For the understanding of the following arguments a short digression on quaternions and its usefulness for describing nonrelativistic spin-$\frac{1}{2}$ fermions will be helpful:

Quaternions [15, 16, 17] are a non-commutative extension of the complex numbers and play a special role in mathematics, because they form one of only three finite-dimensional division algebra containing the real numbers as a subalgebra. (The other two are the complex numbers and the octonions.) As a vector space they are generated by 4 basis elementes 1, I, J and K which fulfill $I^2 = J^2 = K^2 = IJK = -1$, where K can be obtained as a product $K = IJ$ from I and J. Quaternions are non-commutative in the sense IJ=-JI. Any quaternion q has an expansion of the form

$$\begin{aligned} q &= c_1 + Jc_2 \\ &= r_1 + Ir_2 + Jr_3 + Kr_4 \end{aligned} \tag{1}$$

with real r_i and complex $c_1 = r_1 + Ir_2$ and $c_2 = r_3 - Ir_4$.

There is a one-to-one corresponence between unit quaternions q_0 and SU(2) transformation matrices, because the latter are necessarily of the form $(\alpha, \beta; -\beta^*, \alpha^*)$ with complex α and β fulfilling $|\alpha|^2 + |\beta|^2 = 1$, and can be written as $q_0 = \alpha + J\beta$. Therefore, the action of

SU(2) matrices on spinor fields (c_1, c_2) (c_1 with spin up and c_2 with spin down) can in quaternion notation be rewritten as:

$$c_1 + Jc_2 \to (\alpha + J\beta)(c_1 + Jc_2) \tag{2}$$

For example the unit quaternions I and J corresponding to rotations by π about the x and y-axis amount to $c_1 \to Ic_1, c_2 \to -Ic_2$ and $c_1 \to -c_2, c_2 \to c_1$, respectively. For a general SU(2) transformation one has $c_1 \to \alpha c_1 - \beta^* c_2$ and $c_2 \to \alpha^* c_2 + \beta c_1$, from which e.g. the antisymmetric tensor product combination $c_1 c_2' - c_2 c_1'$ can be shown to be rotationally invariant (spin 0).

To describe spin-$\frac{1}{2}$ bound states one should use the symmetry function of the representation G_1. This function will also be called G_1 in the following and can be given as linear combination of the G_1 representation matrices (=unit quaternions):

$$\begin{aligned}
G_1 \; = \; & g(1,2,3,4) + Ug(2,3,1,4) + U^2 g(3,1,2,4) \\
+ \; & Ig(2,1,4,3) + Sg(3,2,4,1) + R^2 g(1,3,4,2) \\
+ \; & Jg(3,4,1,2) + Rg(1,4,2,3) + T^2 g(2,4,3,1) \\
+ \; & Kg(4,3,2,1) + Tg(4,1,3,2) + S^2 g(4,2,1,3) \\
+ \; & \frac{I+K}{\sqrt{2}} g(3,2,1,4) + \frac{I-J}{\sqrt{2}} g(1,3,2,4) + \frac{J+K}{\sqrt{2}} g(2,1,3,4) \\
+ \; & \frac{1-J}{\sqrt{2}} g(2,3,4,1) + \frac{1-K}{\sqrt{2}} g(3,1,4,2) + \frac{J-K}{\sqrt{2}} g(1,2,4,3) \\
+ \; & \frac{I-K}{\sqrt{2}} g(1,4,3,2) + \frac{1+K}{\sqrt{2}} g(2,4,1,3) + \frac{1+I}{\sqrt{2}} g(3,4,2,1) \\
+ \; & \frac{1+J}{\sqrt{2}} g(4,1,2,3) + \frac{I+J}{\sqrt{2}} g(4,2,3,1) + \frac{1-I}{\sqrt{2}} g(4,3,1,2)
\end{aligned} \tag{3}$$

where $R = \frac{1}{2}(1 - I - J - K), S = \frac{1}{2}(1 - I + J + K), T = \frac{1}{2}(1 + I - J + K)$ and $U = \frac{1}{2}(1 + I + J - K)$. One can see explicitly from this equation, which S_4 permutation $\overline{ijkl}$ is represented in G_1 by which quaternion, because the corresponding quaternion appears as a coefficient of $g(i, j, k, l)$. For example, the permutation $\overline{2341}$ is represented by $\pm(1-J)/\sqrt{2}$, and so on. In other words, the quaternion coefficients $1, I, J, K, (I+K)/\sqrt{2}, ..., R, S, T, ...$ in this equations represent the elements of $\tilde{S}_4$[2].

Due to the 2-fold covering of S_4 each of the real functions $g(i,j,k,l)$ in eq. (3) with its 24 terms is in fact a difference

$$g(i,j,k,l) = p(i,j,k,l) - m(i,j,k,l) \tag{4}$$

so as to obtain the 48 terms needed for a symmetry function of $\tilde{S}_4$.

Eq. (3) should be considered as the spin factor of the 4-tetron bound state, whereas the A_1, A_2, E, T_1 and T_2-functions of the ordinary S_4 representations account for the flavor factor. In fact, working out the quaternion multiplications in eq. (3) and using $K = IJ$ one obtains a representation of the form $G_1 = c_1 + Jc_2$ with c_1 and c_2 decribing the 2 spin directions of the compound fermions, cf. eq. (2). Mathematically, the appearance of 2 complex functions c_1 and c_2 in eq. (3) is merely expression of the fact that for the 2-dimensional representation G_1 4 real(=2 complex) symmetry functions can be constructed, which in eq. (3) are combined in one quaternion function.

Eq. (3) therefore describes a decent fermion state which transforms in the standard way, cf. eq. (2). On the other hand, eq. (3) also inherits the spatial

[2] While $\tilde{S}_4$ itself can be shown to make up the inner shell of D_4-lattices [20], the first half of coefficients in eq. (3) represent even permutations corresponding to $\tilde{A}_4$ which is sometimes called the 'binary tetrahedral group', and generates the F_4 lattice also called the ring of Hurwitz integers (=quaternions with half integer coefficients). The Hurwitz quaternions form a maximal order (in the sense of ring theory) in the division algebra of quaternions with rational components. This accounts for its importance. For example restricting to integer lattice points, which seems a more obvious candidate for the idea of an integral quaternion, one does not get a maximal order and is therefore less suited for developing a theory of left ideals as in algebraic number theory. What Hurwitz realized, was that his definition of integral quaternions is the better one to operate with.

permutation hypothesis (i.e. giving up full SU(2) rotational invariance on the tetron level) in that the function G_1 naturally reacts like a (projective) S_4 representation under permutations of $i, j, k, l \in \{1, 2, 3, 4\}$.

The picture followed here is a sort of molecular approach where one starts with a fixed spatial tetrahedral configuration with 4 distinct permutation ('spin') indices $i, j, k, l \in \{1, 2, 3, 4\}$. Its reaction under permutations (=tetrahedral T_d transformations) of i, j, k, l is dictated by the spatial permutation hypothesis, whereas the behavior (G_1) under full rotational SU(2) is obtained from the requirement that the compound state must be a fermion.

Since we have given up rotational symmetry on the tetron level, the question of how a single tetron t_i^a with index $i \in \{1, 2, 3, 4\}$ transforms into itself under rotations need not be discussed. It is merely necessary to know how compound states transform unter permutations of indices $i, j, k, l \in \{1, 2, 3, 4\}$ and this question is answered by the symmetry function G_1.

In other words: since G_1 is not contained in any tensor product of 4 S_4 representations, one can only interpret the inner symmetry part of the wavefunction as a tensor product of tetron factors, but not the functions $g(i, j, k, l)$ or p and m of eq.(4).

$$g(i, j, k, l) \neq t_i \otimes t'_j \otimes t''_k \otimes t'''_l \tag{5}$$

Nevertheless, I will sometimes use the tensor notation for the sake of illustration. For instance, the complete 'spin' and 'flavor' wave function of quarks and leptons can then plainly be denoted as

$$\begin{array}{rcl} t_1^a \otimes t_2^{b'} \otimes t_3^{c''} \otimes t_4^{d'''} & + & t_1^b \otimes t_2^{c'} \otimes t_3^{a''} \otimes t_4^{d'''} + ... \\ I t_2^a \otimes t_1^{b'} \otimes t_4^{c''} \otimes t_3^{d'''} & + & ... \\ & ... & \end{array} \tag{6}$$

Here in the rows the tetron flavor indices a, b, c, d are permutated in order to obtain the appropriate flavor combination (A_1 of S_4 as an example, for the A_2, T_1 etc flavor representations G_2 and H will come into play), whereas in the columns the tetron 'spin' indices i, j, k, l are permutated in order to obtain the G_1 spin combination.[3]

In summary, eq. (3) should be considered as the spin factor of the quark and lepton states, whereas the A_1, A_2, E, T_1 and T_2-functions of the ordinary S_4 representations account for the inner symmetry 'flavor' factor. (Those functions can be found, for example, in ref. [2].) The full quark and lepton spectrum of table 1 including spatial and inner symmetries can then be written as

$$(A_1 + A_2 + 2E + 3T_1 + 3T_2)_{in} \otimes G_{1sp} = 24G_1 \tag{7}$$

where *in* stands for the inner and *sp* for the spatial part of the wave function, and the factor of 24 on the r.h.s. accounts for the 24 degrees fo freedom of 3 fermion families.[4]

[3]Note that in general, the permutation of the tensor product indices - denoted by primes in eq. (5) - must not be messed up with the permutation of spin states. Only in the case at hand, where 4 different spin states in 4 different tensor factors are considered, there is no difference.

[4]There is a pictorial interpretation of the fermion bound state 'molecules', where the tetrahedron as a whole forms a sort of molecular cluster, and the 24 inner S_4 symmetry configurations can be thought to be realized in ordinary space. Namely, on each of the 4 corners of a tetrahedron a single tetron t_ι^α is located which is composed of a 'nucleus' $\alpha \in 1, 2, 3, 4$ surrounded by a 'cloud' $\iota \in 1, 2, 3, 4$. Inner symmetry transformations act by interchanging the clouds whereas under spatial rotations both nuclei and clouds are transformed simultaneously. In other words, the 24 flavor states $(A_1+A_2+2E+3T_1+3T_2)_{in}$ can be obtained by varying ι for fixed α, whereas unter spatial tetrahedral transformations the G_{1sp}-combination of indices should be chosen with varied ι and α simultaneously.

5 Dark Matter from Tetrons

Dark matter is a hypothetical type of matter that is undetectable by its emitted radiation, but which can be inferred only from gravitational effects. Its presence is postulated to explain the flat rotation curves of spiral galaxies and other evidence of missing mass in the universe. According to present observations, there exists between 4 and 6 times more dark matter than ordinary matter in the universe. Further it is known, that it must be composed of mostly cold, i.e. nonrelativistic, particles.

We have seen in the last section, how the spin-$\frac{1}{2}$ nature of quarks and leptons can be deconstructed using the G_1 representation of the permutation group. It is certainly true that the phenomenological observation of 24 quarks and leptons and their interactions suggests a permutation principle only on the level of *inner* symmetries. However due to the problems which arise in connection with spin and statistics we were naturally lead to consider the possibility that there is a spatial S_4-index as well and that this can be used to understand the spin-$\frac{1}{2}$ nature of quarks and leptons.

In the following I want make use of this procedure to show that there are natural dark matter candidates in the tetron model responsible for the bulk of the observed dark matter in the universe. Namely, if this approach has some meaning it is tempting that besides G_1 also the two other half-integer spin representations of $\tilde{S}_4$ (H and G_2) play a role in nature, or in other words, that together with ordinary (G_1-)matter sets of particle families with spin $\frac{3}{2}$ (H) and spin $\frac{5}{2}$ ($G_2 + H$) should have been produced during cosmogenesis. In fact, eq. (7) naturally extends to

$$(A_1 + A_2 + 2E + 3T_1 + 3T_2)_{in} \otimes (G_1 + G_2 + 2H)_{sp} = 24(G_1 + G_2 + 2H) \quad (8)$$

As before, *in* stands for the inner and *sp* for the spatial S_4 index set and the factor of 24 on the r.h.s. accounts for the 24 'flavor' degrees of freedom of 3 times 3 fermion families for G_1, H and $G_2 + H$ each with particle masses of roughly comparable size.

Next, it will be assumed that - apart from gravity forces - the new (G_2 and H) fermions decouple from ordinary (G_1) fermions, i.e. that spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ matter have interactions completely separate from those of ordinary matter.[5]

Assuming further, that initially all matter fields are produced at uniform rates, one expects a ratio of 1:5 for the relative distribution of matter (including neutrinos) and dark matter in the universe. This ratio is obtained by counting the spin degrees of freedom 2:(4+6) of spin-$\frac{1}{2}$, -$\frac{3}{2}$ and -$\frac{5}{2}$ objects or equivalently from the ratio of dimensions $dim(G_1) : dim(G_2 + 2H)$ and should be considered as one of the main results of the present paper. The fact that only 3 representations are involved has to do with the fact that S_4 is a finte group with a finite number of representations.

[5]It is an interesting question how the interactions among the dark matter (G_2 and H) fermions look like and whether they lead to atomic and molecular binding states similar to what we are used from ordinary matter or whether the spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ quarks will not be confined and exist as free particles. A natural ansatz is to extend the vector boson content of table 2 in a manner compatible with permutation symmetry. In fact one may summarize the content of table 2 as

$$(A_1^\pm + A_2^\pm + 2E^\pm + 3T_1^\pm + 3T_2^\pm)_{in} \otimes T_{1sp} \tag{9}$$

in terms of O_h representations $R^\pm$, where $+$ stands for lefthanded and $-$ for righthanded vector bosons. One may try to extend this expression to include the interactions among the H- and $G_2 + H$-fermion families:

$$(A_1^\pm + A_2^\pm + 2E^\pm + 3T_1^\pm + 3T_2^\pm)_{in} \otimes (A_1 + A_2 + 2E + 3T_1 + 3T_2)_{sp} \tag{10}$$

and has to show that the additional bosons interact only within the H- and G_2+H-fermion families, but not with ordinary (i.e. G_1-) matter.

The idea behind this consideration is, that at Big Bang energies where masses play no role, all 3 matter types (G_1, G_2 and H) are produced in equal amount corresponding to a mass energy ratio of ordinary to dark matter $dim(G_1)$: $dim(G_2 + 2H) = 1 : 5$ and that this ratio has not changed since that time because apart from gravity there are no interactions between the 3 matter types. In other words, all decays and transitions take place only within one of the matter types and do not disturb the ratio 1:5. The same holds true for radiation: when an electron-positron pair annihilates, a photon of type G_1 appears, and this can only annihilate into a fermion-antifermion pair of type G_1. The reason for this lies in the manner in which the photon - and also the gluon and the W-boson - are constructed in the tetron model as scattering states of G_1-fermions only [2].

6 A new Statistics

Eq. (3) reflects the statistical behavior of a 4-tetron conglomerate under permutations of its components. This behavior has a certain similarity to that of fermions but is certainly not identical. While conglomerates of fermions usually transform with the totally antisymmetric representation (like A_2), tetrons go with G_1, which gives a factor of I under the exchange $(1 \leftrightarrow 2, 3 \leftrightarrow 4)$ or $\frac{1}{\sqrt{2}}(J+K)$ under $(1 \leftrightarrow 2)$, whereas a 2-fermion conglomerate in a $A_2 = c_1c_2' - c_2c_1'$ configuration responds with -1 (i.e. antisymmetric) to the exchange of $(1 \leftrightarrow 2)$. See table 3, where the behavior of tetrons and fermions is compared. The fact that tetrons behave more complicated under transpositions $(i \leftrightarrow j)$, has to do with the fact that transpositions in S_4 correspond to relatively complicated space transformations in T_d.

We therefore conclude that tetrons follow their own statistics which is nei-

FERMIONS	TETRONS
compound states:	
boson from 2 fermions:	fermion from 4 tetrons:
complex tensor product	quasi-complex, quaternion tensor product
$A_2 = c_1 c_2' - c_2 c_1'$	$G_1 = g(1,2,3,4) + Ig(2,1,4,3) + J...$
	$= t_1^a t_2^{b'} t_3^{c''} t_4^{d'''} + It_2^a t_1^{b'} t_4^{c''} t_3^{d'''} + ...$
bosonic behavior under rotations	fermionic behavior under rotations
	$G_1 \to (\alpha + J\beta)G_1$
permutation behavior/statistics:	
-1 under $(1 \leftrightarrow 2)$	a factor I under $(1 \leftrightarrow 2, 3 \leftrightarrow 4)$
	a factor $\frac{1}{\sqrt{2}}(J+K)$ under $(1 \leftrightarrow 2)$ etc

Table 3: Comparison between the known fermion behavior and the anticipated tetron behavior.

ther bosonic nor fermionic, and assert, that a sort of 'tetron spin statistics theorem' holds, which allows only bound states in which all tetron flavors are different (cf. the selection rule / exclusion principle mentioned at the end of section 2).

7 Gravitons, Quintessence and the Interaction among Tetrons

In this section I follow the idea that the gravitational field can be described in terms of tetron constituents. This could be either in the form of a van-der-Waals remnant of the interactions among tetrons or, in more concrete terms, of a composite gravitational field, described in terms of tetron interactions.

What is the possible form of the interaction among tetrons? On an effective Lagrangian level it involves 4-tetron product terms like $t_i^a t_j^b t_k^c t_l^d$. It would be desirable to interpret this as an effective interaction which can be traced back to an interaction of 2 tetrons of the form $t_i^a t_j^b B_{kl}^{cd}$, with $i, j, k, l, a, b, c, d \in \{1, 2, 3, 4\}$ and B_{kl}^{cd} being some interaction 'field'. Note that as before, no specific spatial transformation properties can be assigned to a single index i or j. However, in the combination $ijkl$ they will transform under an S_4 representation.

Since gravity is flavor independent, in order to construct it from B-fields, these must not depend on the flavor indices a, b. Therefore the 2-tetron interaction simplifies to

$$L_{ttB} = t_i^a t_j^b B_{kl} \tag{11}$$

In pictorial language the B-field occupies the 6 edges of a tetrahedron.

In concrete terms the graviton will be assumed to be a bound state of two B-fields. Furthermore it should meet the general selection rule / exclusion principle formulated in ref. [2] that every physical field must be a permutation field. Then - in the same way as fermion states were written down with the help of the representation G_1 eq. (3) - the gravitational field can be expanded with the help of spin-2 representation matrices $R_{\mu\nu}(ijkl)$ of S_4 given by the representation $E + T_2$ of S_4:

$$g_{\mu\nu} = R_{\mu\nu}(1234) B_{12} B_{34} + R_{\mu\nu}(2143) B_{21} B_{43} + ... \tag{12}$$

In the following explicit construction the spin-2 representation will be formally calculated from a product of 2 vector representations

$$T_1 \otimes T_1 = A_1 + T_1 + E + T_2 \tag{13}$$

of the spatial S_4-symmetry indices, where A_1, T_1 and $E + T_2$ represent the spin-0, spin-1 and spin-2 contributions to the product, respectively. Furthermore, the temporal gauge $g_{0\mu} = 0$ will be used which, at least in the weak field approoximation, is known to be compatibel with the harmonic gauge often used in relativistic calculations [26].

The metric tensor then takes the form

$$g_{\mu\nu} = \begin{pmatrix} -t_{XX} - t_{YY} - t_{ZZ} & 0 & 0 & 0 \\ 0 & t_{XX} & t_{XY} & t_{XZ} \\ 0 & t_{YX} & t_{YY} & t_{YZ} \\ 0 & t_{ZX} & t_{ZY} & t_{ZZ} \end{pmatrix} \tag{14}$$

Here we have allowed for a nonvanishing g_{00} contribution due to the singlet A_1 which may represent the quintessence scalar ϕ_q [10] appearing in solutions to the dark energy problem and a possible antisymmetric component of $g_{\mu\nu}$ stemming from the spin-1 contribution T_1 on the r.h.s. of eq. (13). The antisymmetric components may play a role in the so-called scalar-vector-tensor model [22] and in gravity with torsion [23]. Making use of the appropriate Clebsch-Gordon coefficients [21] the relation of $g_{\mu\nu}$ eq. (14) to the known S_4 representation matrices [2] is given by

$$A_1 = t_{XX} + t_{YY} + t_{ZZ} \tag{15}$$

$$E_{11} = (t_{XX} - t_{YY})/2 \tag{16}$$

$$E_{12} = (t_{XX} + t_{YY} - 2t_{ZZ})/\sqrt{6} \tag{17}$$

$$T_{2,11} = (t_{XY} + t_{YX})/2 \tag{18}$$

$$T_{1,11} = (t_{XY} - t_{YX})/2 \tag{19}$$

etc. Putting everything together, one obtains for example for the (A_1) quintessence field $\phi_q = \sum_{ijkl} B_{ij} B_{kl}$ where the sum runs over all permutations $\overline{ijkl} \in S_4$. Similarly for the spatial components of $g_{\mu\nu}$ from the other representations E, T_1 and T_2.

It should be noted that, instead of using 2-B-field bound states, in the lattice interpretation given in the appendix one may be more general and assume that the graviton and its companions are excitations within the permutation lattice of the general form

$$1_{in} \otimes (A_1 + A_2 + 2E + 3T_1 + 3T_2)_{sp} \tag{20}$$

Since - in contrast to eq. (8) - there is no inner symmetry index, only one A_1, one A_2, one T_1 and one $E + T_2$ field emerge on the ground state level. This corresponds to a scalar field ϕ_q, an axial scalar ϕ_a, a spin-1 vector U_μ and a spin-2 tensor field. In the massless limit the transversal modes of the spin-1 and spin-2 excitations will vanish and a graviton and a vector field each with 2 helicities appear.

Having constructed the compound states one can try to write down their effective interactions. The requirement of local Lorentz invariance more or less fixes the Lagrangian to be [10, 22, 24, 26]

$$L = \frac{1}{2}\sqrt{-g}M_P^2 R + L(\phi_q) + L(\phi_a) + L(U_\mu) + L_{WW} \tag{21}$$

where R is the Ricci scalar associated with the graviton, g is the determinant of the (symmetric) metric tensor and $M_P = 1/\sqrt{8\pi G}$ the reduced Planck mass.

$$L(\phi_q) = \frac{1}{2}\partial_\mu \phi_q \partial^\mu \phi_q - V(\phi_q) \tag{22}$$

denotes the quintessence part of the Lagrangian [10, 24, 26]. Similarly for $L(\phi_a)$ and $L(U_\mu)$, whereas L_{WW} denotes interactions among the various fields[22].

Exploring the phenomenology of eqs. (21) and (22) requires a form for the potential $V(\phi_q)$. In order to account for the dark energy component of the total cosmic mass energy, this is usually chosen in such a way that the field stress-energy tensor approximates the effect of a cosmological constant[10, 24, 25].

8 A Tetron Plasma in the very early Universe

According to the cosmological Standard Model the universe began in a state, in which spacetime and physical laws have no real meaning. This so called Planck era lasted about 5,4 10^{-44} s. Only after that spacetime and matter came into being in the process of the Big Bang, and the laws of physics came into action.

As discussed in section 4 the question how a single tetron behaves under spacetime transformations is not well put, because single tetrons cannot be isolated spatially. Therefore I want to develop a picture that in the Planck era the universe consisted of a countable set of a large number of tetrons and B-fields - just a set, with no spatial properties, but possibly with interactions governed by permutation symmetry - and that the physical history of the universe as a spacetime manifold began only, when the tetrons formed S_4 bound states, which transform under representations of the rotation or Lorentz group.

I will call the state before the advent of bound states a tetron plasma - although one may object that 'plasma' is perhaps not the right word for a set of tetrons without a metric space, so that for example particle velocities, energies and probably even temperature cannot be defined. As a countable, practically infinite set it has a S_∞ permutation symmetry, which in the process of bound state formation gets broken to S_4. About the nature of the symmetry breaking $S_\infty \to S_4$ one can only speculate. It may have to do with Bott periodicity which honours spatial dimensions of 3 and 7, because in these dimensions division algebra structures can be imposed on the corresponding vector spaces (cf. the discussion at the end of the appendix).

The appearance of S_4-symmetry and of a 3-dimensional space are actually correlated. Namely, according to eqs. (12) and (3) 4-tetron and 2-B-field aggregates constitute points in space by defining a transition from a set on which only permutation operations act ($ijkl... \rightarrow$ permutations of $ijkl...$) to a continuus space where rotational symmetry transformations $R_{\mu\nu}(ijkl)$ (eq. (12)) and I, J, K etc (eq. (3)) are defined. In that sense it may be said that tetron interactions constitute 3-dimensional space.

The transition of the universe from a tetron plasma to the later radiation and matter phases could be related to cosmic inflation, because a tetron plasma governed by tetron statistics during the Planck era could account for the pressure required in the inflationary scenario, by means of the enormous bindung energies set free when quarks, leptons and radiation states are formed and space is blown up from a discrete set(=tetron plasma), where distances are not defined to a curved semidiscrete manifold, where the extension of bound states is roughly given by the Planck scale.

Unfortunately, in its present stage the tetron model does not provide a suitable dynamical scheme and therefore does not have enough quantitative predictive power to compete with current Lagrangian approaches [13, 11, 12] to inflation. In the Lagrangian models the effects of inflation are described by an (effective) Lagrangian containing a scalar inflaton field with a definite dynamics, which is able to quantitatively explain the mechanism which drives the rapid expansion in the inflation period (for a comprehensive review see e.g. ref. [13]). This field may or may not be one of the Higgs fields appearing in standard particle physics models and with minimal [11] or non-minimal [12] coupling to gravity. Prior to the expansion period, the inflaton is at a higher energy state. A suitable potential or random quantum fluctuations then generate a repulsive force and trigger a phase transition whereby the inflaton field releases its potential energy as matter and radiation as it settles to its lowest energy state.

The Lagrangian approach to inflation can be interpreted in different ways. One way (preferred by the present author) is to argue that these models (of which there are hundreds) provide a convenient method of parametrizing the early universe but that, because they are fundamentally semi-classical, are unlikely to be a true description of the physics underlying the very early universe. The other (probably more common) approach is to argue that the inflaton is the true source of inflation and that its identity may be found by considering one of the extensions of the standard model based on grand unified theories, supergravity or string theory, from which then definite quantitative predictions can be obtained.

In comparison, the tetron model arguments in favor of inflation are only qualitative in nature. Nevertheless, they may lead to a microscopic understanding of an effective inflaton interaction, once a model for the dynamical behavior of tetrons and of a tetron plasma is developed.

9 Summary

In summary, the tetron model modifies the standard Big Bang scenario in various respects. Prior to the epoche of radiation, quark-gluon plasma etc governed by GUT or Standard Model interactions there may a tetron plasma governed by tetron statistics, where distances, angles and a metric do not exist but arise only when tetronic bound states are formed. The formation of these bound states sets free an an enormous amount of binding energy and introduces the pressure needed for inflation in the early universe.

Furthermore, it was shown how the tetron model yields the physical particles, i.e. fermions of the form $\sim t^4$, radiation $\sim (\bar{t}t)^4$ and gravitational interactions $\sim B^2$ as well as more speculative fields of spin $\frac{3}{2}$ and spin $\frac{5}{2}$ which may serve as dark matter candidates.

There are several objections which may be raised against the tetron model. One is that at its current state it relies mainly on group theoretical arguments and not much can be said about the dynamical behavior of tetrons. What seems to be certain, however, is that due to the discrete-like features of the model the ultraviolet treatment of the tetron theory will be quite different from the renormalization one usually encounters at small distances in quantum field theories.

Further, one could suspect that the tetron model contradicts the Weinberg-Witten theorem [28] which states that no massless (composite or elementary) particles with spin greater than one are consistent with any renormalizable Lorentz invariant quantum field theory (excluding only nonrenormalizable theories of gravity and supergravity). However, in the case at hand the theorem does not apply, because we have abandoned Lorentz symmetry from the start replacing it by the spatial permutation hypothesis (cf. section 3) or by a spatial 'permutation' lattice (cf. the appendix).

10 Appendix: An alternative Approach using Lattices

The tetrahedral or permutation group S_4 is not only the symmetry group of a regular tetrahedron, but also of a tetrahedral lattice or of a fluctuating

S_4-permutation lattice.

It is therefore tempting to assume, that the inner symmetry space of tetrons is not continuous (with a continuous symmetry group) but has instead the discrete structure of such a tetrahedral lattice. The observed quarks and leptons can then be interpreted as excitations on this lattice and characterized by representations of the lattice symmetry group S_4, i.e. by $A_1 + A_2 + 2E + 3T_1 + 3T_2$, just as in the 'classical' tetron model presented in the main text. In this picture the original dynamics is governed by some unknown lattice interaction instead of by four real tetron constituents.

The lattice ansatz also naturally explains the selection rule mentioned in section 2 (that all physical states must be permutation states), just because only representations of the permutation group S_4 are allowed.

In the following I will make the additional assumption that not only the inner symmetry is discrete but that physical space is a lattice, too. The reason for this is that although theories with a discrete inner symmetry over a continuous base manifold have been examined [29] they seem to me rather artificial because they will usually lead to domain walls and other discontinuities.

More precisely, in the spirit of the spatial permutation hypthesis (section 3) instead of a fixed spatial lattice the existence of a spatial permutation lattice with symmetry group S_4 will be assumed where the lattice points are not a priori fixed but may be fluctuating due to quantum effects. The lattice spacing would be typically of the order of the Planck scale with the extension of the bound states slightly larger.

One could ask, why the (inner) lattice structure is seen in the flavour spectrum part of eq. (7) whereas the spatial part G_{1sp} to a human observer appears as spin-$\frac{1}{2}$ representations of the *continuous* rotation group. The point is that with respect to the spatial lattice present physical experiments always work at distances much larger than the lattice spacing ($\cong M_P$) whereas for the inner symmetry lattice we do *not* encounter the continuum limit, so that the representations $A_{1,2}$, E and $T_{1,2}$ remain relevant for the particle spectrum.

A drawback of the lattice picture as compared to the tetron constituent model, is that it is still less specific and there is a larger amount of arbitrariness concerning the origin of the observed spectrum $(A_1 + A_2 + 2E + 3T_1 + 3T_2)_{in}$ for quarks and leptons. These are interpreted as lattice excitations, and one may, for example, assume the existence of 'elementary' excitations g_{1in}, t_{1in} and h_{in} on the inner symmetry lattice (transforming with respect to the representations G_1, T_1 and H, respectively) from which the quark and lepton spectrum is built according to

$$g_{1in} \otimes t_{1in} \otimes h_{in} = (A_1 + A_2 + 2E + 3T_1 + 3T_2)_{in} \tag{23}$$

However, apart from the fact that there are other possibilities of tensor products that yield the same result, the physical meaning of the 'elementary' excitations is rather unclear.

There is a similar situation in the spatial sector of the model, where one would like to obtain the spectrum $(G_1 + G_2 + 2H)_{sp}$ from some elementary tensor product. Again, it turns out that there are several different combinations of elementary excitations leading to the required result.

One may speculate whether a unification of the spatial and inner symmetry sector could remedy the arbitrariness. What I have in mind is a compactification scenario where one starts with a n-dimensional lattice (or n+1 in a relativistic scenario to include a time variable), n-3 dimensions of which being compactified. The most natural choice seems to be n=7 because it allows spinorial structures which is inherited to the n=3 base manifold in the process of compactification. Due to lack of time I have not yet analyzed this promising possibility in detail.

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